Enrollment No./Seat No.:

GUJARAT TECHNOLOGICAL UNIVERSITY

Bachelor of Engineering - SEMESTER - III EXAMINATION - WINTER 2025

Subject Code: BE03000251 **Date: 22-12-2025**

Subject Name: Probability and Statistics

Time: 10:30 AM TO 01:00 PM **Total Marks: 70**

Instructions

1. Attempt all questions.

2. Make suitable assumptions wherever necessary.

3. Figures to the right indicate full marks.

4. Simple and non-programmable scientific calculators are allowed.

			Marks
Q.1	**(a)**	A committee of 5 is to be selected from a group of 6 men and 9 women. If the selection is made randomly, what is the probability that the committee consists of 3 men and 2 women?	**03**
	(b)	Define with example: (1) Discrete random variable (2) Continuous random variable.	**04**
	(c)	A bin contains 3 types of disposable flashlights. The probability that a type 1 flashlight will give more than 100 hours of use is 0.7, with the corresponding probability for type 2 and type 3 flashlights being 0.4 and 0.3 respectively. Suppose that 20% of the flashlights in the bin are type 1, 30% are type 2, and 50% are type 3. (a) What is the probability that a randomly chosen flashlight will give more than 100 hours of use? (b) Given that a flashlight lasted more than 100 hours, what is the conditional probability that it was a type 2 flashlight?	**07**
Q.2	**(a)**	Suppose that a random variable X has an exponential distribution with mean equal to 10. Determine $(a) P(X > 10) \quad (b) P(X < 30)$	**03**
	(b)	Suppose that the number of accidents occurring on a highway each day is a Poisson random variable with mean equal to 3. Find the probability that 3 or more accidents occur today.	**04**
	(c)	The phone lines to an airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on successive calls are independent. Assume that 10 calls are placed to the airline. (a) What is the probability that for exactly 3 calls the lines are occupied? (b) What is the probability that for at least one call the lines are not occupied? (c) What is the expected number of calls in which the lines are occupied?	**07**
		OR	
	(c)	It is known that 5% of the books bound at a certain bindery have defective bindings. Find the probability that 2 of 100 books bound by this bindery have defective bindings using (a) the formula for the binomial distribution (b) the Poisson approximation to the binomial distribution.	**07**

Q.3 **(a)** Calculate median for the following frequency distribution. **03**

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	8	7	12	28	20	10	10

(b) Calculate the rank correlation coefficient between marks assigned to 9 students by judges X and Y in a certain competitive test as shown in table below. **04**

Marks by judge X	53	42	60	45	41	37	38	25	27
Marks by judge Y	68	43	38	77	48	35	30	25	50

(c) Find the coefficient of skewness for the following data. **07**

Profit(Rs. Lakhs)	70-80	80-90	90-100	100-110	110-120	120-130	130-140	140-150
Number of customers	12	18	35	42	50	45	30	8

OR

(a) Define (1) moments about mean (2) moments about origin and write the relationship between them. **03**

(b) Assume that the current measurements in a strip of wire follow a normal distribution with a mean of 10 milliamperes and a standard deviation of 2 milliamperes. What is the probability that a current measurement is between 9 and 11 milliamperes? Use $P(0 < Z < 0.5) = 0.1915$ **04**

(c) Find Karl Pearson's coefficient of correlation for the following data. **07**

x	15	16	17	18	19	20
y	80	75	60	40	30	15

Q.4 **(a)** In 64 randomly selected hours of production, the mean and the standard deviation of the number of acceptable pieces produced by an automatic stamping machine are 1038 and 146 respectively. At the 0.05 level of significance, does this enable us to reject the null hypothesis $\mu = 1000$ against the alternative hypothesis $\mu > 1000$? Use critical value $z_{0.05} = 1.645$. **03**

(b) It is desired to determine whether there is less variability in the silver plating done by Company 1 than in that done by Company 2. If independent random samples of size 12 of the two companies' work yield $s_1 = 0.035$ mil and $s_2 = 0.062$ mil, test the null hypothesis $\sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $\sigma_1^2 < \sigma_2^2$ at the 0.05 level of significance. Use critical value $F_{0.05}(11, 11) = 2.82$ **04**

(c) An investigation of two types of bulldozers showed that 50 failures of one type of bulldozer took on an average 6.8 hours to repair with a standard deviation of 0.85 hours, while 50 failures of the other type of bulldozer took on an average 7.3 hours to repair with a standard deviation of 1.2 hours. Test the null hypothesis $\mu_1 - \mu_2 = 0$ (namely, the hypothesis that on an average, it takes an equal amount of time to repair either kind of bulldozer) against the alternative hypothesis $\mu_1 - \mu_2 \neq 0$ at the level of significance $\alpha = 0.10$. Use critical value $z_{0.05} = 1.645$ **07**

OR

(a) A manufacturer of submersible pumps claims that at most 30% of the pumps require repairs within the first 5 years of operation. If a random sample of 120 of these pumps includes 47 which required repairs within the first 5 years, test the null hypothesis $p = 0.30$ against the alternative hypothesis $p > 0.30$ at the 0.05 level of significance. Use critical value $z_{0.05} = 1.645$ **03**

(b) A random sample of 6 steel beams has a mean compressive strength of 58,392 psi (pounds per square inch) with a standard deviation of 648 psi. Use this information and the level of significance $\alpha = 0.05$ to test whether the true average compressive strength of the steel from which this sample came is 58,000 psi. Assume normality. Use critical value $t_{0.025,5} = 2.571$ **04**

(c) The number of defects in printed circuit boards is hypothesized to follow Poisson distribution. A random sample of 60 printed boards has been collected and the following number of defects observed. **07**

Number of defects	0	1	2	3
Observed frequency	32	15	9	4

Use the chi-square distribution to test the claim that the number of defects follows the Poisson distribution. Use $\chi^2_{0.05,1} = 3.84$

Q.5 **(a)** Write the probability density function of gamma distribution. What are the mean and variance of gamma distribution? **03**

(b) Suppose the probability density function of a random variable X is given by **04**

$$f(x) = \begin{cases} \frac{x}{8}, 3 < x < 5 \\ 0, otherwise \end{cases}$$

Determine $(a) P(X < 4)$ and $(b) P(4 < X < 5)$

(c) Fit a curve of the form $y = ae^{bx}$ to given data. **07**

x	0	5	10	15	20
y	100	200	450	950	2000

OR

(a) Define independent events. If A and B are independent events with $P(A) = 0.26$ and $P(B) = 0.45$, find $(a) P(A \cap B)$ $(b) P(A \cap B')$ where B' denotes the complement of B **03**

(b) A random sample of size 15 from bivariate normal population gave correlation coefficient $r = 0.8254$. Does this indicate the existence of correlation in the population at $\alpha = 0.05$ level of significance? Critical value of t for $\frac{\alpha}{2} = 0.025$ with 13 degrees of freedom is 2.160. **04**

(c) Determine the straight line of best fit to given data. **07**

x	2	2	6	8	10
y	0	1	2	3	3

***