Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III(NEW) EXAMINATION – SUMMER 2023

Subject Code:3130107 Date:24-07-2023

Subject Name:Partial Differential Equations and Numerical Methods

Time:02:30 PM TO 05:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Marks

Q.1 **(a)** Find the real root of the equation $f(x) = x^3 - x - 1 = 0$ correct to two decimal places by iterative method. **03**

(b) Solve the given system of Linear equations by using Gauss Elimination method: **04**

$$3x + y - z = 3, 2x - 8y + z = -5, x - 2y + 9z = 8$$

(c) Explain the Newton-Raphson method with its graphical interpretation. Also, compute the real roots of the equation $f(x) = xe^x - \cos x = 0$ correct upto three decimal places using Newton-Raphson method. **07**

Q.2 **(a)** Using method of least squares fit a straight line to the following data: **03**

x	2003	2004	2005	2006	2007
y	35	56	79	80	40

(b) Fit the curve $y = ax^b$ to the following data. **04**

x	61	26	7	2.6
y	350	400	500	600

(c) Solve the following system of equation using Gauss-Seidel method: **7**

$$2x + y + z = 4, x + 2y + z = 4, x + y + 2z = 4.$$

OR

(c) Compute $f(9.2)$ from the following table using the Newton's divided difference formula. **7**

x	8	9	9.2	11
$f(x)$	2.079442	2.197225	2.251292	2.397895

Q.3 **(a)** State the Newton's forward interpolation formula. **03**

(b) Use Trapezoidal rule to evaluate $\int_4^8 \frac{dx}{x}$, using for equal sub-intervals. **04**

(c) Explain Euler's method briefly and using it evaluate $y(1)$ for $\frac{dy}{dx} = x + y|y(0) = 1$ by taking $h = 0.2$ using improved Euler's method. **07**

OR

Q.3 **(a)** Evaluate $\int_0^1 e^{-x^2} dx$ using Simpson's-$\frac{1}{3}$ rule by taking $h = 0.1$ $n = 10$. **03**

(b) Find the positive root of the equation $x - 2\sin x = 0$ by the secant method starting from $x_0 = 2, x_1 = 1.9$. **04**

(c) **07**

Apply Runge –Kutta fourth order method to obtain $y(1)$ by taking $h = 0.2$ for the following Initial value problem $\frac{dy}{dx} = x + y|y(0) = 1$.

Q.4 **(a)** Using Newton's forward interpolation formula compute $\cosh 0.56$ from the following table **03**

x	0.5	0.6	0.7	0.8
$\cosh x$	1.127626	1.185465	1.255169	1.337435

(b) State the Lagrange's linear partial differential equation of first order and hence solve $\frac{y^2 z}{x} p + xzq = y^2$. **04**

(c) Solve the following equation by the method of separation of variables. $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ given $(x, 0) = 6e^{-3x}$. **07**

OR

Q.4 **(a)** Solve **03**

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y.$$

(b) Using Taylor's method $\frac{dy}{dx} = x^2 - y | y(0) = 1$ at $x = 0.1, 0.2$. **04**

(c) Using Lagrange's interpolation formula, find the value of $x = 10$, from the following table: **07**

x	5	6	9	11
y	12	13	14	16

Q.5 **(a)** Form a partial differential equation by eliminating the arbitrary constants from $2z = \frac{x^2}{a^2} + \frac{y^2}{b}$. **03**

(b) Obtain the general solution of $\sqrt{p} + \sqrt{q} = 1$. **04**

(c) Using the method of separation of variables, solve **07**

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

OR

Q.5 **(a)** Form a partial differential equation by eliminating the arbitrary function form $f(x + y + z, x^2 + y^2 + z^2) = 0$. **03**

(b) Solve by using Charpit's method. $(p^2 + q^2)y = qz$. **04**

(c) Solve the following IVP **07**

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$u(x, 0) = f(x) \; for \; 0 < x < l.$$

$$\frac{\partial u}{\partial t}(x, 0) = 0 \; for \; 0 < x < l.$$