

Enrollment No./Seat No.:

## GUJARAT TECHNOLOGICAL UNIVERSITY

Bachelor of Engineering - SEMESTER - 1/2 EXAMINATION - WINTER 2025

Subject Code: BE02000011

Date: 23-12-2025

Subject Name: Mathematics – 2

Time: 02:30 PM TO 05:30 PM

Total Marks: 70

### Instructions

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

|  | Marks |
|--|-------|
| <b>Q.1 (a)</b> Check whether the function $e^{\bar{z}}$ analytic or not at any point ?   | 03    |
| <b>(b)</b> Solve $x \log x \frac{dy}{dx} + y = 2 \log x$   | 04    |
| <b>(c)</b> Diagonalize the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$  | 07    |
| <b>Q.2 (a)</b> State Cauchy integral formula and Evaluate $\oint_C \frac{dz}{z(z+3)}$ Where C is the circle $ z  = 1$  | 03    |
| <b>(b)</b> Solve $y'' - 4y = 0, y(0) = 2, y'(0) = -1$  | 04    |
| <b>(c)</b> (i) Evaluate $\int_C (x - y + ix^2) dz$ Where C is a straight line from $z=0$ to $z=1+i$ .<br>(2) If $\sin(\alpha + i\beta) = x + iy$ then prove that $x^2 \operatorname{sech}^2 \beta + y^2 \operatorname{cosech}^2 \beta = 1$ | 07    |
| <b>OR</b>  |       |
| <b>(c)</b> (i) Evaluate $\int_C \frac{e^z}{(z-1)^4} dz$ where C is the circle $ z  = 3$<br>(ii) Using CR equations show that $z z $ is no where analytic.  | 07    |
| <b>Q.3 (a)</b> Solve $\frac{dy}{dx} + 2xy = xe^{-x^2}$   | 03    |
| <b>(b)</b> Find the rank of the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$  | 04    |

- (c) Check whether  $u = x^2 - y^2 - y$  is harmonic or not. If so find the harmonic conjugate  $v$  of  $u$ . 07

**OR**

- (a) Test for exactness and solve  $(\tan y + x) dx + (x \sec^2 y - 3y) dy = 0$  03

- (b) Solve the following linear system by Gauss elimination method. 04  
 $x + 2y - z = 2, x - y + z = 4, 2x + y - z = 5$

- (c) Define Harmonic function. Check whether  $u = x^3 - 3xy^2$  is Harmonic or not if so construct the analytic function  $f(z) = u + iv$  07

- Q.4** (a) Solve  $(xy^2 - x) dx = (y + x^2y) dy$  03

- (b) 
$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$
 04  
 Find inverse of the matrix by Cayley Hamilton theorem.

- (c) State Cauchy residue theorem and Evaluate  $\oint_C \frac{z^3 - z^2 + z - 1}{z^3 + 4z} dz$ , Where  $C$  is 07  
 (a)  $|z| = 1$ , (b)  $|z| = 3$

**OR**

- (a) Solve  $x \frac{dy}{dx} + \frac{y^2}{x} = y$  03

- (b) 
$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$
 04  
 Find the inverse of the matrix using Gauss Jordan method.

- (c) Expand  $f(z) = \frac{1}{(z+1)(z-2)}$  in Laurent's series in the region 07  
 (1)  $|z| < 1$  (2)  $1 < |z| < 2$  (3)  $|z| > 2$

- Q.5** (a) Solve  $(x^2 + y^2 + x) dx + xy dy = 0$  03

- (b) Solve  $y'' - 2y' + y = 4 \frac{e^x}{x^2}$  04

- (c) Solve  $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$  by the method of variation of Parameters. 07

**OR**

- (a) Solve  $y = 3x + \log p$  03

- (b) Solve  $y'' + y' - 6y = 6x + 3x^2 - 6x^3$  by undetermined coefficient method 04

- (c) Find the power series solution of  $y'' + x^2y = 0$  about  $x = 0$ . 07

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