Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II EXAMINATION – WINTER 2024

Subject Code:3110014 **Date:03-01-2025**

Subject Name:Mathematics - 1

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** Find the equation of tangent plane and normal line to the surface $xyz = 6$ at $(1,2,3)$. **03**

(b) Evaluate $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$. **04**

(c) Solve the following system by Gauss Elimination method: **07**

$$x+y+2z=9$$
$$2x+4y-3z=1$$
$$3x+6y-5z=0$$

Q.2 **(a)** Find the directional derivative of $f(x,y,z)=xy^2+yz^3$ at the point $(2,-1,1)$ in the direction $\hat{i}+2\hat{j}+2\hat{k}$. **03**

(b) Expand $x^4-3x^3+2x^2-x+1$ in powers of $(x-3)$. **04**

(c) Find the Fourier series of $f(x)=x^2 \quad 0<x<\pi$, $=0 \quad \pi<x<2\pi$. **07**

OR

(c) Find the Fourier series of $f(x)=x+|x|$ in the interval $-\pi<x<\pi$. **07**

Q.3 **(a)** Show that the sequence $\{u_n\}$, where $u_n=\frac{\sin n}{n}$ converges to zero. **03**

(b) Evaluate $\int_0^3 \frac{dx}{(x-1)^{\frac{2}{3}}}$. **04**

(c) A rectangular box without a lid is to be made from 12 m^2 of cardboard. Find the maximum volume of such box. **07**

OR

Q.3 **(a)** Evaluate $\int_0^1 (\log x)^5\,dx$. **03**

(b) Find the cosine series for $f(x)=\pi-x$ in the interval $0<x<\pi$. **04**

(c) Find the extreme values of the function $x^3+xy^2+21x-12x^2-2y^2$. **07**

Q.4 **(a)** Test the convergence of the series $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+1}$. **03**

(b) Evaluate the integral $\int_0^{\frac{\pi}{2}} \int_0^{1-\sin\theta} r^2 \cos\theta \, dr d\theta$. **04**

(c) Find the length of the parabola $x^2 = 4y$ which lies inside the circle $x^2 + y^2 = 6y$ **07**

OR

Q.4 **(a)** Evaluate $\int_0^2 \int_1^z \int_0^{yz} xyz \, dx \, dy \, dz$. **03**

(b) Test the convergence of the series $\sum \frac{(n+1)^n x^n}{n^{n+1}}$. **04**

(c) Change the order of integration and evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dydx$. **07**

Q.5 **(a)** Test the convergence of the series $\sum_{n=1}^{\infty} n^2 e^{-n^3}$. **03**

(b) Find a and b such that $A = \begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ has 3 and -2 as eigenvalues. **04**

(c) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$. **07**

OR

Q.5 **(a)** Evaluate $\int_0^{\infty} \frac{dv}{(1+v^2)(1+\tan^{-1} v)}$. **03**

(b) Using Gauss Jordan method find inverse of $\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$. **04**

(c) Show that the matrix $\begin{bmatrix} 1 & -2 & 0 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ is not diagonalizable. **07**

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