Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III(OLD) EXAMINATION – WINTER 2022

Subject Code:130002 Date:16-02-2023

Subject Name:Advanced Engineering Mathematics

Time:02:30 PM TO 05:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 (a) (i)Solve $\frac{dy}{dx}+y\tan x=\sin 2x$ **03**

(ii) Find the solution of differential equation $ye^{x}dx+(2y+e^{x})dy=0,$ where $y(0)=-1$ **04**

(b) Find the series solution of differential equation $\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$ **07**

Q.2 (a) (i) Solve $x^2ydx=(x^3+y^3)dy$ **03**

(ii) Find the solution by variation of parameter **04**

$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=e^{x}$

(b) Solve $\frac{d^2y}{dx^2}-y=x\sin x+e^{x}+x^2e^{x}$ **07**

OR

(b) (i) Find the Laplace transform of $te^{t}\sin t$ **03**

(ii) Find the inverse Laplace transform of $\frac{s^3}{s^4-81}$ **04**

Q.3 (a) (i) Define 1 Error function, 2 Gamma function, 3 Heaviside's unit step function. **03**

(ii) Obtain the half range sine series for $f(x)=2x,$ in $0\prec x\prec 1$ **04**

(b) Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos\left(\log(1+x)\right)$ **07**

OR

Q.3 (a) (i) Solve $\frac{dy}{dx}+\tan x\tan y=\cos x\sec y$ **03**

(ii) Find the solution using method of undetermined coefficients. **04**

$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=3e^{-2x}$

(b) Find the series solution by Frobinus method near $x=0$ **07**

$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$

Q.4 **(a)** (i) Solve $p^2 - q^2 = x - y$ **03**

(ii) solve $p(1+q) = qz$ **04**

(b) Using Laplace transformation solve the initial value problem **07**

$y'' + 2y' + 5y = e^{-t}\sin t, \quad y(0) = 0, y'(0) = 1$

OR

Q.4 **(a)** **(i) Find the Laplace transformation of** $t^2 e^{3t} \sin 4t$ **03**

(ii)State convolution theorem and using it find the inverse Laplace transformation of $\frac{1}{s\left(s^2+4\right)}$ **04**

(b) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ subject to the condition $u(x,0) = 4e^{-4x}$ **07**

Q.5 **(a)** (i) State second shifting theorem and using it find the inverse Laplace transformation of $\frac{e^{-3s}}{s^2+8s+25}$. **03**

(ii) Find the Inverse L aplace transformation of $\tan^{-1}\left(\frac{2}{s}\right)$ **04**

(b) Obtain the Fourier series of $f(x) = x - x^2$, in the interval $-\pi \prec x \prec \pi$ **07**

OR

Q.5 **(a)** (i) Form a partial differential equation by eliminating the arbitrary function. **03**

$z = xy + f(x^2 + y^2)$

(ii) Find the Laplace transformation of $\frac{1-\cos t}{t}$ **04**

(b) Find the Fourier integral representation of the function **07**

$$f(x) = \begin{cases} 1, & |x| \prec 1 \\ 0, & |x| \succ 1 \end{cases}$$ and evaluate $\int_0^\infty \frac{\sin\omega}{\omega} d\omega$

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