Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV EXAMINATION – SUMMER 2025

Subject Code:3140708 Date:17-05-2025

Subject Name:Discrete Mathematics

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Prove that: $(A \cap B) \cup (A - B) = A$ and $A \cup (B - A) = A \cup B$	**03**
	(b)	Check whether the relation R defined in the $\{1,2,3,4,5,6\}$ as $R = \{(a,b) : b = a + 2\}$ is reflexive, symmetric or transitive.	**04**
	(c)	**(I)** Define connected graph, Boolean matrix and Euler path.	**03**
		(II) If $f(x) = \frac{x+1}{x-1}$ and $g(x) = \frac{x-1}{x+1}$ then find $g \circ f(x)$ and $f \circ g(x)$	**04**
Q.2	**(a)**	Find total number of diagonals in a polygon with n sides.	**03**
	(b)	If R be the relation defined in Q -the set of rational numbers by $R = \left\{ \left(\frac{a}{b}, \frac{c}{d} \right) \in Q \times Q : ad = bc \right\}$, then show that R is an equivalence relation.	**04**
	(c)	Solve the recurrence relation $S_n - 6S_{n-1} + 9S_{n-2} = 3^{n+1}$	**07**
		OR	
	(c)	Solve the recurrence relation $S_n + 3nS_{n-1} = 0$, $S_0 = 1$.	**07**
Q.3	**(a)**	Define complete graph and find order and size of the graph K_{2025}	**03**
	(b)	Show that the proposition $\lfloor (p \to q) \wedge (q \to r) \rfloor \to (p \to r)$ is tautology.	**04**
	(c)	Show that $(R, +, \times)$ is an integral domain, where $R = \left\{ a + b\sqrt{11} \middle\| a, b \in I \right\}$	**07**
		OR	
Q.3	**(a)**	Draw Hasse diagram for $P = \{1,2,3,5,11,13,17,19,23\}$ and $\leq$ is a relation such that $x \leq y \Leftrightarrow x \mid y$.	**03**
	(b)	Check whether the relation R defined by $R = \{(a,b) : a \leq b^3, a \ \& \ b \text{ are real numbers}\}$ is reflexive, symmetric or transitive.	**04**
	(c)	**(I)** Does there exist a graph with 20 edges and each vertex of degree 3?	**03**

(II) Show that the function $f: R \to R$ such that $f(x) = x^3 + x$ is a bijection. **04**

Q.4 **(a)** Prove that $(1\times 2)+(2\times 3)+....+(n\times(n+1)) = \frac{n(n+1)(n+2)}{3}$ **03**

(b) If G is an abelian group with n elements $g_1, g_2, ..., g_n$ then show that $(g_1 g_2 .....g_n)^2 = e$, where e is the identity element of G **04**

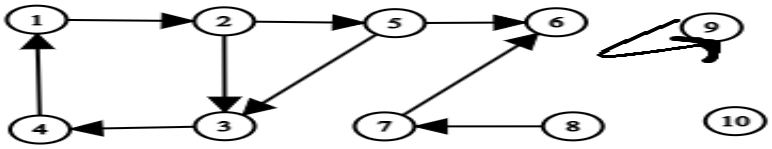
(c) Define Lattice. And consider a poset $(\{3,\ 5,\ 9,\ 15,\ 24,\ 45\},\ |\ )$, where $|$ denotes "divides" is a lattice. Then **07**

(i) Draw its Hasse Diagram. **(ii)** Find its maxima, minima, greatest and least elements when they exist. (iii) Find maxima, minima, greatest and least elements of the set M = {3, 9, 15}, when they exist.

OR

Q.4 **(a)** Find total number of vertices of a full ternary tree with n levels. **03**

(b) Find a Node base for the following graph: **04**



(c) Show that arbitrary intersection of subgroups of a group is a subgroup of a group. Will union of two subgroups be also subgroup of group? **07**

Q.5 **(a)** Show that the set $\{1,\ -1,\ i,\ -i\}$ is a group with respect to multiplication. **03**

(b) Show that in a lattice if $a \le b \le c$, then **04**
(I) $a \oplus b = b * c$ and (II) $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$

(c) Find transitive closure by Wars hall's Algorithm if $A = \{1,2,3,4,5\}$ and $R = \{(1,4),(2,1),(2,5),(2,4),(4,3),(5,3),(3,2)\}$ **07**

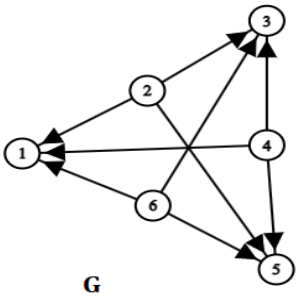
OR

Q.5 **(a)** Show that $H = \left\{ \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} : a \in R \right\}$ is a subgroup of G, where G is the group of all non-singular lower triangular matrices of order 2×2 over R with usual matrix multiplication. **03**

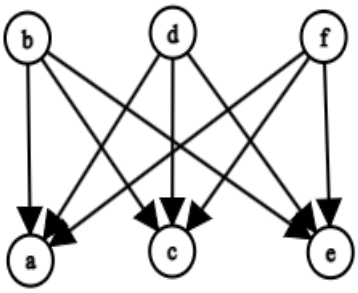
(b) **(I)** Draw a complete bipartite graph which is not regular. **04**

(II) draw a graph which is regular but not bipartite.

(c) Define Isomorphic graphs. Check whether the following graphs are isomorphic? **07**



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Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2024

Subject Code:3140708 Date:01-07-2024

Subject Name: Discrete Mathematics

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Define injective function. Given $A = \{2, 5, 6\}$, $B = \{3, 4, 2\}$, find $(A - B)$ and $(B - A)$.	**03**
	(b)	Determine the relation $\leq$ (less than or equal) on the set $\mathbb{Z}$ of integers are reflexive, symmetric, anti-symmetric, transitive.	**04**
	(c)	(i) Check whether the function $f(x) = x^3 - 2$, for $x \in \mathbb{R}$ is invertible function. If so, find $f^{-1}(x)$.	**03**
		(ii) Prove that a tree with n vertices has $n - 1$ edges.	**04**
Q.2	**(a)**	Identify the statement $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is tautology or contradiction without constructing the truth table.	**03**
	(b)	Let G be the subset of 2 × 2 real matrices with a nonzero determinant. Check whether G is group under matrix multiplication. If so, is it abelian group?	**04**
	(c)	(i) Symbolize the expression "John is a bachelor and this painting is red".	**03**
		(ii) Express the following using predicate, quantifier and logical connectives. Also verify the validity of the consequence. Everyone who graduates gets a job. Ram is graduated. Therefore, Ram got a job.	**04**
		OR	
	(c)	Use a truth table to determine whether the following argument form is valid. $p \rightarrow q$ $p \rightarrow r$ $\therefore p \rightarrow q \vee r$	**07**
Q.3	**(a)**	Let g be a homomorphism from a group $(G, *)$ to a group (H, Δ). Show that $g(e_G) = e_H$ and for any $a \in G$, $g(a^{-1}) = (g(a))^{-1}$.	**03**
	(b)	Prove that : $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.	**04**
	(c)	(i) Prove that every cyclic group is abelian.	**03**
		(ii) Consider the set of positive integers $\mathbb{N}$. Check which of $(\mathbb{N}, +)$ and $(\mathbb{N}, \times)$ are semigroup and which are monoid?	**04**
		OR	

Q.3 **(a)** Find left cosets and right cosets of $H = \{0,3\}$ in the group $(\mathbb{Z}_6, +_6)$. **03**

(b) (i) Suppose repetitions are not allowed, how many four digit numbers can be formed from six digits 1,2,3,5,7,8? **04**
(ii) How many of such numbers less than 4000?
(iii) How many in (i) are even?
(iv) How many in (i) are divisible by 10?

(c) Show that $(R, +, \times)$ is an integral domain, where **07**
$R = \{a + b\sqrt{5} \,/\, a,\ b \in \mathbb{Z}\}$.

Q.4 **(a)** Let $S = \{1, 2, 3, 4\}$ and $R = \{(1,1), (1,4), (2,2), (2,3), (3,2), (3,3), (4,1), (4,4)\}$. **03**
Draw the graph of R and hence write partition of S.

(b) Define Lattice. Draw the Hasse diagram of (S_{12}, D), where D is the relation of **04**
"division" in $\mathbb{N}$ such that for any $a, b \in \mathbb{N}$, aDb iff a divides b and S_{12} is the set of all divisors of 12.

(c) Let $\langle L, \leq \rangle$ be a lattice. Show that for $a, b, c \in L$, following inequalities holds. **07**
(i) $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$ and
(ii) $a * (b \oplus c) \geq (a * b) \oplus (a * c)$.

OR

Q.4 **(a)** Let the POSET $(\rho(A), \leq)$ where $A = \{a, b, c\}$, relation is subset. Find **03**
(i) Upper bound of $\{\{\ \}, \{a\}, \{c\}\}$,
(ii) GLB of $\{\{\ \}, \{a\}, \{c\}\}$, if exist,
(ii) LUB of $\{\{\ \}, \{a\}, \{c\}\}$, if exist.

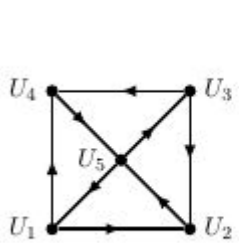
(b) Solve $a_n = a_{n-1} + a_{n-2}, a_0 = 0,\ a_1 = 1$. **04**

(c) Given the relation matrices M_R and M_S, find $M_{R \circ S}$, $M_{\tilde{R}}$, $M_{\tilde{S}}$, $M_{\widetilde{R \circ S}}$, and **07**
show that $M_{\widetilde{R \circ S}} = M_{\tilde{S}} \circ M_{\tilde{R}}$.

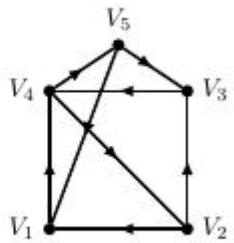
$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}.$$

Q.5 **(a)** Define Isolated node, Binary tree and Regular graph. **03**

(b) Check whether the following graphs are isomorphic or not. **04**

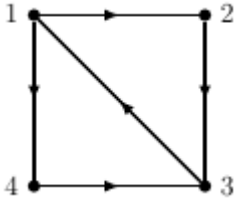


$G(V, E)$



$H(V, E)$

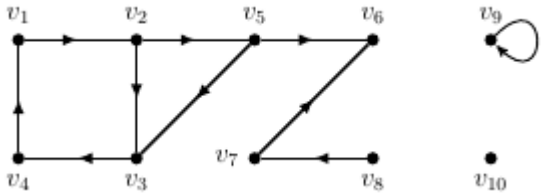
(c) Define path matrix. Warshall's algorithm to obtain path matrix from the adjacency matrix of following graph **07**



OR

Q.5 **(a)** A graph G has 15 edges, 3 vertices of degree 4 and other vertices of degree 3. Find the number of vertices in G. **03**

(b) Find all the node base of the given digraph. Also find $d(V_3, V_6), d(V_6, V_3)$. **04**



(c) Draw binary trees whose post-order produced the string d-e-c-g-j-h-f-b-l-n-q-r-p-m-k-a and pre-order produced the string a-b-d-h-e-i-j-c-f-g-k and in-order produced the string h-d-b-i-e-j-a-f-c-k-g. **07**

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Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– IV(NEW) EXAMINATION – SUMMER 2023

Subject Code:3140708 Date:17-07-2023

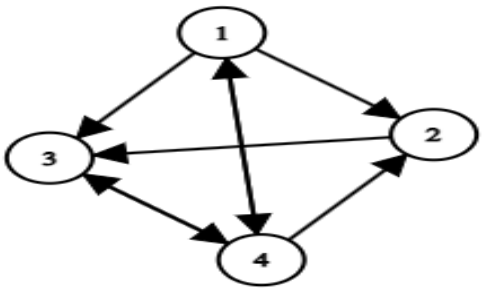
Subject Name:Discrete Mathematics

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1	**(a)**	If $A=\{x\|x=3^n,\ n\le 6, n\in N\}$ $B=\{x\|x=9^n,\ n\le 4, n\in N\}$ then find $A\cup B$, $A\cap B$ and $A-B$.	**03**
	(b)	Check whether the relation R defined in the $\{1,2,3,4,5,6\}$ as $R=\{(a,b):b=a+1\}$ is reflexive, symmetric or transitive.	**04**
	(c)	**(I)** Prove that $p\to q$ and $(\neg p\vee q)$ are logically equivalent.	**03**
		(II) Let $Y=\{n^2:n\in N\}$ and $f:N\to Y$ defined as $f(n)=n^2$. Show that f is invertible and find the inverse of f.	**04**
Q.2	**(a)**	In a survey of 400 students in a school, 100 were found as drinking apple juice, 150 as drinking orange juice and 75 drinking both apple and orange juice. Find how many students drink neither apple nor orange juice.	**03**
	(b)	Let $f:R\to R$ be function defined by $f(x)=ax+b$. Find values of a and b for which $f\circ f=I_R$	**04**
	(c)	If $R=\{(a,b):\|a-b\|=1\}$ and $S=\{(a,b):a-b\ is\ even\}$ be two relations on $A=\{1,2,3,4\}$. Then (I) Find matrices of R and S, (II) Find digraph of R and S (III) Find the relation RS	**07**
		OR	
	(c)	**(I)** Find n if ${}^nC_3+{}^{n+2}C_3={}^nP_3$	**03**
		(II) From 4 professors and 6 students, a committee of 3 is to be formed. In how many ways, this can be done, if the committee contains **(1)** at most 1 professor **(2)** at least 2 professors.	**04**
Q.3	**(a)**	Find reachable set of all the vertices and node base of following graph.	**03**



(b) Show that in a lattice if $a \le b \le c$, then **04**
(I) $a \oplus b = b * c$ and (II) $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$

(c) Solve the recurrence relation which represents the Fibonacci sequence **07**
$F_n = F_{n-1} + F_{n-2}$ with $F_0 = F_1 = 1$.

OR

Q.3 **(a)** The Indian cricket team consist of 16 players. It includes 2 wicket keepers and 5 bowlers. In how many ways eleven player can be selected if we have to select on wicket keeper and at-least 4 bowlers? **03**

(b) Prove that any graph has even number of odd vertices. **04**

(c) Find the generating function of $a_{n+2} = a_{n+1} + a_n$, where $a_0 = a_1 = 1$ for $n \ge 1$ **07**

Q.4 **(a)** If G is an abelian group with n elements $g_1, g_2, ..., g_n$ then show that $(g_1 g_2 ..... g_n)^2 = e$, where e is the identity element of G **03**

(b) Draw a binary tree whose post-order produced the string **04**
$d - g - e - b - i - j - h - f - c - a$ and pre-order produces the string
$j - b - a - c - d - i - h - e - g - f$

(c) Define Lattice. And draw the Hasse diagram representing the partial ordering $\{(A, B) : A \subseteq B\}$ on the power set $P(S)$ where $S = \{1, 2, 3\}$. Find the maximal, minimal, greatest and least elements of the poset. **07**

OR

Q.4 **(a)** Define Isolated vertex, Pendent vertex and Size of a graph **03**

(b) Let G be a graph with n vertices and m edges such that vertices have degree k or $k+1$. Prove that if G has N_k vertices of degree k and N_{k+1} vertices of degree $k+1$ then $N_k = (k+1)n - 2m$ **04**

(c) Show that G is an abelian group under usual matrix addition, where **07**
$G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a, b, c, d \in R \right\}$

Q.5 **(a)** Show that the only right coset of a subgroup H in a group G that is also subgroup of G is H itself. **03**

(b) Dose there exists a 4- regular graph with 6 vertices? If so, construct the graphs. **04**

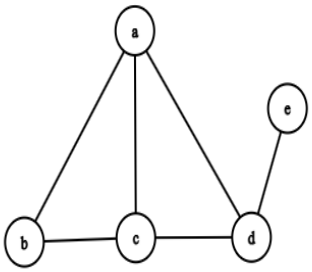
(c) Show that $(R, +, \times)$ is an integral domain, where $R = \left\{ a + b\sqrt{5} \middle| a, b \in I \right\}$ **07**

OR

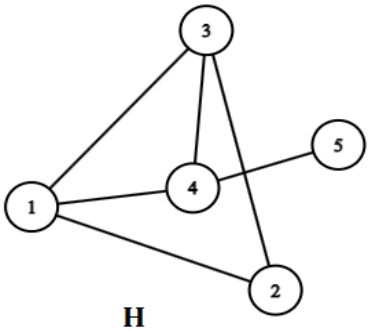
Q.5 **(a)** Define cycle, walk and tree. **03**

(b) Find transitive closure by Warshall's Algorithm if $A = \{1, 2, 3, 4, 5\}$ and **04**
$R = \{(1,2), (3,4), (4,5), (4,1), (1,1)\}$

(c) Define Isomorphic Graphs. Determine whether the following graphs are isomorphic or not. **07**



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Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2022

Subject Code:3140708 Date:02-07-2022

Subject Name:Discrete Mathematics

Time:10:30 AM TO 01:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Determine whether each of these statements is true or false. 1)$0 \in \emptyset$ 2)$\emptyset \subset \{0\}$ 3)$\{0\} \in \{0\}$ 4) $\emptyset \in \{\emptyset\}$ 5)$\{\emptyset\} \in \{\{\emptyset\}\}$ 6) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$	**03**
	(b)	Determine whether f is a function from the set of all bit strings to the set of integers if 1) $f(s)$ is the position of a 0 bit in S. 2) $f(s)$ is the number of a 1 bits in S. Find the range of each of the following functions that assigns: 3) to a bit string the number of one bits in the string 4) to each bit string twice the number of zeros in that string.	**04**
	(c)	1) Find the bitwise OR, and bitwise XOR of the bit string 1111 0000, 1010 1010 2) Show that the function $f: R \to R^+ \cup \{0\}$ defined by $f(x) = \lvert x \rvert$ is not invertible. Modify the domain or codomain of f so that it becomes invertible. 3) Let S be subset of a universal set ∪. The characteristic function f_S : ∪→ {0,1} , $f_S(x) = 1, if\ x \in S\ and\ 0\ is\ x \notin S$. Let A and B be sets. Then show that $f_{A \cap B}(x) = f_A(x).f_B(x)$	**07**
Q.2	**(a)**	Let P(x) be the statement *"$x = x^2$ "*. If the domain consists of the integers, what are the truth values of the following? 1) $\exists x\ P(x)$ 2) $\forall x \neg P(x)$ 3) $\exists x \neg P(x)$	**03**
	(b)	Identify the error or errors in this argument that supposedly shows that if $\exists x\ P(x) \wedge \exists x\ Q(x)$ is true then $\exists x\ (P(x) \wedge Q(x))$ is true. 1. $\exists x\ P(x) \wedge \exists x\ Q(x)$ Premise 2. $\exists x\ P(x)$ Simplification from (1) 3. *P(c)* Existential instantiation from (2) 4. $\exists x\ Q(x)$ Simplification from (1) 5. *Q(c)* Existential instantiation from (2) 6. $P(c) \wedge Q(c)$ Conjunction from (3) and (5) 7. $\exists x\ (P(x) \wedge Q(x))$ Existential generalization	**04**
	(c)	1) Use a truth table to verify the De Morgan's law $\neg(p \vee q) \equiv \neg p \wedge \neg q$ 2) Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.	**07**

OR

(c) 1) Suppose that the domain of Q(x, y, z) consists of triples *x, y, z*, where *x* = 0, 1, or 2, *y* = 0 or 1, *and z* = 0 or 1. Write out following propositions using disjunctions and conjunctions. **07**

i) $\exists z \neg Q(0,0,z)$ ii). $\forall y Q(0,y,0)$

2)

i) Show that $\exists x\, P(x) \wedge \exists x\, Q(x)$ and $\exists x(P(x) \wedge Q(x))$ are not logically equivalent.

ii) Show that $\exists x(P(x) \vee Q(x))$ and $\exists x\, P(x) \vee \exists x\, Q(x)$ are logically equivalent.

Q.3 **(a)** For the relation {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)} on the set $\{1,2,3,4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive..(Justify your answer if the property is not satisfied) **03**

(b) For the following relations on the set of real numbers, **04**

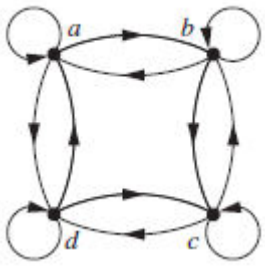
$R_1 = \{(a,b) \in R^2 | a \geq b\}$, $R_2 = \{(a,b) \in R^2 | a \leq b\}$

$R_3 = \{(a,b) \in R^2 | a \neq b\}$ find

1) $R_1 \oplus R_3$ 2) $R_2 o\, R_3$

(c) 1) Draw the Hasse diagram for the poset ({2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72}, |). Hence find $glb(\{60,72\})$ and all maximal elements. **07**

2) Determine whether the relation with the directed graph shown is an equivalence relation.



OR

Q.3 **(a)** Suppose that the relations R_1 and R_2 on a set *A* are represented by the matrices **03**

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

What are the matrices representing $R_1 \cup R_2$ and $R_1 \cap R_2$?

(b) Use Warshall's algorithm to find the transitive closures of the relation $R = \{(1,4),(2,1),(2,3),(3,1),(3,4),(4,3)\}$ on $\{1,2,3,4\}$ **04**

(c) 1) Draw the Hasse diagram for the poset({2, 4, 5, 10, 12, 20, 25}, |). Hence, find the are maximal and minimal elements. **07**

2) Which of these collections of subsets are partitions of {1, 2, 3, 4, 5, 6}? Justify your answer if it is not a partition.

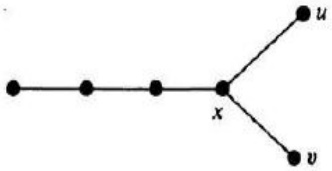
i) {1, 2}, {2, 3, 4}, {4, 5, 6}

ii) {1, 4, 5}, {2, 6}

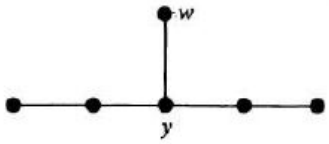
Q.4 **(a)** Explain Path and Circuit of a graph. **03**

(b) 1) Define Isomorphic Graphs **04**

2) Verify the following graphs are Isomorphic or not (Justify).



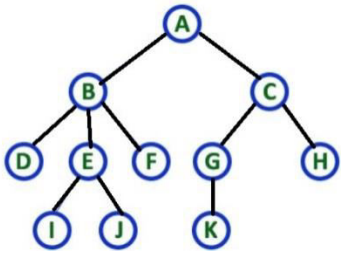
Graph -1



Graph-2

(c) 1) Define Subtree and Degree of a Node **07**

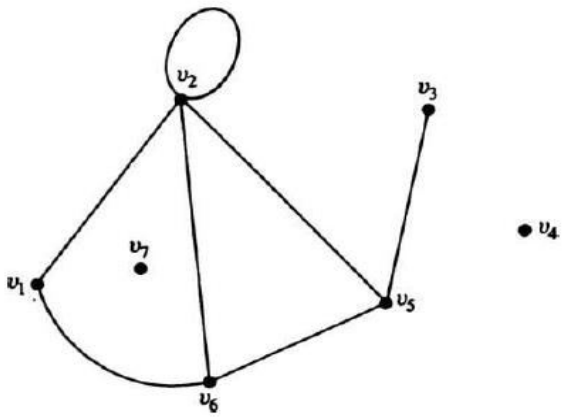
2) Determine degree of the each node for the following tree.



OR

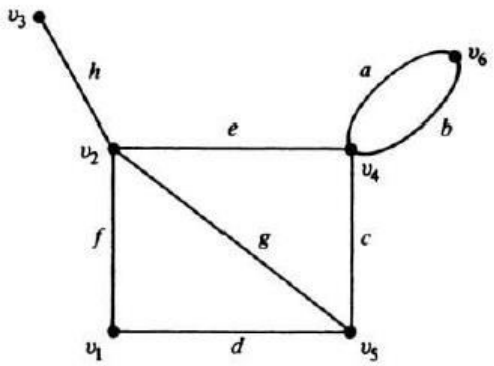
Q.4 **(a)** 1) Define: i) Isolated vertex ii) Null graph **03**

2) Identify Isolated vertex/vertices from the following graph



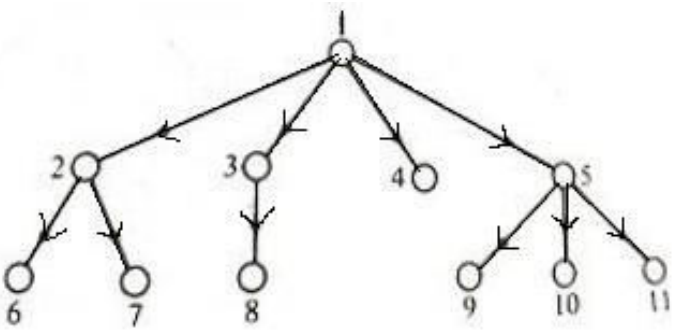
(b) 1) Define incidence Matrix of a Graph **04**

2) Find incidence matrix for the following graph



(c) 1) Define: M-ary Tree and Binary Tree. **07**

2) Represent the following directed tree as Binary tree



Q.5 **(a)** 1) Define: SemiGroups. **03**

2) Let S = {a, b, c, d}. The following multiplication table, define operations · on S. Is ⟨S, ·⟩semigroup? Justify

·	a	b	c	d
a	a	b	c	d
b	b	a	a	b
c	c	b	a	a
d	d	a	a	a

(b) Let $H = \{1, -1\}$ and $G = \{1, -1, i, -i\}$. $(H,\times)$ is a sub-group $(G,\times)$. Then find all left cosets and right cosets of H in G. **04**

(c) 1) Define:Ring. **07**

2) Write elements of the ring $\langle z_{10},\ +_{10},\ \times_{10}\rangle$. And find $-3, -8, 3^{-1}, 4^{-1}$

OR

Q.5 **(a)** Consider the set Q of a rational numbers. Let $*$ be the operation on Q defined by $a * b = a + b - ab$. Find **03**

1) $3 * 4$ 2) the identity element for $*$.

(b) Write the equivalence classes for congruence modulo 4 i.e.. z_4 **04**

Let the subset H={[0],[2]} is a subgroup of $G = \langle z_4, +_4 \rangle$. Then determine all left cosets of H in G.

(c) We are given the ring $\langle \{a, b, c, d\}, +, \cdot \rangle$ the operations $+$ and $\cdot$ on R are as shown in the following table. **07**

+	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

·	a	b	c	d
a	a	a	a	a
b	a	c	a	c
c	a	a	a	a
d	a	c	a	a

1) Does it have an identity?

2) What is the zero of this ring?

3) Find the additive inverse of each of its elements

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