Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–III EXAMINATION – WINTER 2025

Subject Code:3130005 **Date:31-12-2025**

Subject Name: Complex Variables and Partial Differential Equations

Time:10:30 AM TO 01:00 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Show that $\left|\frac{z}{|z|} - 1\right| \le |\arg(z)|$	**03**
	(b)	Prove that u $= x^2 - y^2$ and v $= \frac{y}{x^2+y^2}$ are harmonic functions of (x, y) but are not harmonic conjugates	**04**
	(c)	Find the bilinear transformation which maps the points $z = 1, i, -1$ into $w = i, 0, -i$ respectively. Find invariant points and image of $|z| < 1$ under this mapping.	**07**
Q.2	**(a)**	Evaluate the integral $\int_C \bar{z}dz$; where C is right half of the circle $z = 2e^{i\theta}(-\frac{\pi}{2} < \theta < \frac{\pi}{2})$	**03**
	(b)	Evaluate $\int_0^{3+i} z^2 dz$ along the parabola $x = 3y^2$	**04**
	(c)	Explain inversion transformation. Find the image of ¼ ≤ y ≤ ½ under mapping w = 1/z. Show this translation on graph	**07**
		OR	
	(c)	State Cauchy integral theorem and Cauchy integral formula. Evaluate $\oint_c \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)(z-2)}dz$, where $c : |z| = 3$	**07**
Q.3	**(a)**	Determine the poles of the function and the residue at each pole $f(z) = \frac{z^2}{(z-2)^2(z-1)}$	**03**
	(b)	Find the radii of convergence and region of convergence of the following: (i) $\sum_{n=1}^{\infty} \frac{z^n}{2^n+1}$ (ii) $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$	**04**
	(c)	Find the Laurent series of $\frac{1}{z(z^2 - 3z + 2)}$ for region $(i)\, 0 < |z| < 1$ $(ii)\ 1 < |z| < 2$ $(iii)\ |z| > 2$	**07**

OR

Q.3	**(a)**	Define: Singular point, Isolated singular point, Residue. Explain types of isolated singular points.	**03**
	(b)	Expand $f(z) = \frac{(z-1)}{(z+1)}$ as Taylor series about the following points a) z = 0 b) z = 1	**04**
	(c)	Evaluate following real integration using residue theorem. $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+1)} dx$	**07**
Q.4	**(a)**	Eliminate the arbitrary function from the equation $z = xy + f(x^2 + y^2)$	**03**
	(b)	Solve $y^2p - xyq = x(z - 2y)$	**04**
	(c)	Solve following non-linear partial differential equations using Charpit's method $px + qy = pq$	**07**

OR

Q.4	**(a)**	Solve: $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that $z = e^y$ and $\frac{\partial z}{\partial x} = 1$ when x = 0	**03**
	(b)	Solve following non-linear partial differential equations. a) $p(1+q) = qz$ b) $p^2 - q^2 = x - y$	**04**
	(c)	Solve the following Partial differential Equation by Langrage's Method $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$	**07**
Q.5	**(a)**	Find the Particular integral of $4r + 12s + 9t = e^{(3x-2y)}$	**03**
	(b)	Using method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ given that $u(x,0) = 6e^{-3x}$.	**04**
	(c)	Show that u = sin9t sin (1/4)x is a solution of a one dimensional wave equation	**07**

OR

Q.5	**(a)**	Classify following second order homogeneous partial differential equations as elliptic, parabolic or hyperbolic a) $\frac{\partial^2 u}{\partial t^2} + 4\frac{\partial^2 u}{\partial x \partial t} + 4\frac{\partial^2 u}{\partial x^2} = 0$ b) $2\frac{\partial^2 u}{\partial t^2} + 4\frac{\partial^2 u}{\partial x \partial t} + 3\frac{\partial^2 u}{\partial x^2} = 0$	**03**
	(b)	Solve $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y$	**04**
	(c)	Derive the solutions of one dimensional wave equation .	**07**

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