Enrollment No./Seat No.:

GUJARAT TECHNOLOGICAL UNIVERSITY

Bachelor of Engineering - SEMESTER - III EXAMINATION - WINTER 2025

Subject Code: BE03000211 **Date: 22-12-2025**

Subject Name: Mathematics 3 (Mechanical)

Time: 10:30 AM TO 01:00 PM **Total Marks: 70**

Instructions

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Marks

Q.1 (a) (1) State Dirichlet's conditions for convergence of Fourier series. **03**
(2) Shw that $1+\triangle=E$.
(3) What is the order of error in the trapezoidal rule ?

(b) (1) Show that $u=e^{x}cosy$ is a solution of Laplace equation. **04**
(2) Form PDE by eliminating arbitrary constants a and b from $z=(x+a)(y+b)$.

(c) (1) Solve $\frac{dy}{dx}=3+2xy$ where $y(0)=1$ for $x=0.1$ by Picard's method. **07**
(2) Find the Laplace transform of f(t) defined as $f(t)=\begin{cases} t, & \text{if } 0<t<4 \\ 5, & \text{if } t>4. \end{cases}$

Q.2 (a) Solve: $(y+z)p+(z+x)q=x+y$. **03**

(b) Solve: $\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x \partial y}+2\frac{\partial^2 z}{\partial y^2}=x+y.$ **04**

(c) Find the Fourier series of $f(x)=x^2$ when $-\pi \le x \le \pi$ and $f(x+2\pi)=f(x)$. **07**

OR

(c) Find the Fourier series of $f(x)=\begin{cases} 8 & \text{if } 0<x<2, \\ -8 & \text{if } 2<x<4, \end{cases}$ and $f(x+4)=f(x)$. **07**

Q.3 (a) Using Newton's backward interpolation formula, find $y(300)$ using following observations. **03**

x	50	100	150	200	250
y	618	724	805	906	1032

(b) Find the Lagrange's interpolation polynomial from the following data. **04**

x	0	1	4	5
f(x)	1	3	24	39

(c) Solve by the method of separation of variables $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$ given that $u=3e^{-y}-e^{-5y}$ when $x=0$. **07**

OR

(a) Solve: $p^2+q^2=x^2+y^2$. **03**

(b) Solve: $2\frac{\partial^2 z}{\partial x^2}-5\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=5sin(2x+y)$ **04**

(c) Compute $f(9.2)$ from the following values using Newton's divided difference formula. **07**

x	8	9	9.5	11
f(x)	2.079442	2.197225	2.251292	2.397895

Q.4 (a) Given that **03**

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=1.1$ using method of numerical differentiation.

(b) Find $\int_{10}^{16} ydx$ by Simpson's $\frac{1}{3}-$ rule for the following values. **04**

x	10	11	12	13	14	15	16
y	1.02	0.94	0.89	0.79	0.71	0.62	0.55

(c) A rod of length L with insulated side is initially at uniform temperature 100 C. Its ends are suddenly cooled at 0 C and kept at that temperature. Find the temperature $u(x,t)$. **07**

OR

(a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using trapezoidal rule with $h=0.2$. **03**

(b) Evaluate $\int_0^3 \frac{dx}{1+x}$ with $n=6$ using Simpson's $\frac{3}{8}-$ rule. **04**

(c) Given $\frac{dy}{dx}=\frac{x^2+y^2}{10}, y(0)=1$, using fourth order Runge-Kutta method, find $y(0.2)$ with $h=0.1$. **07**

Q.5 (a) Using Euler's method, find $y(0.2)$ for given $\frac{dy}{dx}=y-\frac{2x}{y}, y(0)=1$ in two steps. **03**

(b) Find: (1) $L[2t^3+e^{-2t}+t^{\frac{4}{3}}]$ (2) $L[e^{-3t}sin^2t]$ **04**

(c) Find: $L^{-1}\left[\frac{s^3}{s^4-81}\right]$ **07**

OR

(a) Apply Runge-Kutta method of second order to find an approximate value of *y* at *x* = *0.01* given that $\frac{dy}{dx}=x^2+y, y(0)=1$ with *h* = *0.01*. **03**

(b) Find: (1) $L[tcos(4t+3)]$ (2) $L\left[\frac{e^{-at}-e^{-bt}}{t}\right]$ **04**

(c) Using Laplace transforms solve the initial value problem **07**

$$\frac{d^2y}{dt^2}+y=sin2t, y(0)=2, \left(\frac{dy}{dt}\right)_{t=0}=1$$

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