Enrollment No./Seat No.:

GUJARAT TECHNOLOGICAL UNIVERSITY

Bachelor of Engineering - SEMESTER - 1/2 EXAMINATION - WINTER 2025

Subject Code: 3110015 **Date: 03-01-2026**

Subject Name: Mathematics - 2

Time: 02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions

2. Make suitable assumptions wherever necessary.

3. Figures to the right indicate full marks.

			Marks
Q.1	**(a)**	If ϕ is a scalar field, prove that curl(grad ϕ)=0.	**03**
	(b)	Determine whether the vector field $\bar{u} = y^2\hat{\imath} + 2xy\,\hat{\jmath} - z^2\hat{k}$ is solenoidal or irrotational at the point (1,2,1)?	**04**
	(c)	Verify Green's theorem in the plane for $\bar{F} = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$ taken around the rectangle bounded by the lines $x = \pm 1, y = 0$ and $y = 2$.	**07**
Q.2	**(a)**	Define order and degree of the ordinary differential equation. Give an example of ordinary differential equation: (i) first order first degree, (ii) first order higher degree.	**03**
	(b)	Find the Laplace transform of (i) $t\cos^2 t, (ii)\frac{\sin t}{t}$.	**04**
	(c)	Apply the method of undetermined coefficients to solve the differential equation $(D^2 - 2D + 5)y = 5x^3 - 6x^2 + 6x$.	**07**
		OR	
	(c)	Apply the variation of parameters method to solve the differential equation $(D^2 - 3D + 2)y = e^x$.	**07**
Q.3	**(a)**	State first shifting theorem for Laplace transform. Find the Laplace transform of $e^{-3t}(t^2 + \sin t)$.	**03**
	(b)	Find the general solution of the differential equation $x^2y\,dx - (x^3 + xy^2)dy = 0$.	**04**
	(c)	Apply Laplace transform to solve the initial value problem: $y'' - 6y' + 9y = t^2e^{3t}, y(0) = 2, y'(0) = 6$.	**07**
		OR	
	(a)	State convolution theorem for Laplace transform. Find the inverse Laplace transform of $\frac{1}{s(s+1)}$.	**03**
	(b)	Check whether the given differential equation is exact or not: $(x^4 - 2xy^2 + y^4)dx - (2x^2y - 4xy^3 + \sin y)\,dy = 0$.	**04**
	(c)	Find the Fourier cosine integral representation of $f(x) = \frac{\pi}{2}e^{-x}, x > 0$.	**07**

Q.4 **(a)** Find the general solution of the differential equation: $y' + 2y\tan x = \sin x$. **03**

(b) Solve the Euler-Cauchy differential equation : $(4x^2D^2 + 16xD + 9)y = 0$. **04**

(c) Solve: (i) $(D^2 - 4)y = e^{2x} + e^{-4x}$. **07**

(ii) $(D^3 - 3D^2 - D + 3)y = 1$.

OR

(a) Solve: $x^2p^2 + xyp - 6y^2 = 0$. **03**

(b) If $y_1 = x$ is one solution of $x^2y'' + xy' - y = 0$, find the second solution. **04**

(c) Solve (i) $(D^2 + 9)y = \cos 4x$, (ii) $(D^3 - 5D^2 + 8D - 4)y = e^{-x}$. **07**

Q.5 **(a)** Classify the singular points of the differential equation $x^2y'' + xy' - 2y = 0$. **03**

(b) Find the series solution of $y'' = 2y'$ in powers of x. **04**

(c) Using the power-series method, solve $(1 - x^2)y'' - 2xy' + 2y = 0$. **07**

OR

(a) Discuss about ordinary point, singular point, regular singular point and irregular singular point for the differential equation $x^3(x-1)y'' + 3(x-1)y' + 7xy = 0$. **03**

(b) Derive recurrence relation for the Bessel's polynomial $J_n(x)$. **04**

(c) Find the series solution of $2x(x-1)y'' - (x+1)y' + y = 0$ at $x = 0$. **07**

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