Enrollment No./Seat No.:

GUJARAT TECHNOLOGICAL UNIVERSITY

Bachelor of Engineering - SEMESTER - 1/2 EXAMINATION - WINTER 2025

Subject Code: BE01000041/BE01R00041 **Date: 22-12-2025**

Subject Name: Mathematics-I

Time: 02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Define Improper integral of Second kind and test the convergence of the integral $\int_0^1 \frac{5dx}{(x-1)^2}$	**03**
	(b)	Define Gamma function. Prove that $\Gamma(n) = 2\int_0^\infty e^{-x^2} x^{2n-1} dx$	**04**
	(c)	Obtain the Maclaurin's series expansion of $\frac{1}{1-x}$ for $\lvert x \rvert < 1$. Using it, expand $log_e(1-x)$ in powers of x.	**07**
Q.2	**(a)**	Define Beta function. Show that $\beta(m,n) = \beta(n,m)$	**03**
	(b)	Find local extreme value of the function $f(x) = 2x^3 - 6x$	**04**
	(c)	Define geometric series and explain its convergence. Test the convergence for the following series. $(i) \sum_{n=1}^{\infty} \frac{1}{4n^2-1}$ $(ii) \sum_{n=0}^{\infty} 2^{n/2}$	**07**
		OR	
	(c)	Define nth term test for the divergence of a series. Test the convergence for the following series. $(i) \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$ $(ii) \sum_{n=1}^{\infty} nsin\left(\frac{1}{n}\right)$	**07**
Q.3	**(a)**	Evaluate : $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$	**03**
	(b)	Find the area generated by revolving the curve $y = x,\ x \in [0,1]$	**04**
	(c)	Find radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{nx^n}{2^n}$	**07**
		OR	

(a) Evaluate : $\lim_{x \to 0} \left(\frac{1}{x^2} \right)^{\tan x}$ **03**

(b) Find the volumes of the solids generated by revolving the region under the curve $y = \sqrt{x}$ over the interval [1, 4] about the x-axis. **04**

(c) Define Alternating series. Verify whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3 + 1}$ is conditionally convergent or not. **07**

Q.4 (a) Evaluate the limit : $\lim_{(x,y) \to (0,0)} \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}$ **03**

(b) Find Tangent plane and Normal line to the surface $xy^2 + 2yz^2 + 3zx^2 = 6$ at the point (1,1,1) . **04**

(c) Find the shortest and the longest distances of the point (1,1,1) from the sphere $x^2 + y^2 + z^2 = 1$ **07**

OR

(a) Discuss the continuity of $f(x, y)$ at the origin. where, **03**

$$f(x, y) = \begin{cases} \frac{2x^2 y}{x^4 + 2y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

(b) Find the directional derivative of $f(x, y) = x^2 e^{xy}$ at the point (1, 0) in the direction of $\bar{u} = \hat{i} - \hat{j}$ **04**

(c) Find critical points of the function $f(x, y) = x^3 - y^3 - 3x + 3y + 1$. Also find local maxima, local minima and saddle points, if any. **07**

Q.5 (a) Evaluate : $\iint_R (x + y)^2 \, dA \; ; \; R = [0, 1] \times [0, 2]$ **03**

(b) Evaluate the integral $\int \int r\sqrt{a^2 - r^2} dr d\theta$ over the region of the upper half of the circle $r = a \cos\theta$ **04**

(c) Evaluate the integral $\int_0^{\infty} \int_x^{\infty} e^{-y^2} dy dx$ by changing the order of the integral. **07**

OR

(a) Evaluate $\iint_R dA$ where R is the region bounded by the parabolas $y = x^2$ and $x = y^2$. **03**

(b) Evaluate : $\int_0^1 \int_0^1 \int_{\sqrt{x^2+y^2}}^2 z \, dzdydx$ **04**

(c) Find the volume of the region bounded by a paraboloid $z = 1 - x^2 - y^2$, a plane $z = 1$ and XY-plane. **07**

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