Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – WINTER 2023

Subject Code:3130107 Date:12-01-2024

Subject Name:Partial Differential Equations and Numerical Methods

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Marks

Q.1 (a) Find the real root of the equation $f(x) = 2x - \log_{10} x = 0$ correct to four decimal places by iterative method. **03**

(b) Solve the given system of Linear equations by using Gauss Elimination method: **04**

$$3x + y - z = 3, 2x - 8y + z = -5, x - 2y + 9z = 8$$

(c) Explain the Newton-Raphson method with its graphical interpretation. Also, compute the real roots of the equation $f(x) = x\log_{10} x - 1.2$ correct upto three decimal places using Newton-Raphson method. **07**

Q.2 (a) Fit a least square geometric curve $y = ax^b$ to the following data **03**

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

(b) If P is the pull required to lift a load W by means of a pulley block, find a linear approximation of the form $P = mW + c$ connecting P *and* W, using the following data. **04**

P	13	18	23	27
W	51	75	102	119

(c) Solve the following system of equation using Gauss-Seidel method: **7**

$$2x + y + z = 4, x + 2y + z = 4, x + y + 2z = 4.$$

OR

(c) Using Newton's divided difference formula, evaluate $f(8)$ *and* $f(15)$ **7**

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

Q.3 (a) Using Newton's forward Interpolation formula, find $y(8)$ from the following table: **03**

x	0	5	10	15	20	25
y	7	11	14	18	24	32

(b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $\frac{3}{8} - rule$ taking $h = \frac{1}{6}$. **04**

(c) Explain Euler's method briefly and using it evaluate $y(1)$ for $\frac{dy}{dx} = x + y | y(0) = 1$ by taking $h = 0.2$ using improved Euler's method. **07**

OR

Q.3 (a) Use trapezoidal rule to evaluate $\int_0^1 x^3 dx$considering four subintervals. **03**

(b) Find the real root of the equation $xe^x - 3 = 0$ by Regula Falsi method, correct to three decimal places. **04**

(c) Apply Runge –Kutta fourth order method to obtain $y(1)$ by taking $h = 0.2$ for the following Initial value problem $\frac{dy}{dx} = x + y | y(0) = 1$. **07**

Q.4 (a) Using Newton's Backward interpolation formula compute $f(7.5)$ from the following table **03**

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	27	64	125	216	343	512

(b) State the Lagrange's linear partial differential equation of first order and hence solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$. **04**

(c) Solve the following equation by the method of separation of variables. $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$ given $(x, 0) = 4e^{-x}$. **07**

OR

Q.4 (a) Solve **03**

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y.$$

(b) Using Taylor's method to obtain $y(0.2)$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x | y(0) = 0.$ **04**

(c) State Lagrange's interpolation formula and using it find the value of $y(5)$, from the following table: **07**

x	1	2	3	4	7
y	2	4	8	16	128

Q.5 (a) Form a partial differential equation by eliminating the arbitrary constants from $z = (x^2 + a)(y^2 + b)$ **03**

(b) Obtain the general solution of $pq = p + q$ **04**

(c) Using the method of separation of variables, solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u = 0$ subject to condition $u = 0$ and $\frac{\partial u}{\partial x} = 1 + e^{-3y}$ when $x = 0$ $for\ all\ y$. **07**

OR

Q.5 (a) Form a partial differential equation by eliminating the arbitrary function form $f(x + y + z, x^2 + y^2 + z^2) = 0.$ **03**

(b) Solve by using Charpit's method $2zx - px^2 - 2pxy + pq = 0$ **04**

(c) Solve the following IVP **07**

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$u(x, 0) = 0\ for\ 0 < x < l.$$

$$\frac{\partial u}{\partial t}(x, 0) = g(x)\ for\ 0 < x < l.$$

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