Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–III (NEW) EXAMINATION – WINTER 2024

Subject Code:3130005 Date:21-11-2024

Subject Name: Complex Variables and Partial Differential Equations

Time: 10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Find the principal argument $Arg(z)$ when $z=\dfrac{-2}{1+i\sqrt{3}}$.	**03**
	(b)	Find all the values of following: $(1-i)^{\frac{2}{3}}$.	**04**
	(c)	Find the bilinear transformation which maps the points $z=0,-i,-1$ into $w=i,1,0$ respectively.	**07**
Q.2	**(a)**	Show that $f(z)=z^3$ is analytic everywhere.	**03**
	(b)	Evaluate $\int(x^2-iy^2)dz$ along the parabola $y=2x^2$ from (1,2) to (2,8).	**04**
	(c)	Find Laurent's series expansion in powers of z that represent $f(z)=\dfrac{1}{z^2(1-z)}$ for the following domains: (i) $\lvert z\rvert<1$ (ii) $\lvert z\rvert>1$	**07**
		OR	
	(c)	Find the image of the half-plane $x>c$, *when* $c>0$ under the transformation $w=\dfrac{1}{z}$. Show the regions graphically.	**07**
Q.3	**(a)**	If $u+iv$ is analytic, show that $v-iu$ and $-v+iu$ are also analytic.	**03**
	(b)	Show that $u(x,y)=x^2-y^2+x$ is harmonic. Find the corresponding analytic function $f(z)=u+iv$.	**04**
	(c)	Evaluate $\int_C \dfrac{\cos\pi z}{z^2-1}dz$, where C is the rectangle whose vertices are $2\pm i$, $-2\pm i$.	**07**
		OR	
Q.3	**(a)**	Obtain the residue of $f(z)=\dfrac{z-3}{(z+1)(z+2)}$ at it's poles.	**03**
	(b)	Evaluate $\int_C \dfrac{e^{2z}}{(z+1)^4}dz$, where C is the circle $\lvert z\rvert=2$.	**04**
	(c)	Evaluate $\int_0^\infty \dfrac{x\sin x}{x^2+9}dx$ using residue.	**07**
Q.4	**(a)**	Form a partial differential equation for the equation $(x-a)(y-b)-z^2=x^2+y^2$	**03**
	(b)	Solve: $xp+yq=3z$	**04**

	(c)	Solve by Charpit's Method: $p = \left(z + qy\right)^2$	**07**
		OR	
Q.4	**(a)**	Form a partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$	**03**
	(b)	Solve $x(y-z)p + y(z-x)q = z(x-y)$	**04**
	(c)	Solve $(D^2 - 2DD' + D'^2)z = e^{x+2y} + x^3$	**07**
Q.5	**(a)**	Solve: $p - x^2 = q + y^2$	**03**
	(b)	Solve $(D^2 - 2DD' + D'^2)z = \tan(x+y)$	**04**
	(c)	Solve $(D^2 + DD' - 6D'^2)z = \sin(2x+y)$	**07**
		OR	
Q.5	**(a)**	Solve: $(p-q)(z - px - qy) = 1$	**03**
	(b)	Solve: $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 2\frac{\partial^3 z}{\partial y^3} = 0$	**04**
	(c)	Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u,\ u(x,0) = 6e^{-3x}$	**07**

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