Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III(NEW) EXAMINATION – WINTER 2022

Subject Code:3130107 Date:20-02-2023

Subject Name:Partial Differential Equations and Numerical Methods

Time:02:30 PM TO 05:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Marks

Q.1 (a) Compute the real root of the equation $x^4 - x - 9$ by Newton-Raphson method correct to three decimal places. **03**

(b) Solve the given System of Linear equations by using Gauss Elimination method: **04**

$$3x + 4y - z = 8; -2x + y + z = 3;\ x + 2y - z = 2$$

(c) State the difference between Regula false position method and Secant method. Use secant method to find the root of the equation $f(x) = xlogx_{10} - 1.9 = 0$ at the end of 3rd iteration and correct upto 4 decimal places. **07**

Q.2 (a) Using method of least squares fit a straight line to the following data: **03**

x	1	2	3	4	5	6
y	1200	900	600	200	110	50

(b) Using the method of least squares fit the non-linear curve of the form $y = ae^{bx}$ to the following data. **04**

x	0	1	2
y	5.012	10	31.62

(c) Solve the following system of equation using Gauss-Seidel method: **07**

$$8x + y + z = 8, 2x + 4y + z = 4, x + 3y + 5z = 5.$$

OR

(c) Compute $f(9.5)$ from the following table using the Newton's divided difference formula. **07**

x	8	9	9.2	11
$f(x)$	2.079442	2.197225	2.219203	2.397895

Q.3 (a) State the formula for Cubic spline interpolation. **03**

(b) Using Trapezoidal rule to evaluate $\int_0^1 x^3 dx$ considering five subintervals. **04**

(c) Explain Euler's method briefly and using it evaluate $y(0.1)$ for $\frac{dy}{dx} = x + y + xy | y(0) = 1$ by taking $h = 0.025$. **07**

OR

Q.3 (a) State Trapezoidal rule with $n = 10$ and evaluate $\int_0^1 e^x dx$ and also compare with exact answer. **03**

(b) Find the real root of the equation $x^3 - x - 1 = 0$ correct to two decimal places by iteration method. **04**

(c) Apply Runge –Kutta fourth order method to obtain $y(1.2)$ by taking $h = 0.1$ for IVP $\frac{dy}{dx} = x^2 + y^2 | y(1) = 1.5$. **07**

Q.4 **(a)** Using Newton's forward interpolation formula compute $\cosh 0.56$ from the following table **03**

x	0.5	0.6	0.7	0.8
$\cosh x$	1.127626	1.185465	1.255169	1.337435

(b) State the Lagrange's linear partial differential equation of first order and hence solve $pz - qy = z^2 + (x+y)^2$. **04**

(c) Solve the equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables. **07**

OR

Q.4 **(a)** Solve **03**

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y.$$

(b) Using Taylor's method $\frac{dy}{dx} = x + y | y(0) = 1$ at $x = 0.2$ correct to three decimal places. **04**

(c) Using Lagrange's interpolation formula, find the value of $x = 10$, from the following table: **07**

x	5	6	9	11
y	12	13	14	16

Q.5 **(a)** Form a partial differential equation by eliminating the arbitrary constants from $z = (x^2 + a)(y^2 + b)$. **03**

(b) Obtain the general solution of $p^2 + q^2 = 2$.. **04**

(c) Using the method of separation of variables, solve **07**

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \; given \; that \; u(x,0) = 6e^{-3x}.$$

OR

Q.5 **(a)** Form a partial differential equation by eliminating the arbitrary function form $f(x + y + z, x^2 + y^2 + z^2) = 0$. **03**

(b) Solve by using Charpit's method $px + qy = pq$. **04**

(c) Solve the following IVP **07**

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$u(x,0) = f(x) \; for \; 0 < x < l.$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0 = \left.\frac{\partial u}{\partial x}\right|_{x=L} \; for \; t > 0.$$