GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER– III (NEW) EXAMINATION – SUMMER 2022

Subject Code:2130002   **Date:08-07-2022**

Subject Name:Advance Engineering Mathematics

Time:02:30 PM TO 05:30 PM   **Total Marks:70**

Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

                                                                            MARKS

Q.1 **(a)** Solve the differential equation $ye^x\, dx + \left(2y + e^x\right)dy = 0$. **03**

      (b) Find the Laplace Transform of $(t+1)^2\, e^t$ **04**

      (c) Find the Fourier series expansion of the periodic function $f(x) = x - x^2$ in the interval $-\pi \le x \le \pi$. **07**

Q.2 **(a)** Find $L^{-1}\left\{\dfrac{3s+4}{s^2+9}\right\}$ **03**

      (b) Solve $y'' - 2y' + y = 10e^x$ **04**

      (c) Find the Fourier series of the periodic function with a period 2 of **07**
$$f(x) = \begin{cases} \pi, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$$

OR

      (c) Find the power series solution of the equation **07**
$\left(x^2+1\right)y'' + xy' - xy = 0$ about $x = 0$.

Q.3 **(a)** Define Bata function, Gamma function and write the relation between Beta and Gamma function. **03**

      (b) Use convolution theorem to find the inverse Laplace Transform of $\dfrac{1}{(s+1)(s^2+1)}$. **04**

      (c) Solve the differential equation using method of variation of parameters: $y'' + 9y = \tan 3x$. **07**

OR

Q.3 **(a)** Define (1) Rectangle function; (2) Saw tooth wave function . **03**

      (b) Find the half range sine series of $f(x) = x^2$ in the interval $(0, \pi)$. **04**

      (c) Solve the initial value problem using Laplace Transform **07**
$y'' + y' = t^2 + 2t$, $y(0) = 4,\ y'(0) = -2$

Q.4 **(a)** Find the Laplace Transform of $\int\limits_0^t e^{-2t}\, t^3\, dt$. 03

(b) Solve the differential equation $\dfrac{dy}{dx} + \dfrac{2y}{x} = y^2 x^2$. 04

(c) Find the Fourier Integral representation of the function 07
$$f(x) = \begin{cases} 1 - x^2\,, & |x| \le 1 \\ 0\,, & |x| > 1 \end{cases}$$

OR

Q.4 **(a)** Find the inverse Laplace Transform of $\dfrac{e^{-\pi s}}{s^2 - 2s + 2}$. 03

(b) Solve $(mz - ny)\,p + (nx - lz)\,q = ly - mx$. 04

(c) Solve $x^2 y'' + 5xy' + 3y = \dfrac{\log x}{x^2}$ 07

Q.5 **(a)** Form a differential equation for the equation $z = (x-2)^2 + (y-3)^2$. 03

(b) Find the Laplace Transform of $f(t) = \begin{cases} t^2\,, & 0 < t < 1 \\ 4t\,, & t > 1 \end{cases}$ 04

(c) Solve the equation $u_x = 2u_t + u$ given $u(x,0) = 4e^{-4x}$, by the method of separation of variables. 07

OR

Q.5 **(a)** Solve $p^2 + q^2 = x + y$ 03

(b) Solve $\dfrac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0$, $z = e^y$ and $\dfrac{\partial z}{\partial x} = 1$. 04

(c) Solve $\left(D^2 + DD' - 6D'^2\right) z = \sin(2x + y)$. 07

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