Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II EXAMINATION – WINTER 2025

Subject Code:3110014 **Date:17-01-2026**

Subject Name: Mathematics - 1

Time:02:30 PM TO 05:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Evaluate $\lim_{x\to 0}(\cos x)^{\cot x}$	3
	(b)	Discuss the convergence of the series $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\cdots$. If convergent find its sum.	4
	(c)	1. Investigate the convergence of the integral $\int_0^1 \frac{1}{1-x}\,dx$.	3
		2. Find the length of the curve $(x)=\frac{x^3}{12}+\frac{1}{x}$; $1\le x\le 4$.	4
Q.2	**(a)**	Discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{2n+1}{(n+1)^2}$	3
	(b)	Discuss the convergence of the following: 1. $\sum_{n=1}^{\infty}\frac{4^n+5^n}{6^n}$ 2. $\sum_{n=1}^{\infty}\frac{2^n}{n^3+1}$	4
	(c)	Obtain the Fourier series of the function $f(x)=\begin{cases}0 & if\ -\pi<x\le 0\\ x & if\ 0\le x<\pi\end{cases}$	7
		OR	
	(c)	Obtain the Fourier series of the function $f(x)=e^{\lvert x\rvert}$; $-2\le x\le 2$	7
Q.3	**(a)**	If $=f(x-y,y-z,z-x)$, Show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$.	3
	(b)	Find the tangent plane and normal line of the surface $x^3+y^3+z^3=3$ at the point $(1,1,1)$.	4
	(c)	Find the extreme values that the function $f(x,y)=xy$ takes on the ellipse $\frac{x^2}{8}+\frac{y^2}{2}=1$.	7
		OR	
Q.3	**(a)**	$u=2x^3y+y^3z^2$, where $x=rse^t$, $x=rs^2e^{-t}$ and $x=r^2s\sin t$, find $\frac{\partial u}{\partial s}$ at $r=1, s=1, t=0$.	3
	(b)	If $u=tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\sin 2u$.	4
	(c)	Examine the function $f(x,y)=2x^4+y^2-x^2-2y$ for maxima and minima.	7
Q.4	**(a)**	Evaluate $\int_0^1\int_0^x e^{\frac{y}{x}}\,dy\,dx$.	3
	(b)	Evaluate $\iint_R f(x,y)dA$, where $f(x,y)=6x^2+2y$ and R is the region bounded by $y=x^2$ and $y=4$.	4

(c) By using changing the order of integration, evaluate $\int_0^1 \int_{-\sqrt{1-y^2}}^{1-y} y\, dx\, dy$. **7**

OR

Q.4 **(a)** Evaluate $\int_0^{\pi/2} \int_0^{a\cos\theta} r\sin\theta\ dr\, d\theta$. **3**

(b) Evaluate $\int_0^1 \int_0^{1-y} \int_0^{1-y-z} z\, dx\, dz\, dy$. **4**

(c) Evaluate $\iint_R x\, dx\, dy$, where R is the region bounded by tringle with vertices $(0,0), (0,1)$ and $(1,1)$, using the transformations $x = u$, $y = uv$. **7**

Q.5 **(a)** Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. **3**

(b) Expand $\ln(1 + sinx)$ in powers of x up to x^3. **4**

(c) Solve the following system using Gauss-Jordan method. **7**

$x - 2y - z + 3w = 1$
$2x - 4y + z = 5$
$x - 2y + 2z - 3w = 4$

OR

Q.5 **(a)** Using row echelon form find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$. **3**

(b) Discuss the convergence of the series $\frac{1}{1+3} + \frac{2}{1+3^2} + \frac{3}{1+3^3} + \frac{4}{1+3^4} + \cdots$. **4**

(c) Find the modal matrix P and diagonal matrix D for the matrix $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$. **7**

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