Enrollment No./Seat No.:

GUJARAT TECHNOLOGICAL UNIVERSITY

Bachelor of Engineering - SEMESTER - IV EXAMINATION - WINTER 2025

Subject Code: 3140708 **Date: 20-11-2025**

Subject Name: Discrete Mathematics

Time: 02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Draw the graph with 4 nodes and 7 edges.	**03**
	(b)	Define simple graph, degree of a vertex, finite and infinite graphs and complete graph.	**04**
	(c)	(i) If A = B = {1, 2, 3, 5, 6, 10, 15, 30} and relation R defined as a/b, where a∈A and b∈B. Find the relation matrix. (ii) Solve the recurrence relation $a_n - 3a_{n-1} = 2$, $n \geq 1$, $a_0 = 1$.	**07**
Q.2	**(a)**	Let A, B and C be the sets such that $(A \cap B \cap C) = \varnothing$, $(A \cap B) \neq \varnothing$, $(A \cap C) \neq \varnothing$, $(B \cap C) \neq \varnothing$. Draw the corresponding venn diagram.	**03**
	(b)	Define subgroup of a group. Also, show that the subset H of a group of integers I is a subgroup under addition, where H = {...., -2m, -m, 0, m, 2m, ...}.	**04**
	(c)	How many integers between 1 to 2000 are divisible by 2, 3, 5 or 7.	**07**
		OR	
	(c)	Using venn diagram, prove the following (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$	**07**
Q.3	**(a)**	Check whether the proposition $((p \vee q) \wedge \sim p) \rightarrow q$ is contradiction, tautology or contingency.	**03**
	(b)	Show that the proposition $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$ is a tautology.	**04**
	(c)	Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 18, 24} be ordered by the relation x divides y. Show that the relation is partial ordering and draw the Hasse diagram.	**07**
		OR	
	(a)	Show that the propositions $p \rightarrow q$ and $\sim p \vee q$ are logically equivalent.	**03**
	(b)	Negate (opposite) each of the following statements (i) $\forall$ x, $\lvert x \rvert = x$ (ii) $\exists$ x, $x^2 = x$ (iii) If there is a riot, then someone killed. (iv) It is day light and all the people are arisen.	**04**

(c) Solve the following recurrence relation **07**

(i) $a_n - 7a_{n-1} + 10a_{n-2} = 0$ given that $a_0 = 0$, $a_1 = 3$.

(ii) $a_n - 4a_{n-1} + 4a_{n-2} = 0$ given that $a_0 = 1$, $a_1 = 6$.

Q.4 **(a)** Explain Partially order relation, Poset and Hasse diagram with example(s) **03**

(b) Explain the regular graph and complete bipartite graph with example(s). **04**

(c) Define tree and root. Also draw a tree with 10 vertices which has vertices either of degree 1 or of degree 3. Is it possible to draw a same type of tree with 11 vertices? **07**

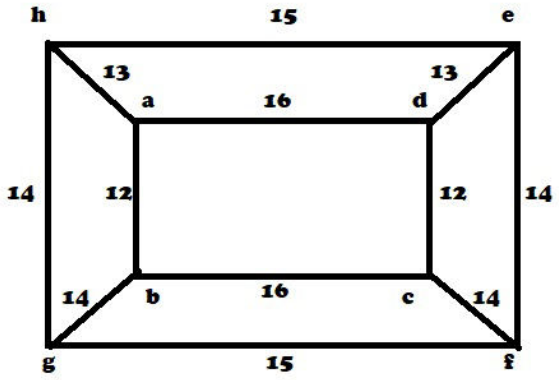
OR

(a) Let n be a positive integer, S_n be the set of all divisors of n. Let D denote the relation of division. Draw the lattices for (i) n = 24, (ii) n = 30 **03**

(b) Draw the following grapghs **04**

(i) $K_{3,4}$ (ii) $K_{4,4}$ (iii) $K_{5,4}$ (iv) K_5

(c) Find minimum spanning tree for the weighted graph given below **07**



Q.5 **(a)** Define abelian group for any non empty set. **03**

(b) If G = {0, 1, 2, 3, 4, 5, 6, 7} and operation $+_8$ is an addition modulo 8, then show that $(G, +_8)$ is an abelian group. **04**

(c) Consider the set **(C, +, .),** where **C** is the set of complex numbers and + and . are ordinary addition and multiplication operation. Show that **(C, +, .**) is a field. **07**

OR

(a) Let Q_+ be the set of all positive rational numbers and $*$ be a binary operation on Q_+ defined by $a*b = ab/3$, then show that $(Q_+, *)$ is an abelian group. **03**

(b) Define cyclic group and show that the following groups are cyclic groups **04**

(i) Third roots of unity i.e. $G = \{1, \omega, \omega^2\}$, $\omega^3 = 1$

(ii) $G = \{1, -1, i, -i\}$, where $i^2 = -1$

(c) Show that $R = \{a+b\sqrt{2}; a, b \in I\}$ is an integral domain but not a field under the binary operations + (usual addition) and . (usual multiplication). **07**

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Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–IV (NEW) EXAMINATION – WINTER 2024

Subject Code:3140708 Date:26-11-2024

Subject Name:Discrete Mathematics

Time:02:30 PM TO 05:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Define onto function. Check whether the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ is one-one and onto.	**03**
	(b)	For the relation R $= \{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$ on the set $A = \{1,2,3,4\}$. Check whether it is reflexive, symmetric, anti-symmetric, transitive.	**04**
	(c)	(i) A history class contains 8 male students and 6 female students. Find the number n of ways that the class can elect: (a) 1 class representative (b) 2 class representatives, 1 male and 1 female (c) 1 president and 1 vice president. (ii) Define elementary cycle, loop, tree and pendent vertex.	**03** **04**
Q.2	**(a)**	Show that if every element in a group is its own inverse, then the group must be abelian.	**03**
	(b)	Identify the statement $(p \to q) \leftrightarrows (\neg p \vee q)$ is tautology or contradiction.	**04**
	(c)	Use a truth table to determine whether the following argument form is valid. $p \to q$ $q \to r$ $\therefore p \to r$	**07**
		OR	
	(c)	(i) Symbolize the following expressions "x is the father of the mother of y" where $P(x)$: x is a person. $F(x,y)$: x is the father of y. $M(x,y)$: x is the mother of y.	**03**
		(ii) Show that the premises "Everyone in this discrete mathematics class has taken a course in computer science" and "Marla is a student in this class" imply the conclusion "Marla has taken a course in computer science."	**04**

Q.3 **(a)** Define homomorphism. Let G be the group of real numbers under addition, and let G' be the group of positive real numbers under multiplication. Check whether the mapping $f : G \to G'$ defined by $f(a) = 2^a$ is a homomorphism. **03**

(b) Suppose that 100 mathematics students at a college taking at least one of the languages French, German, and Russian, given the following data: **04**
65 study French, 45 study German, 42 study Russian, 20 study French and German, 25 study French and Russian, 15 study German and Russian.
(1) Find the number of students who study all the three languages.
(2) Find the number of students who study only French.

(c) (i) The subset $H = \{0, 2\}$ is a subgroup of $(\mathbb{Z}_4, +4)$. Is H a normal subgroup? **03**
(ii) Let $\mathbb{Z}_m$ denotes the integers modulo m. Check whether $\mathbb{Z}_m$ is a group under addition. If so, is it abelian group? **04**

OR

Q.3 **(a)** Find all the generators of cyclic group $(\mathbb{Z}_5, +5)$. **03**

(b) Prove that $A \cap \bar{B} = A \cap \bar{C}$ if and only if $A \cap B = A \cap C$. **04**

(c) Consider the ring $\mathbb{Z}_{30} = \{0, 1, 2, \ldots, 29\}$ of integers modulo 30. **07**
(a) Find -8 and -17.
(b) Find $11^{-1}, 13^{-1}$ and 14^{-1}.
(c) Let $f(x) = 2x + 4$. Find the roots of $f(x)$ over $\mathbb{Z}_{30}$.

Q.4 **(a)** Let $A = \{1, 2, 3, 4\}$ and the relation $R = \{(1, 1), (1, 4), (4, 1), (4, 4), (2, 2), (2, 3), (3, 2), (3, 3)\}$ on A. Write the matrix of R and check whether the relation is an equivalence. **03**

(b) Let $X = \{2, 3, 6, 12, 24, 36\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y. Find **04**
(i) Lower bound of {3,6},
(ii) Upper bound of {3,6},
(iii) GLB of {3,6}, if exist,
(iv) LUB of {3,6}, if exist.

(c) Solve the recurrence relation using the method of generating function **07**
$a_n - 5\,a_{n-1} + 6a_{n-2} = 3n,\ n \geq 2,\ a_0 = 0,\ a_1 = 2.$

OR

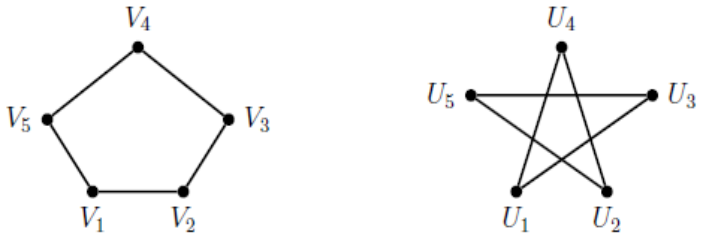
Q.4 **(a)** Let the relation $R = \{(1, 2), (2, 3), (3, 3)\}$ on $A = \{1, 2, 3\}$. Find the transitive closure of R. **03**

(b) Solve $a_n = 2\,a_{n-1} + 3a_{n-2},\ a_0 = 1,\ a_1 = 2.$ **04**

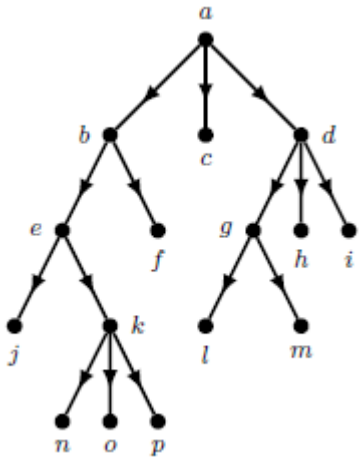
(c) Draw Hasse diagram of $\langle S_{30}, D \rangle$. Prove that $\langle S_{30}, D \rangle$ is a lattice, where D is the relation of "division" in $\mathbb{N}$ such that for any $a, b \in \mathbb{N}$, aDb if and only if a divides b and S_n, $(n \in \mathbb{N})$ is the set of all divisors of n. **07**

Q.5 **(a)** Find the number of edges in a r-regular graph with n vertices. **03**

(b) Check whether the following graphs are isomorphic or not. **04**



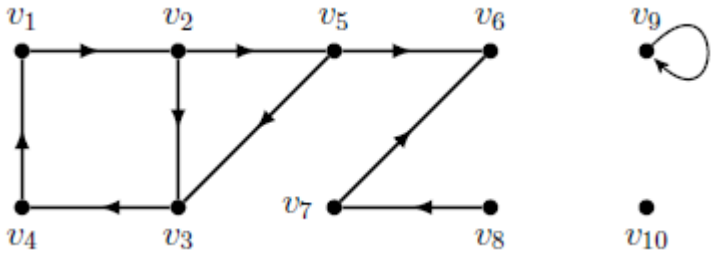
(c) In which order does a pre-order, in-order and post-order traversal visit the vertices of the ordered rooted tree shown in figure? **07**



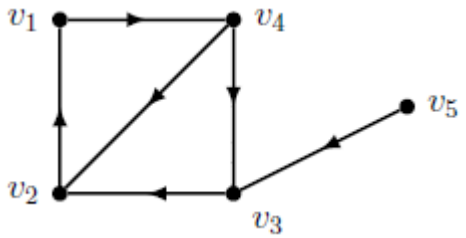
OR

Q.5 **(a)** A tree T has 4 vertices of degree 2, 3 vertices of degree 3, 1 vertex of degree 4. Find the number of pendant vertices in the tree T. **03**

(b) Find reachable set of each node of the given digraph. **04**



(c) Define adjacency matrix. Use Warshall's algorithm to obtain path matrix from the adjacency matrix of **07**



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Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – WINTER 2023

Subject Code:3140708 Date: 06-02-2024

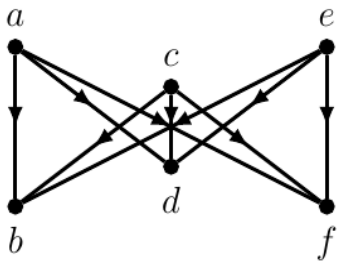
Subject Name: Discrete Mathematics

Time: 10:30 AM TO 01:00 PM Total Marks:70

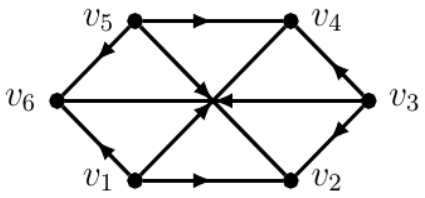
Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	A function $f: \mathbb{R}^+ \to \mathbb{R}$ is defined by $f(x) = x^2 - 8$. Check whether f is one-one and onto.	**03**
	(b)	Determine the relation ∥ (parallel) on the set L of lines in the plane are reflexive, symmetric, anti-symmetric, transitive, irreflexive.	**04**
	(c)	(i) Show that the functions $f(x) = x^3$ and $g(x) = x^{\frac{1}{3}}$ for $x \in \mathbb{R}$ are inverses of one another. (ii) Check whether the following graphs are isomorphic.	**03** **04**



(H)



(G)

Q.2	**(a)**	Identify the statement $(p \wedge q) \wedge \neg(p \vee q)$ is tautology or contradiction.	**03**
	(b)	(i) Show that in a group $(G,*)$, for any $a, b \in G$ if $(a*b)^2 = a^2 * b^2$ then $(G,*)$ must be abelian. (ii) If $(G,*)$ be a group then for any two elements a and b of $(G,*)$, prove that $(a*b)^{-1} = b^{-1} * a^{-1}$.	**04**
	(c)	Use a truth table to determine whether the following argument form is valid. $p \to q$ $q \to r$ $\therefore p \to r$ **OR**	**07**
	(c)	Express the following using predicate, quantifier and logical connectives. Also verify the validity of the consequence. (i) Every computer science major takes discrete mathematics. Natasha is taking discrete mathematics. Therefore, Natasha is a computer science major. (ii) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.	**07**
Q.3	**(a)**	Show that $(\mathbb{Z}_5^*, \times_6)$ is cyclic group, where $\mathbb{Z}_5^* = \mathbb{Z}_5 \backslash \{0\}$.	**03**

(b) Given A $= \{x : x$ is an integer and $1 \leq x \leq 5\}$, $B = \{3,4,5,17\}$, and $C = \{1,2,3,\ldots\}$, find $A \cap B$, $A \cup B$, $A \cap B \cap C$ and $A \cup C$. **04**

(c) (i) Show that every subgroup of a cyclic group is normal **03**

(ii) The subset $H = \{0,2\}$ is a subgroup of $(\mathbb{Z}_4, +_4)$. Find all left and right cosets of H in $(\mathbb{Z}_4, +_4)$. Is H a normal subgroup? **04**

OR

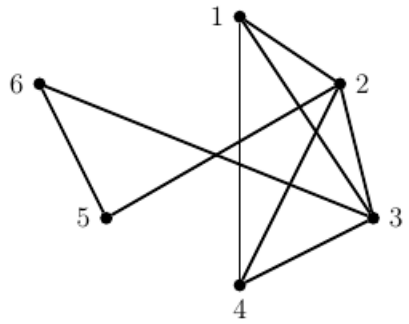
Q.3 **(a)** Define $f: (\mathbb{N} \times \mathbb{N}, *) \to (\mathbb{Q}, \times)$ by $f(a,b) = \frac{a}{b}$. Show that f is a homomorphism. **03**

(b) In a class of 50 students, 12 enrolled for both Mathematics and Science, 32 enrolled for Science. If the students of the class enrolled for at least one of the two subjects, then how many students enrolled for only Mathematics but not Science? **04**

(c) Consider the ring $\mathbb{Z}_{10} = \{0,1,2,\ldots,9\}$ of integers modulo 10. **07**
(a) Find the units of $\mathbb{Z}_{10}$.
(b) Find $-3, -8$ and 3^{-1}.
(c) Let $f(x) = 2x^2 + 4x + 4$. Find the roots of $f(x)$ over $\mathbb{Z}_{10}$.

Q.4 **(a)** Let $X = \{1,2,3,4\}$ and $R = \{\langle x,y\rangle / x > y\}$. Draw the graph of R and also give its matrix. **03**

(b) Find out maximal compatibility blocks of following simplified graph of digraph and write its relation matrix. **04**



(c) Prove that $\langle S_{30}, D\rangle$ is a Boolean algebra. **07**

OR

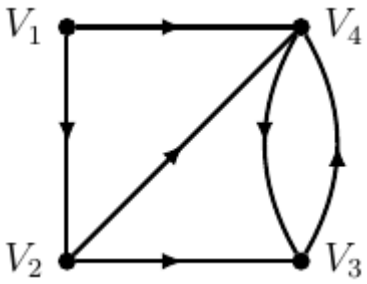
Q.4 **(a)** Define equivalence relation. Let $X = \{1,2,3,4,5\}$, $R = \{\langle x,y\rangle / x$ is divisible by $y\}$. Check whether the relation an equivalence relation? **03**

(b) Let $A = \{a,b,c,d\}$ and $\rho(A)$ its power set. Let $\subseteq$ be the inclusion relation on the elements of $\rho(A)$. Draw the Hasse diagram of $\langle \rho(A), \subseteq\rangle$. **04**

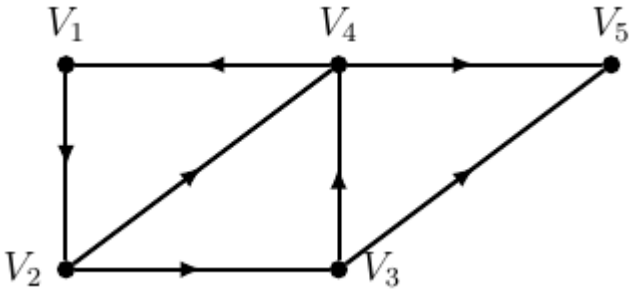
(c) Solve **07**
$a_n = 11a_{n-1} - 39a_{n-2} + 45a_{n-3}, a_0 = 5, a_1 = 11, a_2 = 25.$

Q.5 **(a)** A tree T has 3 vertices of degree 4, 3 vertices of degree 3. Find the number of pendant vertices in tree T. **03**

(b) Find reachable set of each node of the given digraph. Also find $d(V_1, V_3), d(V_3, V_1)$. **04**



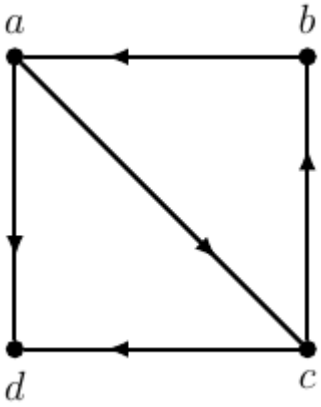
(c) Define Strong, unilateral, week component. Also Find strong, unilateral, week component from the given digraph. **07**



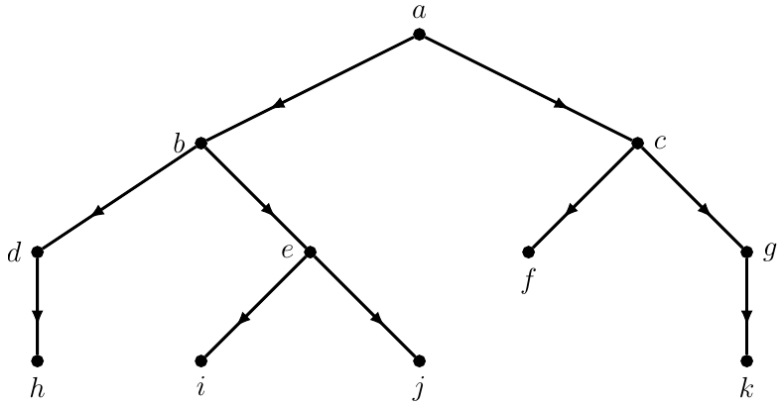
OR

Q.5 **(a)** Define null graph, centre of a graph and complete graph. **03**

(b) Find the path matrix using adjacency matrix of the following graph. **04**



(c) Define tree. In which order does a pre-order, in-order and post-order traversal visit the vertices of the ordered rooted tree shown in figure? **07**



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Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV(NEW) EXAMINATION – WINTER 2022

Subject Code:3140708 Date:16-12-2022

Subject Name:Discrete Mathematics

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	A committee 5 persons, is to be formed from 6 men and 4 women. In how many ways this can be done when (i) at least 2 women are included, (ii) at most 2 women are included.	**03**
	(b)	If $A=\{4,5,7,8,10\}$, $B=\{4,5,9\}$ and $C=\{1,4,6,9\}$, then verify that $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.	**04**
	(c)	Define Functionally complete set of connectives, Principal Disjunctive Normal Form (PDNF). Obtain PDNF for the expression $\neg((p\wedge q)\vee(\neg p\wedge q)\vee(q\wedge r))$	**07**
Q.2	**(a)**	Define Partial Order Relation. Illustrate with an example.	**03**
	(b)	Define one – one function. Show that the function $f:R\to R$, $f(x)=3x-7$ is one – one and onto both. Also find its inverse.	**04**
	(c)	Solve the recurrence relation $a_{n+2}-5a_{n+1}+6a_n=2$ with initial condition $a_0=1$ and $a_1=-1$ using method of undetermined coefficients.	**07**
		OR	
	(c)	Use generating function to solve a recurrence relation $a_n=3a_{n-1}+2$ with $a_0=1$.	**07**
Q.3	**(a)**	Define Partition of a Set. Let $A=\{1,2,3,4,5\}$ and $R=\{(1,2),(1,1),(2,1),(2,2),(3,3),(4,4),(5,5)\}$ be an equivalence relation on A. Determine the partition for R^{-1}, if it an equivalence relation.	**03**
	(b)	Draw Hasse Diagram for the lattice (S_{30},D) where S_{30} is the set of divisors of 30 and D is the relation divides.	**04**
	(c)	Show that the set S of all matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ where $a,b\in R$ is a field with respect to matrix addition and matrix multiplication.	**07**
		OR	
Q.3	**(a)**	Define Semi Group, Monoid. Give an example of an algebraic structure which is semi group but not monoid.	**03**

(b) Consider the a relation R defined on $A=\{1,2,3\}$ whose matrix representation is given below. Determine its inverse R^{-1} and complement R'. **04**

$$M_R=\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}$$

(c) Define free variable and bound variable. **07**
Rewrite the following argument using quantifiers, variables and predicate symbols. Prove the validity of the argument.

"All healthy people eat an apple a day.
Ram does not eat apple a day.
Ram is not a healthy person."

Q.4 **(a)** Define Abelian group and prove that the set $\{0,1,2,3,4\}$ is a finite abelian group under addition modulo 5 as binary operation. **03**

(b) Define even permutation. Show that the permutation $\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 5 & 6 & 2 & 4 & 1 & 3\end{pmatrix}$ is odd and $\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 6 & 3 & 4 & 5 & 2 & 1\end{pmatrix}$ is even. **04**

(c) Define Principal Ideal. Find all the principal Ideal of the ring $[\{0,1,2,3,4,5\},+_6,\times_6]$. **07**

OR

Q.4 **(a)** Define cut vertex. List out all the cut vertices of the graph given in Figure 1. **03**

(b) Define adjacency matrix and path matrix of a graph. Find out adjacency matrix for the graph given in Figure 1. **04**

(c) Define the terms: Simple Graph, Multi – Graph, Weighted Graph, Degree of a vertex, in degree and out degree of a vertex. Illustrate each with an example. **07**

Q.5 **(a)** State Lagrange's theorem and find out all possible subgroups of group $[\{1,-1,i,-i\},\times]$. **03**

(b) Define Eulerian graph, Planar Graph. **04**
Justify whether the graph given in Figure 1 is Planer or not using Euler's formula.

(c) Define Binary tree, Spanning tree, Minimal Spanning tree, Find the minimal spanning tree for the weighted graph given in Figure 2. **07**

OR

Q.5 **(a)** Define Cyclic group, Normal subgroup. Illustrate with an example. **03**

(b) Form a binary search tree for the data 16,24,7,5,8,20,40,3. **04**

(c) Explain Post order traversal. Given the postorder and inorder traversal of a binary tree, draw the unique binary tree. **07**
Postorder: d e c f b h i g a
Inorder: d c e b f a h g i.

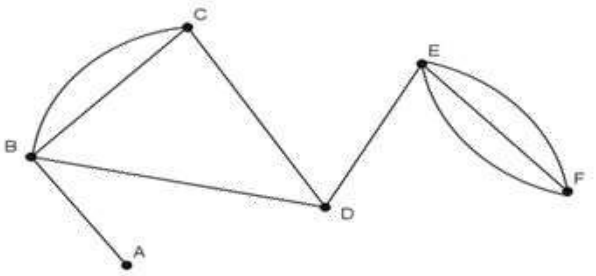


Figure 1

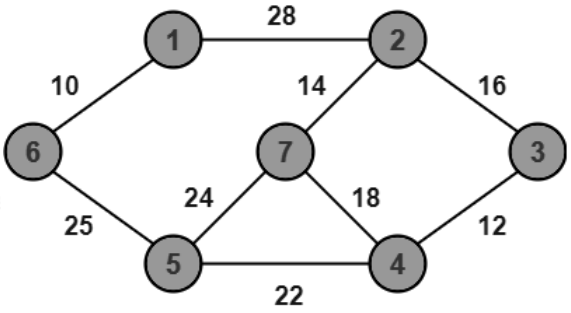


Figure 2

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