Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (OLD) EXAMINATION – WINTER 2023

Subject Code:130002 Date:02-02-2024

Subject Name:Advanced Engineering Mathematics

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1	**(a)**	Find the Fourier series of $f(x) = \|x\|$ in the interval $-\pi < x < \pi$	**07**
	(b)	Solve $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = x^2 \ln x$	**07**
Q.2	**(a)**	(i) Solve $\frac{dy}{dx} - y = e^{2x}$	**03**
		(ii) Solve $(D^2 + a^2)y = \operatorname{cosec} ax$ by the method of variation of parameter	**04**
	(b)	Solve $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \sec x$	**07**
		OR	
	(b)	Solve by using Undetermined Coefficient method $(D^2 + 4)y = 8x^2$	**07**
Q.3	**(a)**	Find power series solution of $y'' + y = 0$	**07**
	(b)	(i) Define 1. Gamma Function 2. Beta Function 3. Signum Function	**03**
		(ii) Show that $\int_0^\infty \frac{\cos \omega x}{\omega^2+1} = \frac{\pi}{2}e^{-x} \quad if \quad x > 0$	**04**
		OR	
Q.3	**(a)**	(i) Find the Laplace transform of $e^{-3t}(2cos5t - 3sin5t)$.	**03**
		(ii) Find the Inverse Laplace transform of $\log\left(\frac{s+a}{s-a}\right)$.	**04**
	(b)	Find power series solution of $y'' + x^2y = 0$	**07**
Q.4	**(a)**	Using the convolution method to find $L^{-1}\left(\frac{1}{s(s^2+4)}\right)$.	**07**
	(b)	Solve $y'' + 4y = 0$, $y(0) = 1$ and $y'(0) = 6$	**.07**
		OR	
Q.4	**(a)**	(i) Find the Laplace transform of $\frac{e^{-t}\sin t}{t}$	**03**
		(2) Solve $y''' - 6y'' + 11y' - 6y = 0$	**04**
	(b)	Obtain the Fourier series of $f(x) = \frac{1}{2}(\pi - x)$ in the interval $0 \le x \le 2\pi$.	**07**
Q.5	**(a)**	Form a partial differential equation by eliminating the arbitrary functions from $f(x^2 + y^2, z - xy) = 0$	**07**
	(b)	Solve $u_x = 4u_y$ subject to the condition $u(0, y) = 8e^{-3y}$ by method of separation of variables	**07**
		OR	
Q.5	**(a)**	Form a partial differential equation by eliminating the arbitrary constants from $z = (x - a)^2 + (y - b)^2$	**07**
	(b)	Solve $(D^2 + 6DD' + 9D'^2)z = 6x + 2y$	**07**

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