Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE- SEMESTER–I & II(NEW)EXAMINATION – SUMMER 2022**

**Subject Code:3110014 Date:02-08-2022**

**Subject Name:Mathematics - 1**

**Time:10:30 AM TO 01:30 PM Total Marks:70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

| | | | Marks |
|---|---|---|---|
| **Q.1** | **(a)** | Is $\sum_{n=1}^{\infty}\frac{1}{n^p}$ convergent for $p>1$? Justify your answer. | **03** |
| | **(b)** | (1) Find $\lim_{x\to a}\frac{\sin x-\sin a}{(x-a)^2}$ | **02** |
| | | (2) Is $\int_0^{\infty}\frac{\sin^2 x}{x^2}$ convergent? Justify your answer. | **02** |
| | **(c)** | (1) Find the length of curve $f(x)=\frac{x^3}{12}+\frac{1}{x},\ 1\le x\le 4$. | **04** |
| | | (2) Prove that $Gamma\,(n)=(n-1)\,Gamma\,(n-1)$. | **03** |
| **Q.2** | **(a)** | Investigate the convergence of $\sum_{1}^{\infty}\frac{n^2}{7^n}$. | **03** |
| | **(b)** | Investigate the convergence of $\sum_{n=1}^{\infty}\frac{2^n\,(n\,!)^2}{(2n)\,!}$ | **04** |
| | **(c)** | Find Fourier series of $f(x)=x^2,\ -\pi<x<\pi$. | **07** |
| | | **OR** | |
| | **(c)** | Find Fourier series of $f(x) = x,\ -1<x<0$; $= 2,\ 0<x<1$. | **07** |
| **Q.3** | **(a)** | Find the derivative of $f(x,y)=x^2+xy+y^2$ in the direction $\hat{i}+\hat{j}$ at $P(1,1)$. | **03** |
| | **(b)** | Find the tangent plane of $z=e^x\cos y$ at $P(0,0,0)$. | **04** |
| | **(c)** | Find local extreme values of $f(x,y)=xy-x^2-y^2-x$. | **07** |
| | | **OR** | |
| **Q.3** | **(a)** | Explain second derivative test for local extreme values. | **03** |
| | **(b)** | Let $f=\ln r$, where $\bar{r}=x\,\hat{i}+y\,\hat{j}+z\,\hat{k}$ and $r=\lvert\bar{r}\rvert$. Find $grad\ f$. | **04** |
| | **(c)** | Determine the minimum value of $x^2yz^2$ subject to the condition $x+y+2z=5$ using method of Lagrange multipliers. | **07** |
| **Q.4** | **(a)** | Evaluate $\int_{y=0}^{1}\int_{x=0}^{2}\frac{1}{\sqrt{4-x^2}\sqrt{1-y^2}}\,dxdy$. | **03** |

**(b)** Evaluate the integral $\int_{0}^{2}\int_{x/2}^{1}\frac{1}{3}e^{y^2}\,dy\,dx$ by change of order. **04**

**(c)** (1) Find the area of the region covered by $x=1,\ x=4,\ y=0$ and $y=\sqrt{x}$. **04**

(2) Evaluate $\int_{x=0}^{1}\int_{y=0}^{x^{1/4}}\int_{z=0}^{y^2}\sqrt{z}\ dz\ dy\ dx$ **03**

**OR**

**Q.4** **(a)** Evaluate $\iint_R xy\ dA$ where $R$ is the region bounded by $x$ axis, the ordinate $x=2a$ and the curve $x^2=4ay$. **03**

**(b)** Evaluate the integral $\int_{y=0}^{1}\int_{x=0}^{\cos^{-1}y}e^{\sin x}\ dx\,dy$ by change of order. **04**

**(c)** (1) Change in to polar coordinates then solve $\int_{0}^{2}\int_{0}^{\sqrt{4-x^2}}e^{-(x^2+y^2)}\,dy\ dx$. **04**

(2) Let $x+y=u$ and $y=uv$ are given transformations. Find Jacobian for change of variables. **03**

**Q.5** **(a)** Find characteristic equation of $A=\begin{bmatrix}1 & -1 & 1\\ 0 & 2 & 1\\ 2 & 0 & 1\end{bmatrix}$. **03**

**(b)** Find Maclaurin's series for $f(x)=e^{2x}\sinh x$ and show at least up to $x^4$ term. **04**

**(c)** Solve **07**

$x+y+w=1,\ 2x+z+w=3,\ 2y+z+2w=2$.

**OR**

**Q.5** **(a)** Show that give matrix $A=\begin{bmatrix}0 & 1\\ -2 & -1\end{bmatrix}$ satisfies its Characteristic equation. **03**

**(b)** Show that $\sum_{n=2}^{\infty}\frac{(-1)^n}{\ln n}$ converges. **04**

**(c)** Show that $A=\begin{bmatrix}0 & 1 & 0\\ -2 & -1 & 2\\ -4 & -8 & 7\end{bmatrix}$ is diagonalizable. Find the matrix of eigen vectors and diagonal matrix. **07**

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