Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–IV EXAMINATION – WINTER 2025

Subject Code:3140610 Date:20-11-2025

Subject Name:Complex Variables and Partial Differential Equations

Time:02:30 PM TO 05:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Find $|z|$, $\operatorname{Arg} z$ and $\arg z$ for $z = -\sqrt{3} + i$	**03**
	(b)	Define a bilinear transformation. Find a bilinear transformation which sends the points $z = -i, 1, i$ of the z-plane onto the points $w = -1, 0, 1$ respectively in the w-plane.	**04**
	(c)	Define an analytic function. Show that $u(x, y) = 2x - x^3 + 3xy^2$ is a harmonic function. Determine its harmonic conjugate $v(x, y)$ and the corresponding analytic function $f(z) = u + iv$.	**07**
Q.2	**(a)**	Evaluate $\int_C \frac{z \sin z}{(z+2)^3} dz$, where C: $|z| = 1$.	**03**
	(b)	Evaluate $\int_C \frac{e^{-z}}{z^2(z+3)} dz$, where C: $|z| = 2$.	**04**
	(c)	Evaluate $\int_c (x^2 - iy^2) dz$, along the parabola $y = 2x^2$ from (1, 2) to (2, 8) .	**07**
		OR	
	(c)	Find the radius of convergence of	
		(i) $\sum_{n=0}^{\infty} \frac{(6n+1)^2}{(2n+5)^2} (z - 2i)^n$	**04**
		(ii) $\sum_{n=0}^{\infty} (3 - 4i)^n z^n$	**03**
Q.3	**(a)**	Using the Maclaurin's series expansion, prove that $e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$ for $|z| < \infty$.	**03**
	(b)	Find the Laurent's series expansion of $f(z) = z^2 \sin\left(\frac{1}{z^2}\right)$ in the domain $0 < |z| < \infty$.	**04**
	(c)	State Cauchy's Residue theorem. Using it to evaluate the integral $\int_C \frac{5z-2}{z(z-1)} dz$ counter clockwise around the circle $|z| = 2$.	**07**
		OR	
Q.3	**(a)**	Expand $f(z) = \frac{1+2z^2}{z^3+z^5}$ in powers of z, where $0 < |z| < 1$.	**03**
	(b)	Using Cauchy's Residue theorem, evaluate $\int_0^{2\pi} \frac{1}{1+3\cos^2\theta} d\theta$	**04**
	(c)	Define Isolated singularity, Pole of order-n, Essential singularity and Removable singularity of a complex function $f(z)$. Also identify the nature of singularities for the function $f(z) = z^2 e^{1/z}$.	**07**

Q.4 **(a)** Form a partial differential equation by eliminating arbitrary function from $z = ax + by + a^2 + ab + b^2$ **03**

(b) Using Lagrange's method, solve the partial differential equation $(x - 2z)p + (2z - y)q = y - x$. **04**

(c) Write Charpit's auxiliary equations of the PDE of the form $f(x, y, z, p, q) = 0$. Hence Solve the PDE $px + qy = pq$ by using Charpit's method. **07**

OR

Q.4 **(a)** Form a partial differential equation by eliminating arbitrary function f from $z = e^{ay} f(x + by)$. **03**

(b) Find complete integral of the PDE $p^2 - q^2 = x - y$. **04**

(c) Find complete integrals of the PDEs **07**

(i) $p^2 + q^2 = 1$

(ii) $p + q = z$

Q.5 **(a)** Solve the partial differential equation $4r + 12s + 9t = 0$ **03**

(b) Solve the partial differential equation $\frac{\partial^4 z}{\partial x^4} - 2\frac{\partial^4 z}{\partial x^2 \partial y^2} + \frac{\partial^4 z}{\partial y^4} = 4e^{2x}$ **04**

(c) Solve the PDE by method of separation of variables, Given **07**

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u\ ;\ \ u(x, 0) = 6e^{-x}$$

OR

Q.5 **(a)** Classify the partial differential equation $u_{xx} + yu_{yy} + \frac{1}{2}u_y = 0$, as Hyperbolic, Parabolic or Elliptic. **03**

(b) Solve the partial differential equation **04**

$$(D^3 - 4D^2D' + 4DD'^2)z = 4\sin(2x + y)$$

(c) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is **07**

$$u(x, 0) = \begin{cases} x & ;\ 0 \le x \le 50 \\ 100 - x & ;\ 50 \le x \le 100 \end{cases}$$

Find the temperature $u(x, t)$ at any time.

*****