Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I (NEW) EXAMINATION – WINTER 2023

Subject Code:3110014 Date:11-01-2024

Subject Name:Mathematics - 1

Time:02:30 PM TO 05:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Evaluate $\lim_{x\to 0^+} x\, lnx$	**03**
	(b)	Define beta and gamma functions. What is the relationship between beta and gamma functions?	**04**
	(c)	Solve the following system of linear equations using Gauss-Jordan elimination: $$\begin{aligned} x_3 + x_4 + x_5 &= 0 \\ -x_1 - x_2 + 2x_3 - 3x_4 + x_5 &= 0 \\ x_1 + x_2 - 2x_3 - x_5 &= 0 \\ 2x_1 + 2x_2 - x_3 + x_5 &= 0 \end{aligned}$$	**07**
Q.2	**(a)**	Define rank of the matrix. Find rank(A) if $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$	**03**
	(b)	Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{1}{1+n}\right)^n$	**04**
	(c)	Find eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$	**07**
		OR	
	(c)	Find the Fourier series of the function $f(x) = x + \pi$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$	**07**
Q.3	**(a)**	If $w = x^2 + y^2, x = r - s, y = r + s$, using chain rule, prove that $\frac{\partial w}{\partial s} = 4s$.	**03**
	(b)	Find the directional derivative of $f(x, y, z) == 2x^2 + 3y^2 + z^2$ at point $(2,1,3)$ in the direction of the vector $\boldsymbol{i} - 2\boldsymbol{k}$.	**04**
	(c)	Find local extreme values of the function $f(x, y) = 4x^2 + 9y^2 + 8x - 36y + 24$.	**07**
		OR	
Q.3	**(a)**	Determine whether $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^3+y^3}$ exists and find it if exists.	**03**
	(b)	Find the equation of the tangent plane to $z = 3x^2 - xy$ at the point $(1,2,1)$.	**04**

	(c)	Find the maximum and minimum values of the function $f(x,y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.	**07**
Q.4	**(a)**	Calculate $\iint_R f(x,y)\,dA$ for $f(x,y) = 100 - 6x^2y$ and $R: 0 \le x \le 2, -1 \le y \le 1$	**03**
	(b)	Sketch the region of integration for the integral $\int_0^2 \int_{x^2}^{2x} (4x+2)dydx$ and write an equivalent integral with the order of integration reversed.	**04**
	(c)	Calculate $\iint_R \frac{\sin x}{x} dA$ where R is the triangle in the xy-plane bounded by the x-axis, the line $y = x$ and the line $x = 1$.	**07**
		OR	
Q.4	**(a)**	Evaluate $\int_1^{\ln 8} \int_0^{\ln y} e^{x+y} dxdy$.	**03**
	(b)	Find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant.	**04**
	(c)	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2+y^2)dydx$ by changing into polar coordinates.	**07**
Q.5	**(a)**	Find the Maclaurin series for $\cos x$.	**03**
	(b)	Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} ne^{-n^2}$.	**04**
	(c)	Test the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$.	**07**
		OR	
Q.5	**(a)**	Define monotonic sequence. Is the sequence $\left\{\frac{1}{n^2}\right\}$ monotonic?	**03**
	(b)	Investigate the convergence of the series $\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$.	**04**
	(c)	Find the interval of convergence of the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$	**07**

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