Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – SUMMER 2024

Subject Code:3130107 **Date:16-07-2024**

Subject Name:Partial Differential Equations and Numerical Methods

Time:10:30 AM TO 01:00 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Marks

Q.1 **(a)** Set up a Newton Raphson method for computing the square root x of a given positive number c and apply it to $c = 2$. **03**

(b) Solve the given System of Linear equations by using Gauss Elimination method: **04**

$$8y + 2z = -7; 3x + 5y + 2z = 8;\ 6x + 2y + 8z = 26$$

(c) State the difference between Regula false position method and Secant method. Use secant method to find the positive root of the equation $f(x) = x - 2\sin x = 0$ starting from $x_0 = 2, x_1 = 1.9$ correct upto 6 decimal places. **07**

Q.2 **(a)** Using method of least squares fit a straight line to the following data: **03**

x	0	5	10	15	20	25
y	12	15	17	22	24	30

(b) Using the method of least squares fit the non-linear curve of the form $y = ae^{bx}$ to the following data. **04**

x	1	5	7	9	12
y	10	15	12	15	21

(c) Solve the following system of equation using Gauss-Seidel method: **7**

$$2x - y + 2z = 3, x + 3y + 3z = -1, x + 2y + 5z = 1.$$

OR

(c) Compute $f(9.2)$ from the following table using the Newton's divided difference formula. **7**

x	8	9	9.5	11
$f(x)$	2.079442	2.197225	2.251292	2.397895

Q.3 **(a)** State Newton's forward difference Interpolation formula. **03**

(b) Using Trapezoidal rule to evaluate $\int_0^1 e^{-x^2} dx$ considering n=10 subintervals. **04**

(c) Explain Euler's method briefly and using it evaluate $y(1)$ for $\frac{dy}{dx} = x + y|y(0) = 0$ by taking $h = 0.2$. **07**

OR

Q.3 **(a)** Using Simpson's-$\frac{1}{3}$ rule to evaluate $\int_0^1 e^{-x^2} dx$ considering n=10 subintervals. **03**

	(b)	Find the real root of the equation $x^3 - 3x + 1 = 0$ correct to three decimal places by iteration method using $x_0 = 1$.	**04**
	(c)	Apply Runge –Kutta fourth order method to obtain $y(1.2)$ by taking $h = 0.5$ for IVP $\frac{dy}{dx} = \frac{y-x}{y+x} \| y(0) = 1$.	**07**
Q.4	**(a)**	Using Newton's forward interpolation formula compute $\cosh 0.56$ from the following table	**03**

x	0.5	0.6	0.7	0.8
$\cosh x$	1.127626	1.185465	1.255169	1.337435

(b) State the Lagrange's linear partial differential equation of first order and hence solve $pz - qy = z^2 + (x + y)^2$. **04**

(c) Solve the equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables. **07**

OR

Q.4 **(a)** Solve **03**

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y.$$

(b) Use Euler's method to find $y(2)$ from the differential equation $\frac{dy}{dx} = x + 2y, y(1) = 1$. **04**

(c) Using Lagrange's interpolation formula, find the value of $f(9.2)$, from the following table: **07**

x	9.0	9.5	11.0
y	2.1972	2.2513	2.3979

Q.5 **(a)** Form a partial differential equation by eliminating the arbitrary constants from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}.+$ **03**

(b) Obtain the general solution of $p^2 + q^2 = 2.$. **04**

(c) Using the method of separation of variables, solve **07**

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \; given \; that \; u(x, 0) = 6e^{-3x}.$$

OR

Q.5 **(a)** Form a partial differential equation by eliminating the arbitrary function form $f(x + y + z, x^2 + y^2 + z^2) = 0$. **03**

(b) Solve by using Charpit's method $p^2. -q^2 = x - y$. **04**

(c) Solve the following IVP **07**

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$u(x, 0) = f(x) \; for \; 0 < x < l.$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0 = \left.\frac{\partial u}{\partial x}\right|_{x=L} \; for \; t > 0.$$

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