

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-III (OLD) EXAMINATION – WINTER 2021****Subject Code:130002****Date:15-02-2022****Subject Name: Advanced Engineering Mathematics****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

		MARKS
Q.1	(a) (i) Solve $(x + 1) \frac{dy}{dx} - y = e^{3x}(x + 1)^2$	03
	(ii) Find the half range sine series of $f(x) = e^x$ in $0 < x < \pi$	04
	(b) Find the series solution of $(1 + x^2)y'' + xy' - xy = 0$	07
Q.2	(a) (i) Solve $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$	03
	(ii) Solve $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \operatorname{cosec}x$ by the method of variation of parameter	04
	(b) Solve $\frac{d^3y}{dx^3} + 8y = \operatorname{cosh}2x$	07
OR		
(b)	(i) Find the Laplace transform of $\sinh^5 t$.	03
	(ii) Find the Inverse Laplace transform of $\frac{3s+7}{s^2-2s-3}$	04
Q.3	(a) (i) Define 1. Gamma Function 2. Beta Function 3. Signum Function	03
	(ii) Find the Fourier cosine series for $f(x) = x^2, 0 < x < c$.	04
	(b) Solve $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \log x \cdot \sin(\log x)$	07
OR		
Q.3	(a) (i) Solve $(x + 1) \frac{dy}{dx} - y = e^{3x}(x + 1)^2$	03
	(ii) Solve by using Undetermined Coefficient method $(D^2 - 2D + 3)y = x^3 + \sin x$	04
	(b) Find the series solution of $2x(x - 1)y'' - (x + 1)y' + y = 0; x_0 = 0$	07
Q.4	(a) Find the Inverse Laplace transform of $s \log\left(\frac{s^2+a^2}{s^2+b^2}\right)$	07
	(b) Solve $2 \frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + u$ subject to the condition $u(x, 0) = 4e^{-3x}$ by method of separation of variables.	07
OR		
Q.4	(a) Solve the initial value problem using Laplace Transform $y'' + 3y' + 2y = e^t, \quad y(0) = 1, y'(0) = 0.$	07

- (b) (i) Solve $p^2 + q^2 = npq$ 03
(ii) Solve $pz - qz = z^2 + (x + y)^2$ 04

Q.5 (a) State the convolution theorem and verify it for $f(t) = t$ and $g(t) = e^{2t}$. 07

(b) Obtain the Fourier series of $f(x) = \left(\frac{\pi-x}{2}\right)^2$ in the interval 07
 $0 \leq x \leq 2\pi$. Hence Deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2}$

OR

Q.5 (a) (i) Form a partial differential equation by eliminating the arbitrary 03
functions from $xyz = \phi(x + y + z)$

(ii) Find the Laplace Transform of $f(t) = \sin at$ 04

(b) Find the fourier integral representation of the function 07

$$f(x) = \begin{cases} 2; & |x| < 2 \\ 0; & |x| > 2 \end{cases}$$
