Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – WINTER 2023

Subject Code:3130005 Date:12-01-2024

Subject Name:Complex Variables and Partial Differential Equations

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Marks

Q.1 **(a)** Express $\left(\frac{2+i}{3-i}\right)^2$ into Polar form. **03**

(b) Find and plot the fourth roots of (-1). **04**

(c) Solve $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$ **07**

Q.2 **(a)** Determine a and b such that $u = ax^3 + bxy$ is harmonic. **03**

(b) Discuss the continuity of $f(z)$ at the origin. **04**

$$f(z) = \frac{\bar{z}}{z}, if \quad z \neq 0$$
$$= 0, if \quad z = 0$$

(c) Show that the function $u = e^x cosy$ is harmonic. Find the conjugate function v and express $u + iv$ as an analytic function of z. **07**

OR

(c) Find the bilinear transformation which maps the points $1, -1, \infty$ onto the points $1 + i, 1 - i, 1$ respectively. Also, find its fixed points. **07**

Q.3 **(a)** Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{z^n}{2^n+1}$. **03**

(b) Separate $\left(\sqrt{i}\right)^{\sqrt{i}}$ into real and imaginary parts. **04**

(c) State Cauchy's Integral Theorem and use it to find $\int_c \frac{e^{2z}}{z^2+1} dz$, where C is $|z| = \frac{1}{2}$. **07**

OR

Q.3 **(a)** Evaluate $\int_0^{2+i} z^2\, dz$ along the line $y = \frac{x}{2}$. **03**

(b) Expand $f(z) = cosz$ as a Taylor series about $z = 0$. **04**

(c) Write Cauchy's Integral formula and hence evaluate: **07**

$\int_c \frac{sin\pi z^2 + cos\pi z^2}{(z-1)(z-2)} dz$, where C is $|z| = 3$.

Q.4 **(a)** Classify the singular point $z = 0$ for function $f(z) = \frac{1}{z^4 - 4z^2}$. **03**

(b) Find the complete integral of $p - 3x^2 = q^2 - y$ **04**

(c) Obtain the Laurent's series for the function $f(z) = \frac{1}{z(1-z)}$ in the regions : **07**

(i) $|z + 1| < 1$ (ii) $1 < |z + 1| < 2$ (iii) $|z + 1| > 2$

OR

Q.4 **(a)** Derive partial differential equation by eliminating a and b from $z = (x - a)^2 + (y - b)^2$. **03**

	(b)	Using Cauchy's residue theorem, evaluate $\int_C \frac{5z-2}{z(z-1)}\, dz$; $\lvert z\rvert = 2$.	**04**
	(c)	Solve $x(y-z)p + y(z-x)q = z(x-y)$.	**07**
Q.5	**(a)**	Evaluate $\int_0^\infty \frac{1}{1+x^2}\, dx$ using contour integration.	**03**
	(b)	Solve $\left(D^2 - 4DD' + 4D'^2\right)z = e^{2x+3y}$.	**04**
	(c)	Find the solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ such that $y = a\, cospt\ when\ x = l, and\ y = 0\ when\ x = 0$.	**07**

OR

Q.5	**(a)**	Solve $(2D^2 + 5DD' + 2D'^2)z = 0$.	**03**
	(b)	Solve $p(1+q) = qz$.	**04**
	(c)	Using the method of separation of variable, find the solution of $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u,\ u(x,0) = 6e^{-3x}$.	**07**

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