Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III (NEW) EXAMINATION – SUMMER 2022

Subject Code:3130107 Date:11-07-2022

Subject Name:Partial Differential Equations and Numerical Methods

Time:02:30 PM TO 05:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Marks

Q.1 **(a)** Compute the real root of the equation $x \log_{10} x = 1.2$ correct upto three decimal places. **03**

(b) Solve the given System of Linear equations by using Gauss Elimination method: **04**

$$2x - 6y + 8z = 24; 5x + 4y - 3z = 2;\ 3x + y2z = 16$$

(c) State the difference between Regula false position method and Secant method. Use secant method to find the root of the equation $f(x) = xlog_{10} - 1.9 = 0$ at the end of 3rd iteration and correct upto 4 decimal places. **07**

Q.2 **(a)** Using method of least squares fit a straight line to the following data: **03**

x	1	2	3	4	5	6
y	1200	900	600	200	110	50

(b) Using the method of least squares fit the non-linear curve of the form $y = ae^{bx}$ to the following data. **04**

x	0	1	2
y	5.012	10	31.62

(c) Solve the following system of equation using Gauss-Seidel method: **7**

$$2x + y + z = 4, x + 2y + z = 4, x + y + 2z = 4.$$

OR

(c) Compute $f(9.5)$ from the following table using the Newton's divided difference formula. **7**

x	8	9	9.2	11
$f(x)$	2.079442	2.197225	2.219203	2.397895

Q.3 **(a)** State the formula for Cubic spline interpolation. **03**

(b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's-$\frac{1}{3}$ rule by taking $h = \frac{1}{4}$. **04**

(c) Explain Euler's method briefly and using it evaluate $y(1)$ for $\frac{dy}{dx} = x + y|y(0) = 1$ by taking $h = 0.1$. **07**

OR

Q.3 **(a)** State Trapezoidal rule with $n = 10$ and evaluate $\int_0^1 e^x dx$ and also compare with exact answer. **03**

(b) Find the real root of the equation $2x - \log_{10} x = 7$ correct to three decimal places by iteration method. **04**

(c) Apply Runge –Kutta fourth order method to obtain $y(1)$ by taking $h = 0.2$ for the following Initial value problem $\frac{dy}{dx} = x + y|y(0) = 1$. **07**

Q.4 **(a)** Using Newton's forward interpolation formula compute $\cosh 0.56$ from the following table **03**

x	0.5	0.6	0.7	0.8
$\cosh x$	1.127626	1.185465	1.255169	1.337435

(b) State the Lagrange's linear partial differential equation of first order and hence solve $(x^2 - yz)p + (x^2 - zx)q = (z^2 - xy)$. **04**

(c) Solve the equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables. **07**

Q.4 **(a)** Solve **03**

$$\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial^2 x \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}.$$

(b) Using Taylor's method $\frac{dy}{dx} = x^2 - y|y(0) = 1$ at $x = 0.1, 0.2$. **04**

(c) Using Lagrange's interpolation formula, find the value of $x = 10$, from the following table: **07**

x	5	6	9	11
y	12	13	14	16

Q.5 **(a)** Form a partial differential equation by eliminating the arbitrary constants from $z = (x^2 + a)(y^2 + b)$. **03**

(b) Obtain the general solution of $pq = p + q$. **04**

(c) Using the method of separation of variables, solve **07**

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \text{ given that } u(x,0) = 6e^{-3x}.$$

Q.5 **(a)** Form a partial differential equation by eliminating the arbitrary function form $f(x + y + z, x^2 + y^2 + z^2) = 0$. **03**

(b) Solve by using Charpit's method. $(p^2 + q^2)y = qz$. **04**

(c) Solve the following IVP **07**

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

$$u(x,0) = f(x) \text{ for } 0 < x < l.$$

$$\frac{\partial u}{\partial t}(x,0) = 0 \text{ for } 0 < x < l.$$

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