Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (OLD) EXAMINATION – SUMMER 2021

Subject Code:130002 Date:03/09/2021

Subject Name:Advanced Engineering Mathematics

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1	**(a)**	(i) Solve $\frac{dy}{dx} = \sin(x+y)$.	**03**
		(ii) Solve $(1+x^2)\frac{dy}{dx} + y = e^{tan^{-1}x}$.	**04**
	(b)	Solve the differential equation $\frac{d^2y}{dx^2} + x^2y = 0$ by Power method at $x = 0$.	**07**
Q.2	**(a)**	Solve the partial differential equation $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ subject to condition $u(0,y) = 3e^{-y} - 5e^{-5y}$ by method of Separation Variables.	**07**
	(b)	Solve the differential equation by Frobenius method $x(x-1)y'' + (3x-1)y' + y = 0.$	**07**
		OR	
	(b)	(i) Solve: $(D^2 - 5D + 6)y = x + e^{4x}$.	**03**
		(ii) Solve : $(D^2 + a^2)y = cosec(ax)$ by variation parameter method	**04**
Q.3	**(a)**	Find the Fourier series for the function $f(x) = x + x^2,\ -\pi < x < \pi$.	**07**
	(b)	Obtain Fourier series for the function $f(x) = x+1, \quad -1 < x < 0$ $= x-1, \quad 0 < x < 1.$	**07**
		OR	
Q.3	**(a)**	Express $f(x) = e^x,\ 0 < x < l$ as a half range Fourier Cosine series with period $2l$.	**07**
	(b)	Find the Fourier series of $f(x) = \sqrt{1-cosx}$ in the interval [0, 2π]. Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{4n^2-1} = \frac{1}{2}$.	**07**
Q.4	**(a)**	(i) State the change of scale property of Laplace Transform. If $L\{f(t)\} = \frac{s}{s^2-k^2}$. find $L\{f(3t).\}$	**03**
		(ii) Find the Laplace Transform of $\frac{e^{-2t}\sin(2t)\cosh(t)}{t}$.	**04**
	(b)	Find the Inverse Laplace Transform of (i) $\frac{3s+7}{s^2-2s-3}$ (ii) $ln\left(1+\frac{1}{s^2}\right)$	**07**
		OR	
Q.4	**(a)**	Solve the initial value problem $y'' + 2y' + y = e^{-t},\ y(0) = -1,\ y'(0) = 1$ by using Laplace transform method.	**07**
	(b)	(i) Find Laplace Transform of $tsin^2 3t$.	**03**
		(ii) State convolution theorem. Use it to find Inverse Laplace Transform of $\frac{1}{(s+1)(s^2+1)}$	**04**

Q.5 **(a)** (i) Form the partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$. **03**

(ii) Solve $\left(D^2 - 2DD' + D'^2\right)z = e^{x+2y}$. **04**

(b) (i) Solve $p^2 + q^2 = x + y$. **03**

(ii) Solve $(mz - ny)p + (nx - lz)q = ly - mx$. **04**

OR

Q.5 **(a)** (i) Solve $z = px + qy + n\sqrt{1 + p^2 + q^2}$. **03**

(ii) Solve $(D^2 + 3D + 2)y = e^{2x}sinx$. **04**

(b) (i) Define Heaviside's function **02**

(ii) Express the function $f(x) = sinx, \quad 0 \le x \le \pi$
$= 0, \quad x > \pi$ **05**

as a Fourier sine Integral.

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