Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – SUMMER 2024

Subject Code:3130005 **Date:16-07-2024**

Subject Name: Complex Variables and Partial Differential Equations

Time:10:30 AM TO 01:00 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Represent $\frac{(1+i)^2}{1-i}$ in a+ib or u+iv form and find its modulus and argument	**03**
	(b)	Define a harmonic function. Show that $u(x,y) = x^2 - y^2$ is harmonic and find the corresponding analytic function f(z) = u(x, y) + iv(x, y)	**04**
	(c)	Find the bilinear transformation which maps the points z=i,1, -i onto the points$w$=-i,1,i respectively.	**07**
Q.2	**(a)**	Evaluate $\int_0^{4+2i} \bar{z}\, dz$ along the curve $z = t^2 + it$	**03**
	(b)	Find the centre and radius of convergence of the given power series. $$\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2}(z-3i)^n$$	**04**
	(c)	Define analytic function and If f(z) is analytic and $\left|f(z)\right| = c$ then show that f(z) is constant	**07**
		OR	
	(c)	Evaluate $\oint_C \frac{\cos \pi z^2}{(z-1)(z-2)}\, dz;\ C\ is\ \lvert z\rvert = 3$	**07**
Q.3	**(a)**	Define: Singular point, Isolated singular point, Residue and Explain types of isolated singular points.	**03**
	(b)	Find the Laurent series of $\frac{7z-2}{(z+1)\ z\ (z-2)}$; $1 < \lvert z+1\rvert < 3$.	**04**
	(c)	Evaluate using Cauchy residue theorem $\oint_c \frac{2z+6}{z^2+4}dz$ where C:$\lvert z-i\rvert = 2$	**07**
		OR	
Q.3	**(a)**	Find the pole and its order of following functions 1. $f(z) = \frac{\sin z}{z^4}$ 2. $f(z) = \frac{1}{(z-5)^3(z^2-4)}$	**03**
	(b)	Find the residues at singular points of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$	**04**

	(c)	Evaluate following real integration using residue theorem. $$\int_0^{2\pi} \frac{1}{(2+cos\theta)^2} d\theta$$	**07**
Q.4	**(a)**	Form PDE by eliminating arbitrary Functions $F(x+y+z, x^2+y^2-z^2)=0$	**03**
	(b)	Solve following Linear Partial Differential Equations 1) $xp+yq=x-y$ 2) $(z-y)p+(x-z)q=(y-x)$	**04**
	(c)	Solve following Non- linear partial differential equations using Charpit's method $px+qy=pq$	**07**

OR

Q.4	**(a)**	Find the order of the following PDE (1 to 3). And check whether the equations are linear, quasilinear or nonlinear. 1) $\frac{\partial u}{\partial x}-\frac{\partial u}{\partial t}=0$, 2) $\frac{\partial^2 u}{\partial x^2}-e^{2x}\frac{\partial^2 u}{\partial t^2}=u^3$,3) $u_y u_{yy}+(u_x)^2=0$	**03**
	(b)	Solve following Non- linear partial differential equations. 1) $p^2-q^2=x-y$ 2) $p(1+q)=qz$	**04**
	(c)	Solve following Partial Differential Equations 1) $(x^2-y^2-z^2)p+2xyq=2xz$ 2) $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}-8\frac{\partial^2 z}{\partial y^2}=0$	**07**
Q.5	**(a)**	Classify second order homogeneous partial differential equations as elliptic, parabolic or hyperbolic 1) $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}-12\frac{\partial^2 z}{\partial y^2}=0$ 2) $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}=0$	**03**
	(b)	Using method of separation of variables solve $\frac{\partial u}{\partial t}+u=2\frac{\partial u}{\partial x}$ given that $u(x,0)=4e^{-3x}$	**04**
	(c)	Determine solution of two-dimensional Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$. which satisfy the condition $u(0,y)=u(l,y)=u(x,0)$ and $u(x,b)=sin\frac{n\pi x}{l}$	**07**

OR

Q.5	**(a)**	Solve partial differential equations by direct integration method $\frac{\partial^2 z}{\partial x^2}=\sin x$	**03**
	(b)	Solve second order homogeneous PDEs $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}-12\frac{\partial^2 z}{\partial y^2}=3e^{2x-3y}$	**04**
	(c)	Find the solution of One- Dimensional Wave Equation.	**07**

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