Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I&II EXAMINATION – SUMMER 2025

Subject Code:BE01000041 **Date:05-06-2025**

Subject Name:Mathematics-I

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Investigate the convergence of $\int_0^1 \frac{1}{1-x}\,dx$.	**03**
	(b)	Define beta and gamma functions. What is the relationship between beta and gamma functions?	**04**
	(c)	The region between the curve $y = \sqrt{x}, 0 \le x \le 4$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume.	**07**
Q.2	**(a)**	Evaluate $\lim\limits_{x\to 0} \frac{3x - sinx}{x}$ using L'Hospital rule.	**03**
	(b)	Find the Taylor series generated by $f(x) = e^x$ at $x = 2$.	**04**
	(c)	Find the Maclaurin series for $f(x) = (1+x)^k$ where k is any real number. Using it, find the Maclaurin series for the function $\frac{1}{1-x}$.	**07**
		OR	
	(c)	Find the local extreme values of the function $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$.	**07**
Q.3	**(a)**	Test the the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ for convergence or divergence.	**03**
	(b)	Test the convergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$.	**04**
	(c)	Define absolutely convergent series and conditionally convergent series. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$.	**07**
		OR	
Q.3	**(a)**	Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n}{n!}$.	**03**
	(b)	Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.	**04**
	(c)	For what values of x does the series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$ converge?	**07**
Q.4	**(a)**	If $f(x,y) = \frac{xy^2}{x^2+y^4}$, does $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exist?	**03**
	(b)	If $z = x^2y + 3xy^4$, where $x = \sin 2t$ and $y = \cos t$, find $\frac{dz}{dt}$ when $t = 0$.	**04**
	(c)	Find the extreme values of the function $f(x,y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.	**07**

OR

Q.4 **(a)** Find the equation of the tangent plane at the point $(-2,1,-3)$ to the ellipsoid $\frac{x^2}{4}+y^2+\frac{z^2}{9}=3$. **03**

(b) Find the directional derivative of the function $f(x,y)=x^2y^3-4y$ at the point $(2,-1)$ in the direction of the vector $\boldsymbol{v}=2\boldsymbol{i}+5\boldsymbol{j}$. **04**

(c) Find the local extreme values of the function $f(x,y)=xy-x^2-y^2-2x-2y+4$. **07**

Q.5 **(a)** Evaluate $\int_0^3\int_0^2(4-y^2)dydx$ **03**

(b) Evaluate $\iint_R\ (3x+4\,y^2)dA$ where R is the region in the upper half-plane bounded by the circles $x^2+y^2=1$ and $x^2+y^2=4$. **04**

(c) Sketch the region of integration, reverse the order of integration and evaluate the integral $\int_0^1\int_x^1 \sin(y^2)\ dydx$. **07**

OR

Q.5 **(a)** Evaluate the triple integral $\iiint_B\ xyz^2\,dV$ where B is the rectangular box given by
$B=\{(x,y,z): 0\le x\le 1, -1\le y\le 2, 0\le z\le 3\}$. **03**

(b) Find the area of the region R bounded by $y=2x^2$ and $y^2=4x$. **04**

(c) Find the volume of the solid that lies under the paraboloid $z=x^2+y^2$ and above the region R in the xy-plane bounded by the lines $y=2x$ and the parabola $y=x^2$. **07**

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