Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II EXAMINATION – WINTER 2024

Subject Code:BE01000041 **Date:04-01-2025**

Subject Name:Mathematics-I

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1	**(a)**	Find $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{sin^2x}\right)$	**03**
	(b)	Write second derivative test for local extrema and find the extrema of $f(x) = x^4 - 4x^3 + 10$	**04**
	(c)	Expand e^x into the power series of x . Find number of terms required to calculate e with an error less than 10^{-4} .	**07**
Q.2	**(a)**	Define Beta and Gamma functions and evaluate $\int_0^\infty \frac{x^3(1+x^2)}{(1+x)^{10}}dx$	**03**
	(b)	Discuss the convergence of $\int_{-\infty}^{\infty}\frac{1}{1+x^2}dx$	**04**
	(c)	The circle $x^2 + y^2 = a^2$ is rotated about x −axis to generate a sphere, find its volume and sketch the region.	**07**
		OR	
	(c)	Find the surface area of a solid generated by the revolution of the circle $r = 2acos\theta$ about the initial line.	**07**
Q.3	**(a)**	Determine whether $\lim_{(x,y)\to(0,0)}\frac{x^6-y^2}{x^3y}$ exists, find it if exists.	**03**
	(b)	Applying the chain rule find $\frac{dw}{dt}$ $if\ w = x^2y - y^2$, where $x = sint$ and $y = e^t$ also find $\frac{dw}{dt}$ at $t = 0$	**04**
	(c)	Find the extreme value of the function $f(x,y) = x^3 + y^3 + 3x^2 - 3y^2$	**07**
		OR	
Q.3	**(a)**	If $u = \tan^{-1}(y/x)$ prove that $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$	**03**
	(b)	Find the tangent plane and normal line of the surface $f(x,y,z) = x^2 + y^2 + z - 9 = 0$ at point $(1, 2, 4)$	**04**
	(c)	Applying Lagrange multipliers to determine the dimension of a rectangular box, open at the top, having volume $32\ ft^3$, and requiring the least amount of material for its construction.	**07**
Q.4	**(a)**	State sandwich theorem for the sequence and discuss the convergence of the sequence $\left\{\frac{\cos n}{n}\right\}$	**03**
	(b)	Test the convergence of the series $\sum_{n=0}^{\infty}\left(\frac{4^n+3}{5^n}\right)$, also find the sum of the series if it is convergent.	**04**
	(c)	Applying appropriate test find the interval of convergence and radius of convergence of the series $\sum_{n=0}^{\infty}\frac{(x-5)^n}{n^2}$	**07**
		OR	

Q.4	**(a)**	Investigate the convergence of the series $\sum_{n=0}^{\infty} \frac{4^n n! n!}{(2n!)}$	**03**
	(b)	Discuss the convergence of the series and find it's sum $\sum_{n=0}^{\infty} \frac{1}{n(n+1)}$	**04**
	(c)	Applying appropriate test find the interval of convergence and radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n(n+1)}$	**07**
Q.5	**(a)**	Evaluate $\iint_R y^2 x \, dA$ over the rectangular region $R = \{(x, y): -3 \le x \le 2, \ 0 \le y \le 1\}$	**03**
	(b)	Use double integration to determine the area of the region R inclosed between the parabola $y = \frac{1}{2}x^2$, and the line $y = 2x$, also sketch the region.	**04**
	(c)	Applying change of the order of the integration to evaluate $\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2} \, dx \, dy$, also sketch the region.	**07**
		OR	
Q.5	**(a)**	Evaluate $\int_0^1 \int_x^1 \int_0^{y-x} dzdydx$	**03**
	(b)	Evaluate the following integral by changing in to the polar co ordinate. $\iint_R e^{x^2+y^2} dA$ where R is the semi circular region bounded by the x-axis and the curve $y = \sqrt{1 - x^2}$	**04**
	(c)	A thin plate covers the triangular region bounded by the x-axis and the line $x = 1$ and $y = 2x$ in the first quadrant. The plate density at the point (x, y) is $\delta(x, y) = 6x + 6y + 6$ find the plate's mass, first moments and the centre of mass about the coordinate axis.	**07**

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