SECTION-02 - BE-02

APTITUDE TEST (MATHEMATICS & SOFT SKILL)

MATHEMATICS

1. Determinant and Matrices
2. Trigonometry
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ENGLISH

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MATHEMATICS

1. Determinant and Matrices

[A] Determinant

[1] Introduction :

In standard 10, we have learned the 'Cross Multiplication' method for the solution of system of linear equations in two variables.

The setps for finding the solution of the system of linear equations $ax + by + m = 0$ and $cx + dy + n = 0$ using cross-Multiplication mthod were as follows :

Step-1 : $ax + by + m = 0$

$cx + dy + n = 0$

Step-2 : $\dfrac{x}{\begin{array}{cc} b & m \\ d & n \end{array}} = \dfrac{y}{\begin{array}{cc} a & m \\ c & n \end{array}} = \dfrac{1}{\begin{array}{cc} a & b \\ c & d \end{array}}$

Step-3 : $\dfrac{x}{bn - dm} = \dfrac{y}{an - cm} = \dfrac{1}{ad - bc}$

Step-4 : If $ad - bc \neq 0$ then,

$x = \dfrac{bn - dm}{ad - bc}$ and $y = \dfrac{an - cm}{ad - bc}$

The expression $ad - bc$ plays an important role in the above solution. The British mathematician **Cayley** introduced a speical arrangement $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ to express the number $ad - bc$ $(a, b, c, d \in R)$. Weather the solution of the equation is unique or not is determine by the value of the above expression so such expression is called '(second order) Determinant'. Thus, the determinant is a special way of expressing real numbers, so the determinant is a number.

[2] Second Order Determinant :

If a, b, c and d are four real numbers, then $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is called second order determinant.

→ a, b, c and d are called the elements of the determinant.

→ $a \quad b$ and $c \quad d$ are called the first row and second row of the determinant respectively.

→ $\begin{array}{c} a \\ c \end{array}$ and $\begin{array}{c} b \\ d \end{array}$ are called the first column and second column of the determinant respectively.

→ $\begin{array}{cc} a & \\ & d \end{array}$ and $\begin{array}{cc} & b \\ c & \end{array}$ are called the principal diagonal and the secondary diagonal of the determinant respectively.

- **Value or expansion of second order determinant :**

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

= (Product of the elements on the principal diagonal) – Product of the elements on the secondary diagonal)

e.g. $\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix} = (3 \times 5) - (4 \times 2)$

$= 15 - 8$

$= 7$

Example-1 : Find the value of the following determinants.

(i) $\begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix}$ (ii) $\begin{vmatrix} x-1 & x \\ x & x+1 \end{vmatrix}$

(iii) $\begin{vmatrix} 5+\sqrt{3} & 3-\sqrt{5} \\ 3+\sqrt{5} & 5-\sqrt{3} \end{vmatrix}$ (iv) $\begin{vmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{vmatrix}$

(v) $\begin{vmatrix} e^{2x} & e^{x} \\ 1 & e^{-x} \end{vmatrix}$

Solution :

(i) $\begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix} = (2 \times 4) - ((-3) \times 5)$

$= 8 - (-15)$

$= 8 + 15$

$= 23$

(ii) $\begin{vmatrix} x-1 & x \\ x & x+1 \end{vmatrix} = (x-1)\cdot(x+1) - (x \cdot x)$

$= x^2 - 1 - x^2 = -1$

$(\because (a-b)(a+b) = a^2 - b^2)$

(iii) $\begin{vmatrix} 5+\sqrt{3} & 3-\sqrt{5} \\ 3+\sqrt{5} & 5-\sqrt{3} \end{vmatrix}$

$= (5+\sqrt{3})(5-\sqrt{3}) - (3-\sqrt{5})(3+\sqrt{5})$

$= (5^2 - (\sqrt{3})^2) - (3^2 - (\sqrt{5})^2)$

$(\because (a-b)(a+b) = a^2 - b^2)$

$= (25 - 3) - (9 - 5)$

$= 22 - 4$

$= 18$

(iv) $\begin{vmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{vmatrix}$

$= (\sin\theta)(\sin\theta) - (-\cos\theta)(\cos\theta)$

$= \sin^2\theta + \cos^2\theta$

$= 1$ $(\because$ Identity $\sin^2\theta + \cos^2\theta = 1)$

(v) $\begin{vmatrix} e^{2x} & e^{x} \\ 1 & e^{-x} \end{vmatrix} = e^{2x} \cdot e^{-x} - e^{x} \cdot 1$

$= e^{2x-x} - e^{x}$

$= e^{x} - e^{x}$

$= 0$

Example-2 : Do as directed :

(1) If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 5$, then find the value of $\begin{vmatrix} 3a & 3b \\ 3c & 3d \end{vmatrix}$.

(2) If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 3$, then find the value of $\begin{vmatrix} 5a & b \\ 5c & d \end{vmatrix}$.

Solution :

(1) Here $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 5$

$\therefore$ $ad - bc = 5$ ...(i)

Now, $\begin{vmatrix} 3a & 3b \\ 3c & 3d \end{vmatrix} = (3a)(3d) - (3b)(3c)$

$= 9ad - 9bc$

$= 9(ad - bc)$

$= 9(5)$ $(\because$ from result (i))

$= 45$

(2) Here $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 3$

$\therefore$ $ad - bc = 3$ ...(i)

Now, $\begin{vmatrix} 5a & b \\ 5c & d \end{vmatrix} = (5a)(d) - (b)(5c)$

$= 5ad - 5bc$

$= 5(ad - bc)$

$= 5(3)$ $(\because$ from result (i))

$= 15$

Example-3 : Solve the equation :

(i) $\begin{vmatrix} x & 2 \\ 2 & 1 \end{vmatrix} = 0$

(ii) $\begin{vmatrix} x & 1 \\ 4 & 2 \end{vmatrix} = 0$

Solution :

(i) Here, $\begin{vmatrix} x & 2 \\ 2 & 1 \end{vmatrix} = 0$

$\therefore$ $(x)(1) - (2)(2) = 0$

$\therefore\ x - 4 = 0$

$\therefore\ x = 4$

(ii) Here, $\begin{vmatrix} x & 1 \\ 4 & 2 \end{vmatrix} = 0$

$\therefore\ (x)(2) - (1)(4) = 0$

$\therefore\ 2x - 4 = 0$

$\therefore\ 2x = 4$

$\therefore\ x = \frac{4}{2} = 2$

[3] Third Order Determinant :

If $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2$ and c_3 are nine real numbers, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ is called third order determinant.

→ Nine real numbers a_i, b_i, c_i $(i = 1, 2, 3)$ are called the elements of the determinant.

→ a_1, a_2, a_3; b_1, b_2, b_3 and c_1, c_2, c_3 are called first row, second row and third row respectively.

→ $\begin{matrix} a_1 \\ b_1 \\ c_1 \end{matrix}$; $\begin{matrix} a_2 \\ b_2 \\ c_2 \end{matrix}$ and $\begin{matrix} a_3 \\ b_3 \\ c_3 \end{matrix}$ are called first column, second column and third column respectively.

Example-7 : Solve the following equations :

(i) $\begin{vmatrix} a & 1 & -2 \\ 4 & 4 & 2 \\ 1 & 3 & 1 \end{vmatrix} = 0$ (ii) $\begin{vmatrix} x & 2 & 3 \\ 5 & 0 & 7 \\ 3 & 1 & 2 \end{vmatrix} = 30$

Solution :

(i) Here, $\begin{vmatrix} a & 1 & -2 \\ 4 & 4 & 2 \\ 1 & 3 & 1 \end{vmatrix} = 0$

$\therefore\ a\begin{vmatrix} 4 & 2 \\ 3 & 1 \end{vmatrix} - 1\begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} - 2\begin{vmatrix} 4 & 4 \\ 1 & 3 \end{vmatrix} = 0$

$\therefore\ a(4 - 6) - 1(4 - 2) - 2(12 - 4) = 0$

$\therefore\ a(-2) - 1(2) - 2(8) = 0$

$\therefore\ -2a - 2 - 16 = 0$

$\therefore\ -2a = 18$

$\therefore\ a = \frac{18}{-2} = -9$

(ii) Here, $\begin{vmatrix} x & 2 & 3 \\ 5 & 0 & 7 \\ 3 & 1 & 2 \end{vmatrix} = 30$

Since one element in the second row is zero, the expansion along the second row will be easier. Hence expanding along second row,

$-5\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} + 0\begin{vmatrix} x & 3 \\ 3 & 2 \end{vmatrix} - 7\begin{vmatrix} x & 2 \\ 3 & 1 \end{vmatrix} = 30$

$\therefore\ -5(4 - 3) + 0 - 7(x - 6) = 30$

$\therefore\ -5(1) - 7x + 42 = 30$

$\therefore\ -5 - 7x + 42 = 30$

$\therefore\ -7x + 37 = 30$

$\therefore\ -7x = 30 - 37$

$\therefore\ -7x = -7$

$\therefore\ x = 1$

Multiple Choice Questions (MCQs) (Solution with Explanation)

1. If $\begin{vmatrix} x & 2 \\ -2 & 2 \end{vmatrix} = 2$, then $x =$ ....

(A) 2 (B) −1 (C) −2 (D) −3

Ans. : (B)

Explanation : $\begin{vmatrix} x & 2 \\ -2 & 2 \end{vmatrix} = 2$

$\therefore\ (x)(2) - (2)(-2) = 2$

$\therefore\ 2x + 4 = 2$ $\therefore\ 2x = -2$

$\therefore\ x = -1$

2. If $\begin{vmatrix} x & -4 \\ y & 4 \end{vmatrix} = 20$, then $x + y =$ ....

(A) 4 (B) 5 (C) −4 (D) −5

Ans. : (B)

Explanation : $\begin{vmatrix} x & -4 \\ y & 4 \end{vmatrix} = 20$

$\therefore$ $(x)(4)-(-4)(y)=20$

$\therefore$ $4x+4y=20$

$\therefore$ $4(x+y)=20$

$\therefore$ $x+y=5$

3. If $\begin{vmatrix} x-1 & 6 \\ 2 & x+1 \end{vmatrix}=12$, then $x=$ ....

(A) ± 3 (B) 0

(C) ± 5 (D) ± 1

Explanation : $\begin{vmatrix} x-1 & 6 \\ 2 & x+1 \end{vmatrix}=12$

$\therefore$ $(x-1)(x+1)-(6)(2)=12$

$\therefore$ $x^2-1-12=20$

$\therefore$ $x^2=25$

$\therefore$ $x=\pm 5$

4. $\begin{vmatrix} \log_{15}5 & -1 \\ \log_{15}3 & 1 \end{vmatrix}=$ ....

(A) 1 (B) -1 (C) 0 (D) 2

Ans. : (A)

Explanation : $\begin{vmatrix} \log_{15}5 & -1 \\ \log_{15}3 & 1 \end{vmatrix}=\log_{15}5+\log_{15}3$

$=\log_{15}(5\times 3)=\log_{15}15=1$

5. $\begin{vmatrix} \log_2 3 & 1 \\ -1 & \log_3 2 \end{vmatrix}=$ ....

(A) 0 (B) 2 (C) 1 (D) -1

Ans. : (B)

Explanation : $\begin{vmatrix} \log_2 3 & 1 \\ -1 & \log_3 2 \end{vmatrix}=\log_2 3\cdot\log_3 2+1$

$\log_2 3\cdot\dfrac{1}{\log_2 3}+1=1+1=2$

Multiple Choice Questions (MCQ's) with (Final Answers)

1. $\begin{vmatrix} 3 & -8 \\ 2 & 0 \end{vmatrix}=$ ....

(A) -13 (B) 19 (C) -16 (D) 16

2. $\begin{vmatrix} 4 & -8 \\ 1 & 0 \end{vmatrix}=$ ....

(A) 8 (B) -8 (C) 9 (D) -9

3. $\begin{vmatrix} -1 & 3 \\ -4 & 6 \end{vmatrix}=$ ....

(A) -18 (B) 6 (C) -6 (D) 18

4. $\begin{vmatrix} 3 & -2 \\ -4 & 4 \end{vmatrix}=$ ....

(A) -20 (B) 20 (C) 4 (D) -4

5. $\begin{vmatrix} x & -y \\ y & x \end{vmatrix}=$ ....

(A) $2x+2y$ (B) x^2-y^2

(C) x^2+y^2 (D) $(x+y)^2$

6. $\begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}=$ ....

(A) $2\cos\theta$ (B) 1 (C) $2\sin\theta$ (D) 0

7. If $\begin{vmatrix} 3x & 9 \\ 2 & 6 \end{vmatrix}=0$, then $x=$ ....

(A) 2 (B) 1 (C) 0 (D) -1

8. $\begin{vmatrix} \tan\theta & \sec\theta \\ \sec\theta & \tan\theta \end{vmatrix}=$ ....

(A) 0 (B) $2\sec\theta$ (C) -1 (D) 1

9. The value of the minor of 6 in $\begin{vmatrix} -1 & 6 & -2 \\ 5 & 0 & 7 \\ 4 & 1 & -3 \end{vmatrix}$ is ..... .

(A) 13 (B) -13 (C) 43 (D) -43

10. The value of cofactor of 5 in $\begin{vmatrix} -1 & 6 & -2 \\ 5 & 0 & 7 \\ 4 & 1 & -3 \end{vmatrix}$ is ..... .

(A) 16 (B) -16 (C) 20 (D) -20

Answers

(1) D	(2) A	(3) B	(4) C	(5) C
(6) B	(7) B	(8) C	(9) D	(10) A

[B] Matrix

[4] Matrix :

Any rectangular arrangement or an array of numbers enclosed in brackets such as [ ] or ( ) is called a matrix. We shall consider only real matrices, i.e. elements or entries of the matrices will be real numbers only.

Thus, $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ is a matrix. It has two rows and three columns. So we say that it is a 2×3 matrix. In general an $m \times n$ matrix is a matrix having m rows and n colums. It can be written as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Here a_{ij} is the element of the matrix in 'i^{th}' row and 'j^{th}' column. In a compact form, we can write this matrix as $[a_{ij}]_{m \times n}$, $1 \le i \le m$, $1 \le j \le n$. We denoted matrices by A, B, C, ... etc. $m \times n$ is known as the order of the matrix.

- **Do as directed :**

(1) If $\begin{bmatrix} x+y & 3 \\ -7 & x-y \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ -7 & 2 \end{bmatrix}$ **, then find** x **and** y**.**

(2) If $\begin{bmatrix} a+2b & 3c+2d \\ 2a-b & c-d \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$**, then find the values of** a, b, c **and** d**.**

Solution :

(1) Here $\begin{bmatrix} x+y & 3 \\ -7 & x-y \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ -7 & 2 \end{bmatrix}$

$\therefore$ By comparing the corresponding elements of two matrices

$x + y = 8$ and $x - y = 2$

$\therefore$ $(x + y) + (x - y) = 8 + 2$

$\therefore$ $2x = 10$

$\therefore$ $x = 5$

Now, by substituting $x = 5$ in $x + y = 8$,

$5 + y = 8$ $\quad \therefore y = 8 - 5 = 3$

$\therefore$ $x = 5$ and $y = 3$

(2) Here $\begin{bmatrix} a+2b & 3c+2d \\ 2a-b & c-d \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$

$\therefore$ By comparing the corresponding elements of two

matrices

$a + 2b = 4$, $2a - b = 3$, $3c + 2d = 0$, $c - d = -5$

Adding the equation $2a - b = 3$, after multiplying it by 2 into the equation $a + 2b = 4$,

$(a + 2b) + 2(2a - b) = 4 + 2\,(3)$

$\therefore$ $a + 2b + 4a - 2b = 4 + 6$

$\therefore$ $5a = 10$ $\quad \therefore a = 2$

Substituting $a = 2$ in the equation $a + 2b = 4$

$2 + 2b = 4$ $\quad \therefore 2b = 2$ $\quad \therefore b = 1$

Similarly, by solving the equation $3c + 2d = 0$ and $c - d = -5$, we have $c = -2$ and $d = 3$.

$\therefore$ $a = 2$, $b = 1$, $c = -2$ and $d = 3$

[5] Types of Matrices :

(1) **Row Matrix :** A $1 \times n$ matrix $[a_{11}, a_{12}, a_{13}, \ldots, a_{1n}]$ is called a row matrix. (A row matrix has only one row and any number of columns.)

For example, $M = [5 \quad -2 \quad 4 \quad 7]$ is a row matrix of order 1×4.

(2) **Column Matrix :** An $n \times 1$ marix $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \\ a_{n1} \end{bmatrix}$ is called

a column matrix. A column matrix has only one column and any number of rows.

For example, $N = \begin{bmatrix} -7 \\ 6 \\ 3 \end{bmatrix}$ is a column matrix of order 3×1.

(3) **Square Matrix :** An $n \times n$ matrix is called a square matrix. A square matrix has the number of rows equal to the number of columns.

For example, $A = \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$ is a square matrix of order 2×2 and $A = \begin{bmatrix} -2 & 1 & 3 \\ 4 & 0 & 5 \\ 7 & -8 & 9 \end{bmatrix}$ is a square matrix of order 3×3.

C = [5] is also a square matrix of order 1×1.

(4) **Diagonal Matrix :** If in a square matrix $A = [a_{ij}]$, we have $a_{ij} = 0$ wherever $i \neq j$, then A is called a diagonal matrix. This is square matrix in which all entries are zero except possibly those on the principal diagonal.

Thus, $A = \begin{bmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & \cdots & 0 \\ 0 & 0 & a_{33} & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \cdots & a_{nn} \end{bmatrix}$ is a diagonal matrix. It is also denoted by

diag $[a_{11} \quad a_{22} \quad a_{33} \quad \cdots \quad a_{nn}]$.

For example, $P = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is a diagonal matrix. It is also denoted by diag [4 5 0].

(5) **Zero Matrix :** If all elements of a matrix are zero, then that matrix is known as zero matrix. We denote zero matrix by $[0]_{m \times n}$ or $O_{m \times n}$.

For example, $O_{3\times2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, $O_{2\times2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ etc. are zero matrices.

[6] Addition of two matrices and properties of addition :

(1) **Addition of two Matrices :**

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are both $m \times n$ matrices, then their sum is defined as $[a_{ij} + b_{ij}]$. It is denoted by A + B.

Thus, $A + B = [a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$

For the sum of two matrices, both should be of order $m \times n$. Their sum will also be a $m \times n$ matrix.

For example, $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+2 & 1-1 \\ 2-1 & 3+1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 0 \\ 1 & 4 \end{bmatrix}$

But if A = [5 3] and $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, then A + B is not possible, because the order of A is 1×2 and the order of B is 2×1. Thus A + B is not possible as the order of A and B are different.

(2) **Properties of addition of Matrices :**

(i) **Commutative Law for Addition :** For $m \times n$ matrices A and B, A + B = B + A

(ii) **Associative Law for addition :** For $m \times n$ matrices A, B and C, (A + B) + C = A + (B + C)

(iii) The Identity Matrix for Addition : For $m \times n$ matrix A and zero matrix $O_{m \times n}$, $A + O = O + A = A$

(iv) Existence of Additive inverse : Corresponding to an $m \times n$ matrix $A = [a_{ij}]$, we have another matrix $[-a_{ij}]$, so that $[a_{ij}] + [-a_{ij}] = 0$.

Here $[-a_{ij}]$ is called the additive inverse of $A = [a_{ij}]$. It is denoted by $-A$.

Thus, $A = [a_{ij}]$, then $-A = [-a_{ij}]$ and $A + (-A) = (-A) + A = 0$

Definition of A – B : If $A = [a_{ij}]$ and $B = [b_{ij}]$ both matrices are of order $m \times n$, then

$A - B = A + (-B) = [a_{ij}] + [-b_{ij}] = [a_{ij} - b_{ij}]$.

For example, if $A = \begin{bmatrix} 2 & 3 & -1 \\ 5 & 4 & 6 \end{bmatrix}$ and

$B = \begin{bmatrix} 3 & 4 & -2 \\ 4 & 6 & 7 \end{bmatrix}$,

then $-B = \begin{bmatrix} -3 & -4 & 2 \\ -4 & -6 & -7 \end{bmatrix}$.

$\therefore \quad A - B = A + (-B)$

$$= \begin{bmatrix} 2 & 3 & -1 \\ 5 & 4 & 6 \end{bmatrix} + \begin{bmatrix} -3 & -4 & 2 \\ -4 & -6 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 3-4 & -1+2 \\ 5-4 & 4-6 & 6-7 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & 1 \\ 1 & -2 & -1 \end{bmatrix}$$

[7] Product of a matrix with a scalar and its properties :

(1) Product of a matrix with scalar :

If $A = [a_{ij}]$ is an $m \times n$ matrix and $k \in R$, then the matrix $[k\,a_{ij}]$ is called the product of the matrix A by the scalar k. It is denoted by kA.

Thus, $A = [a_{ij}]$, then $kA = [ka_{ij}]$

In kA, every element of A gets multiplied by k.

For example $A = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$ then

$$3A = 3\begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix} \quad \textbf{(GTU : June 2016)}$$

$$= \begin{bmatrix} 3 & 12 \\ 9 & -6 \end{bmatrix}$$

(2) Properties of Multiplication of a matrix by a Scalar :

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are $m \times n$ matrices and $k, l \in R$, then

(i) $k(A + B) = kA + kB$

(ii) $(k + l)A = kA + lA$

(iii) $(kl)A = k(lA)$

(iv) $1 \cdot A = A$

(v) $(-1)A = -A$

- **Do as directed :**

(1) **If** $A = \begin{bmatrix} 3 & 0 & x \\ -3 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 5 \\ -2 & y & 4 \end{bmatrix}$ **and** $2A = 3B$**, then find x and y.**

(2) **If** $A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$**, then find** $2A + 3B$**.**

(3) **If** $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 5 & 0 \end{bmatrix}$ **and** $X + A + B = 0$**, the find matrix X.**

Solution :

(1) Here $2A = 3B$

$$\therefore \quad 2\begin{bmatrix} 3 & 0 & x \\ -3 & 5 & 6 \end{bmatrix} = 3\begin{bmatrix} 2 & 0 & 5 \\ -2 & y & 4 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} 6 & 0 & 2x \\ -6 & 10 & 12 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 15 \\ -6 & 3y & 12 \end{bmatrix}$$

By comparing the corresponding elements of two matrices.

$2x = 15$ and $3y = 10$

$\therefore \quad x = \frac{15}{2}$ and $y = \frac{10}{3}$

(2) $2A + 3B = 2\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & 6 \end{bmatrix}$

$= \begin{bmatrix} 4+3 & -2+6 \\ 4+0 & 0+6 \end{bmatrix}$

$= \begin{bmatrix} 7 & 4 \\ 4 & 6 \end{bmatrix}$

(3) Here, $X + A + B = 0$

$\therefore \quad X = -A - B$

$= (-1)A + (-1)B$

$= (-1)\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \end{bmatrix} + (-1)\begin{bmatrix} 3 & -2 & 4 \\ 1 & 5 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1 & -2 & -1 \\ -3 & -4 & -2 \end{bmatrix} + \begin{bmatrix} -3 & 2 & -4 \\ -1 & -5 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1-3 & -2+2 & -1-4 \\ -3-1 & -4-5 & -2+0 \end{bmatrix}$

$\therefore \quad X = \begin{bmatrix} -4 & 0 & -5 \\ -4 & -9 & -2 \end{bmatrix}$

[8] Transpose of a matrix and its properties :

(1) Transpose of a matrix :

If all the rows of a matrix A are converted into corresponding columns, the matrix so obtained is called the transpose of A. It is denoted by A^T or A'.

If $A = [a_{ij}]_{m \times n}$, then $A^T = [a_{ij}]_{n \times m}$.

For example, if $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 5 & -6 \end{bmatrix}$, then

$A^T = \begin{bmatrix} 1 & 4 \\ -2 & 5 \\ 3 & -6 \end{bmatrix}$.

Some properties of addition and Multiplication by a scalar regarding transpose of matrix :

(i) $(A+B)^T = A^T + B^T$

(ii) $(A^T)^T = A$

(iii) $(kA)^T = kA^T, \; k \in R$

(2) Symmetric Matrix :

For a square matrix, $A = [a_{ij}]_{n \times n}$, if $A^T = A$, then A is called a symmetric matrix. Thus, in a symmetric matrix $a_{ij} = a_{ji}$ for all i and j.

For example, $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$, then $A^T = A$

$\therefore$ A is a symmetric matrix.

(3) Skew–Symmetric Matrix :

For a square matrix $A = [a_{ij}]_{n \times n}$, if $A^T = -A$, then A is called a skew-symmetric matrix. Thus, in a skew-symmetric matrix, $a_{ij} = -a_{ji}$ for all i and j.

For example, $A = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -5 \\ -3 & 5 & 0 \end{bmatrix}$, then

$A^T = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 5 \\ 3 & -5 & 0 \end{bmatrix} = -A$

$\therefore$ A is a skew-symmetric matrix.

- **Find the Transpose of following matrices :**

(1) $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 4 & 2 \end{bmatrix}$

(2) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(3) $A = \begin{bmatrix} 1 & -3 & 4 \\ -2 & 1 & 2 \end{bmatrix}$

Solution :

(1) Here, the first row is 1 2, second row is 3 1 and third row is 4 2. By converting all these three rows into the corresponding columns, the first column

is $\begin{matrix}1\\2\end{matrix}$, second column is $\begin{matrix}3\\1\end{matrix}$ and the third column is $\begin{matrix}4\\2\end{matrix}$.

$$\therefore \quad A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 4 & 2 \end{bmatrix}, \text{ then } A^T = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \end{bmatrix}.$$

(2) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\qquad \therefore A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

(3) $A = \begin{bmatrix} 1 & -3 & 4 \\ -2 & 1 & 2 \end{bmatrix}$ $\qquad \therefore A^T = \begin{bmatrix} 1 & -2 \\ -3 & 1 \\ 4 & 2 \end{bmatrix}$

[9] Multiplication of Matrices and Properties of Matrix Multiplication :

(1) Multiplication of Matrices :

The product AB of two matrices A and B is defined only if the number of columns of A is equal to the number of rows of B.

Thus if $A = [a_{ij}]_{m \times p}$ and $B = [b_{ij}]_{p \times n}$, then their product $AB = [c_{ij}]_{m \times n}$ where

$$c_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj} = a_{i1}\, b_{1j} + a_{i2} + b_{2j} + ... + a_{in}\, b_{nj}$$

To obtain the entry in i^{th} row and j^{th} column of matrix AB, we multiply elements of the i^{th} row $\begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{ip} \end{bmatrix}$ of the matrix A with corresponding elements of the j^{th} column $\begin{bmatrix} b_{1j} \\ b_{2j} \\ b_{3j} \\ \\ b_{pj} \end{bmatrix}$ of the matrix B and then we take the sum of all these products.

For example, $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & -2 \\ -1 & 1 \end{bmatrix}$

Here, A is a 2 × 3 matrix and B is a 3 × 2 matrix. Number of columns of A = 3 = Number of rows of B. So their product AB is possible and AB will be 2 × 2 matrix.

$$\text{Let, } AB = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix}$$

$$\text{Now, } AB = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -1 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix}$$

To obtain the element c_{11}, we have to multiply the first row $[2 \quad -1 \quad 3]$ of the matrix A with corresponding elements of the first column $\begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$ of the matrix B and then we have to take sum of all these products.

$$\text{Thus, } c_{11} = \begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$$

$$= (2)(4) + (-1)(2) + (3)(-1)$$

$$= 8 - 2 - 3$$

$$= 3$$

Similalry, we can find the remaining elements c_{12}, c_{21} and c_{22} also.

Thus,

$$AB = \begin{bmatrix} (2)(4)+(-1)(2)+(3)(-1) & (2)(3)+(-1)(-2)+(3)(1) \\ (1)(4)+(4)(2)+(-1)(-1) & (1)(3)+(4)(-2)+(-1)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 8-2-3 & 6+2+3 \\ 4+8-1 & 3-8-1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 11 \\ 13 & -6 \end{bmatrix}$$

(2) Properties of Matrix Multiplication :

(1) Matrix multiplication is not commutative i.e for matrices A and B, it is not necessary that AB is equal to BA.

(2) **Associative Law :**

For matrices $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{n\times p}$ and $C = [c_{ij}]_{p\times q}$, then A(BC) = (AB) C.

(3) **Distributive Laws :**

(i) For matrices $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{n\times p}$ and $C = [c_{ij}]_{n\times p}$, then A (B + C) = AB + AC.

(ii) For matrices $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{m\times n}$ and $C = [c_{ij}]_{n\times p}$, then (A + B) C = AC + BC.

(4) If $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{n\times p}$, then $(AB)^T = B^T A^T$.

(3) **Unit Matrix/Identity Matrix :**

A square matrix in which all elements on principal diagonal are 1 and the rest of them are 0 is called an identity or a unit matrix. Identity matrix is denoted by I.

Thus, $I = [a_{ij}]_{n\times n}$

where $a_{ij} = 1, \quad i = j$

$= 0, \quad i \neq j$

Thus, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix. It is denoted by $I_{2\times 2}$ or I_2.

Thus I_n is an $n \times n$ identity matrix.

If $A = [a_{ij}]_{n\times n}$, then $AI_n = I_nA = A$.

(4) **Scalar Matrix :**

If $k \in R$, then kI_n is called a scalar matrix.

Thus, $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 5I_3$

Here, $k = 5 \in R$, so A is a scalar matrix.

Note : $A^2 = A \cdot A$, $A^3 = A^2 \cdot A$, ..., $A^n = (A^{n-1}) \cdot A$ and $I^2 = I^3 = I^4 = ... = I^n = I$.

- **Do as directed :**

(1) If $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$, then find AB.

(2) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then find A^2.

Solution :

(1) $AB = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$= \begin{bmatrix} (2)(3) + (1)(-5) & (2)(-1) + (1)(2) \\ (5)(3) + (3)(-5) & (5)(-1) + (3)(2) \end{bmatrix}$

$= \begin{bmatrix} 6-5 & -2+2 \\ 15-15 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(2) $A^2 = A \cdot A$

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} (1)(1) + (2)(3) & (1)(2) + (2)(4) \\ (3)(1) + (4)(3) & (3)(2) + (4)(4) \end{bmatrix}$

$= \begin{bmatrix} 1+6 & 2+8 \\ 3+12 & 6+16 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$

[10] Determinant of a Matrix :

If all the entries of a square matrix are kept in their respective places and the determinant of this array is taken, then the determinant so obtained is called the determinant of the given square matrix. If A is a square matrix, then determinant of A denoted by |A| or det A.

Thus, for 3 × 3 matrix

$A = [a_{ij}], |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

and for 2 × 2 matrix $B = [b_{ij}]$, $|B| = \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}$

For example, If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, then $|A| = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}$

$= 8 - 3 = 5$

- If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sec\theta & \tan\theta \\ 0 & \tan\theta & \sec\theta \end{bmatrix}$, then find $|A|$.

Solution : $|A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \sec\theta & \tan\theta \\ 0 & \tan\theta & \sec\theta \end{vmatrix} = \sec^2\theta - \tan^2\theta = 1$

[11] Adjoint Matrix :

For a given square matrix A, if we replace every entry in A by its cofactor as in |A| and than the transpose of this matrix is taken, then the matrix so obtained is called the adjoint of A and it is denoted by *adj* A.

Thus the adjoint matrix of a square matrix $A = [a_{ij}]_{n\times n}$ is $adj\ A = [A_{ji}]_{n\times n}$, where A_{ji} is the cofactor of the element a_{ji} in |A|.

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. If we replace $a_{11}, a_{12}, a_{21}, a_{22}$ by their cofectors $A_{11}, A_{12}, A_{21}, A_{22}$, then we get the matrix $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$. The transpose of this matrix is $\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix}$ which is **adj** A.

$$\text{Thus, adj } A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

Thus, to obtain the adjoint of 2 × 2 matrix, interchange the elements on the principal diagonal and change the sign of the elements on the secondary diagonal.

Similarly, if 3 × 3 matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$,

$$\text{then adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}.$$

- **Some Important Results :**

We shall accept the following results without proof.

(1) For 2 × 2 matrix A, adj (adj A) = A

(2) For 3 × 3 matrix A, A (adj A) = (adj A) A = $|A|\, I_3$

(3) ([illegible] ^{T}) = $(\text{adj } A)^T$

- **Do as directed :**

(1) **If** $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$, **then find adj A.**

Solution :

(1) Here, $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$

$\therefore\ adj\ A = \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}$

(Interchange the elements on principal diagonal of A and change the sign of elements on the secondary diagonal.)

- **Inverse of a Matrix :**

For a square matrix A, if there exists another square matrix B, such that AB = I = BA, then B is called an inverse matrix of A.

It is clear that if B is an inverse of A, then A is an inverse of B. Inverse of A is denoted by A^{-1}.

- **Non–Singular Matrix and Singular Matrix :**

A square matrix A is said to be non-singular if it has an inverse matrix. If A is a non-singular matrix, then A^{-1} is also non-singular matrix and $(A^{-1})^{-1} = A$.

Singular Matrix : A matrix which is not non-singular is called a singular matrix.

- **Some Important results for Inverse of a matrix :**

We shall accept the following results without proof.

(1) If inverse of matrix A exists, then it is unique.

(2) A square matrix A is non-singular if and only if $|A| \neq 0$.

(3) If A is a non-singular matrix, then its inverse matrix

$$A^{-1} = \frac{1}{|A|}(\text{adj } A).$$

(4) $|A^{-1}| = |A|^{-1}$

(5) $(AB)^{-1} = B^{-1}A^{-1}$

(6) $(A^T)^{-1} = (A^{-1})^T$

- **Find the inverse of the following matrices :**

(1) $A = \begin{bmatrix} -8 & 4 \\ -6 & 3 \end{bmatrix}$

(2) $A = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$

(3) $A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$

Solution :

(1) Here, $|A| = \begin{vmatrix} -8 & 4 \\ -6 & 3 \end{vmatrix} = -24 + 24 = 0$

$\therefore$ A is a singular matrix

$\therefore$ A^{-1} does not exist.

(2) Here,

$$|A| = \begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix} = \sin^2\theta + \cos^2\theta = 1 \neq 0$$

$\therefore$ A^{-1} exists

Now, adj $A = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{1}\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} = A^T$$

(3) Here, $|A| = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} = 3 + 4 = 7 \neq 0$

$\therefore$ A^{-1} exists.

Now, adj $A = \begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$

$$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$= \frac{1}{7}\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{7} & \frac{2}{7} \\ -\frac{2}{7} & \frac{1}{7} \end{bmatrix}$$

[12] Solution of System of linear Equations in two variables :

In the previous standards, we have learned the method of elimination, method of substitution, cramer's method for the solution of system of linear equations in two vairables. Now we will look at how the matrix is useful to solve the system of linear equations.

Let,

$a_{11}x + a_{12}y = b_1$

$a_{21}x + a_{22}y = b_2$

are the system of two linear equations of two unknown x and y.

We can represent this system of equations in the matrix form as follows.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad \text{... (i)}$$

If we take $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$,

then (i) can be written as follows.

$AX = B$ ... (ii)

Now if A is a non-singular matrix, i.e. $|A| \neq 0$, then A^{-1} exists, so multiplying by A^{-1} on both the sides of the equation (i)

$A^{-1}(AX) = A^{-1}B$

$\therefore$ $(A^{-1}A)X = A^{-1}B$

$\therefore$ $IX = A^{-1}B$

$\therefore$ $X = A^{-1}B$ or $\frac{1}{|A|}(\text{adj } A)B$

Now, if $A^{-1}B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$, then

from $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$\therefore$ $x = c_1$ and $y = c_2$ is the unique solution of the system of linear equations.

- **Solve the following system of linear equations using matrix method :**

(1) $2x - y = 4,\ 3x + y = 1$

Solution :

(1) Here, $2x - y = 4$

$3x + y = 1$

$\therefore$ $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = 2 + 3 = 5 \neq 0$

$\therefore$ System of equation has a unique solution.

Now, adj $A = \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}$

$\therefore$ $X = A^{-1}B = \frac{1}{|A|}$ (adj A) B

$\therefore$ $X = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} (1)(4) + (1)(1) \\ (-3)(4) + (2)(1) \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 4 + 1 \\ -12 + 2 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 5 \\ -10 \end{bmatrix}$

$\therefore$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\therefore$ $x = 1$ અને $y = -2$

Multiple Choice Questions (MCQs) (Solution with Explanation)

(1) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 4 & 2 \end{bmatrix}$, then $A^T = $ ....

(A) A

(B) $\begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & -3 & 4 \\ -2 & 1 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 2 & 4 \end{bmatrix}$

Ans. : (B)

Explanation :

Here, the first row is 1 2, second row is 3 1 and third row is 4 2. By converting all these three rows into the corresponding columns, the first column is $\begin{matrix} 1 \\ 2 \end{matrix}$, second column is $\begin{matrix} 3 \\ 1 \end{matrix}$ and the third column is $\begin{matrix} 4 \\ 2 \end{matrix}$.

$\therefore$ $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 4 & 2 \end{bmatrix}$, then $A^T = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \end{bmatrix}$.

(2) If $A = \begin{bmatrix} 3 & 0 & x \\ -3 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 5 \\ -2 & y & 4 \end{bmatrix}$ and $2A = 3B$, then $y = $ ....

(A) 5 (B) -5 (C) $\frac{15}{2}$ (D) $\frac{10}{3}$

Ans. : (D)

Explanation : Here, 2A = 3B

$\therefore\ 2\begin{bmatrix} 3 & 0 & x \\ -3 & 5 & 6 \end{bmatrix} = 3\begin{bmatrix} 2 & 0 & 5 \\ -2 & y & 4 \end{bmatrix}$

$\therefore\ \begin{bmatrix} 6 & 0 & 2x \\ -6 & 10 & 12 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 15 \\ -6 & 3y & 12 \end{bmatrix}$

By comparing the corresponding elements of two matrices

$2x = 15$ and $3y = 10$

$\therefore\ x = \frac{15}{2}$ and $y = \frac{10}{3}$

(3) If $\begin{bmatrix} x+y & 3 \\ -7 & x-y \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ -7 & 2 \end{bmatrix}$, then (x, y) = ...

(A) (8, 2) (B) (2, 8)
(C) (5, 3) (D) (3, 5)

Ans. : (C)

Explanation : Here, $\begin{bmatrix} x+y & 3 \\ -7 & x-y \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ -7 & 2 \end{bmatrix}$

$\therefore$ By comparing the corresponding elements of two matrices

$x + y = 8$ and $x - y = 2$

$\therefore\ (x + y) + (x - y) = 8 + 2$

$\therefore\ 2x = 10$

$\therefore\ x = 5$

Now, substitute $x = 5$ in $x + y = 8$

$5 + y = 8 \quad \therefore\ y = 8 - 5 = 3$

$\therefore\ x = 5$ and $y = 3$

(4) If $\begin{bmatrix} 2x-3 & x-5 \\ -3 & 5 \end{bmatrix}$ is a symmetric matrix, then x = ...

(A) –5 (B) –3 (C) –2 (D) 2

Ans. : (D)

Explanation : Here $A = \begin{bmatrix} 2x-3 & x-5 \\ -3 & 5 \end{bmatrix}$ is a symmetric matrix.

$\therefore\ A^T = A$

$\therefore\ \begin{bmatrix} 2x-3 & -3 \\ x-5 & 5 \end{bmatrix} = \begin{bmatrix} 2x-3 & x-5 \\ -3 & 5 \end{bmatrix}$

By comparing the corresponding elements of two matrices.

$x - 5 = -3 \qquad \therefore\ x = 5 - 3 = 2$

(5) If $A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$, then 2A + 3B = ...

(A) $\begin{bmatrix} 7 & 4 \\ 0 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 7 & 4 \\ 4 & 6 \end{bmatrix}$

(C) $\begin{bmatrix} 7 & 4 \\ 6 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 7 & 0 \\ 0 & 4 \end{bmatrix}$

Ans. : (B)

Explanation : $2A + 3B = 2\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & 6 \end{bmatrix}$

$= \begin{bmatrix} 4+3 & -2+6 \\ 4+0 & 0+6 \end{bmatrix}$

$= \begin{bmatrix} 7 & 4 \\ 4 & 6 \end{bmatrix}$

(6) If $\begin{bmatrix} 0 & x & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = [4]$, then x = ...

(A) 2 (B) 4 (C) 5 (D) 6

Ans. : (C)

Explanation : $\begin{bmatrix} 0 & x & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = [4]$

$\therefore\ [(0)(1) + (x)(2) + (-2)(3)] = [4]$

$\therefore\ [0 + 2x - 6] = [4]$

$\therefore\ [2x - 6] = [4]$

$\therefore\ 2x - 6 = 4$

$\therefore\ 2x = 10 \quad \therefore x = 5$

(7) If $\begin{bmatrix} x & 3 \\ y & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 15 \\ 12 \end{bmatrix}$, then $y =$ ....

(A) 4 (B) 9

(C) 3 (D) 2

Ans. : (C)

Explanation : $\begin{bmatrix} x & 3 \\ y & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 15 \\ 12 \end{bmatrix}$

$\therefore \begin{bmatrix} (x)(2) + (3)(3) \\ (y)(2) + (2)(3) \end{bmatrix} = \begin{bmatrix} 15 \\ 12 \end{bmatrix}$

$\therefore \begin{bmatrix} 2x+9 \\ 2y+6 \end{bmatrix} = \begin{bmatrix} 15 \\ 12 \end{bmatrix}$

By comparing the corresponding elements of two matrices

$2x + 9 = 15$ and $2y + 6 = 12$

$\therefore\ 2x = 6$ and $2y = 6$

$\therefore\ x = 3$ and $y = 3$

(8) If $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$, then adj A = ...

(A) $\begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$

Ans. : (C)

Explanation : Here, $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$

$\therefore\ adj\ A = \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}$

(Interchange the elements on principal diagonal of A and change the sign of elements on the secondary diagonal.)

(9) If $A = \begin{bmatrix} -3 & 5 & 4 \\ 0 & 4 & 7 \\ -2 & -5 & 9 \end{bmatrix}$, then A(adj A) = ...

(A) 251 I (B) −251 I

(C) 283 I (D) −283 I

Ans. : (B)

Explanation : We know that for a 3×3 matrix A, $A(\text{adj } A) = (\text{adj } A)A = |A|\, I_3$.

$\therefore\ |A| = \begin{vmatrix} -3 & 5 & 4 \\ 0 & 4 & 7 \\ -2 & -5 & 9 \end{vmatrix} = -3(36+35) - 5\,(0+14)$

$= -3\,(71) - 70 + 32$

$= -213 - 38 = -251$

$\therefore\ A(\text{adj } A) = |A|\, I_3 = -251\, I_3$

$= -251 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -251 & 0 & 0 \\ 0 & -251 & 0 \\ 0 & 0 & -251 \end{bmatrix}$

(10) If $A = \begin{bmatrix} -8 & 4 \\ -6 & 3 \end{bmatrix}$, then $A^{-1} =$ ....

(A) $\begin{bmatrix} 3 & -4 \\ 6 & -8 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{1}{8} & \frac{1}{4} \\ -\frac{1}{6} & \frac{1}{3} \end{bmatrix}$ (D) does not exist

Ans. : (D)

Explanation :

Here, $|A| = \begin{vmatrix} -8 & 4 \\ -6 & 3 \end{vmatrix} = -24 + 24 = 0$

$\therefore$ A is a singular matrix

$\therefore\ A^{-1}$ does not exist

(11) $\begin{bmatrix}1 & 1\\2 & 3\end{bmatrix}+\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}=$ ....

(A) $\begin{bmatrix}2 & -1\\-2 & 3\end{bmatrix}$ (B) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

(C) $\begin{bmatrix}-1 & 2\\3 & 2\end{bmatrix}$ (D) $\begin{bmatrix}3 & 0\\1 & 4\end{bmatrix}$

Ans. : (D)

Explanation : $\begin{bmatrix}1 & 1\\2 & 3\end{bmatrix}+\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}$

$=\begin{bmatrix}1+2 & 1-1\\2-1 & 3+1\end{bmatrix}$

$=\begin{bmatrix}3 & 0\\1 & 4\end{bmatrix}$

(12) If $A=\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & -1\\-5 & 2\end{bmatrix}$, then AB = ...

(A) $\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$ (B) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

(C) $\begin{bmatrix}1 & 0\\1 & 0\end{bmatrix}$ (D) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

Ans. : (B)

Explanation : $AB=\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix}\cdot\begin{bmatrix}3 & -1\\-5 & 2\end{bmatrix}$

$=\begin{bmatrix}(2)(3)+(1)(-5) & (2)(-1)+(1)(2)\\(5)(3)+(3)(-5) & (5)(-1)+(3)(2)\end{bmatrix}$

$=\begin{bmatrix}6-5 & -2+2\\15-15 & -5+6\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

(13) If $A=\begin{bmatrix}\sin\theta & \cos\theta\\-\cos\theta & \sin\theta\end{bmatrix}$, then $A^{-1}=$ ....

(A) A^T (B) I (C) 0 (D) A

Ans. : (A)

Explanation : Here,

$|A|=\begin{vmatrix}\sin\theta & \cos\theta\\-\cos\theta & \sin\theta\end{vmatrix}=\sin^2\theta+\cos^2\theta=1\neq 0$

$\therefore$ A^{-1} exists

Now, adj $A=\begin{bmatrix}\sin\theta & -\cos\theta\\\cos\theta & \sin\theta\end{bmatrix}$

$\therefore$ $A^{-1}=\frac{1}{|A|}(\text{adj }A)=\frac{1}{1}\begin{bmatrix}\sin\theta & -\cos\theta\\\cos\theta & \sin\theta\end{bmatrix}$

$\therefore$ $A^{-1}=\begin{bmatrix}\sin\theta & -\cos\theta\\\cos\theta & \sin\theta\end{bmatrix}=A^T$

Multiple Choice Questions (MCQ's) with (Final Answers)

(1) $I_2=$ ....

(A) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ (B) $\begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}$

(C) $\begin{bmatrix}1 & 0\\1 & 0\end{bmatrix}$ (D) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

(2) If $A=\begin{bmatrix}1 & 4\\3 & -2\end{bmatrix}$, then 3A = ...

(A) $\begin{bmatrix}3 & 12\\3 & -2\end{bmatrix}$ (B) $\begin{bmatrix}1 & 4\\9 & -6\end{bmatrix}$

(C) $\begin{bmatrix}3 & 4\\9 & -2\end{bmatrix}$ (D) $\begin{bmatrix}3 & 12\\9 & -6\end{bmatrix}$

(3) Order of the matrix $\begin{bmatrix}2 & 1 & 2\\1 & 1 & 1\end{bmatrix}$ is ....

(A) 2 × 3 (B) 3 × 2

(C) 2 × 2 (D) none of these

(4) Order of the matrix $\begin{bmatrix}2 & 1 & 5\\6 & 0 & 1\end{bmatrix}$ is ....

(A) 3 × 3

(B) 3 × 2

(C) 2 × 3

(D) none of these

(5) Order of the matrix $\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$ is ....

(A) 2×3 (B) 2×2
(C) 3×2 (D) 3×3

(6) Order of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is ....

(A) 2×3 (B) 3×2
(C) 2×2 (D) none of these

(7) Order of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \end{bmatrix}$ is ....

(A) 3×2 (B) 2×3
(C) 2×2 (D) 3×3

(8) Order of the matrix $\begin{bmatrix} 9 & -6 & 8 \\ 3 & 1 & 0 \end{bmatrix}$ is ....

(A) 3×2 (B) 2×3
(C) 3×3 (D) 2×2

(9) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $A^T =$ ....

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(10) If $A = \begin{bmatrix} 1 & -3 & 4 \\ -2 & 1 & 2 \end{bmatrix}$, then $A^T =$ ....

(A) $\begin{bmatrix} -1 & 3 & -4 \\ 2 & -1 & -2 \end{bmatrix}$ (B) A

(C) $\begin{bmatrix} 1 & -2 \\ -3 & 1 \\ 4 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} -2 & 1 & 2 \\ 1 & -3 & 4 \end{bmatrix}$

(11) If $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$, then $A^T =$ ....

(A) A^{-1} (B) A

(C) $\begin{bmatrix} 2 & 4 \\ 1 & 2 \\ 3 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

(12) If $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$, then $A^T =$ ....

(A) A (B) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 6 & 3 \end{bmatrix}$

(13) If A is a square matrix, then $A - A^T$ is a .... matrix.

(A) diagonal (B) column
(C) symmetric (D) skew-symmetric

(**Note :** For explanation, see the solution of ex. 8(3) (ii))

(14) If A is a square matrix, then $A + A^T$ is a .... matrix.

(A) diagonal (B) column
(C) symmetric (D) skew-symmetric

(**Note :** For explanation, see the solution of ex. 8(3) (i))

(15) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} =$ ....

(A) $\begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$ (B) $\begin{bmatrix} 5 & 3 \\ 9 & 7 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 7 \\ 5 & 9 \end{bmatrix}$ (D) none of these

(16) If $A = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$, then $2A - 3I =$ ....

(A) $\begin{bmatrix} 1 & 8 \\ 6 & -4 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 8 \\ 6 & 7 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 8 \\ 6 & -7 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & -8 \\ -6 & 7 \end{bmatrix}$

(17) If $A = [5 \ \ 3]$ and $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, then $A + B =$ ....

(A) $[7 \ \ 7]$ (B) $\begin{bmatrix} 7 \\ 7 \end{bmatrix}$

(C) $[14]$ (D) not possible

(18) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $A^2 =$ ....

(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

(19) If $A = \begin{bmatrix} -7 & 6 \\ 5 & -2 \end{bmatrix}$, then $AI_2 =$ ....

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 7 & -6 \\ -5 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} -7 & 6 \\ 5 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} -2 & -6 \\ -5 & -7 \end{bmatrix}$

(20) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^2 =$ ....

(A) $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 4 \\ 9 & 16 \end{bmatrix}$

(C) $\begin{bmatrix} 7 & 15 \\ 22 & 10 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(21) If $A_{2\times3}$ and $B_{3\times4}$ are matrices then the order of AB is

(A) 4×2 (B) 2×4

(C) 3×3 (D) AB is not possible

(22) If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$, then AB = ...

(A) $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

(23) If $A = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, then AB = ...

(A) not possible (B) [9]

(C) 1×1 (D) $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 6 & 4 \\ 1 & 3 & 2 \end{bmatrix}$

(24) If $A_{3\times4}$ and $B_{4\times4}$ are matrices then the order of AB is ....

(A) 3×4 (B) 4×4

(C) 4×3 (D) 3×3

(25) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then adj A = ...

(A) $\begin{bmatrix} d & b \\ c & a \end{bmatrix}$ (B) $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

(C) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (D) $\begin{bmatrix} -a & c \\ b & -d \end{bmatrix}$

(26) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then adj A = ...

(A) $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$

(27) If A is a non-singular matrix, then ....

(A) $A^T = A$ (B) $A^T = -A$

(C) $|A| \neq 0$ (D) $|A| = 0$

(28) If AB = I, then matrix B = ...

(A) adj A (B) A^T

(C) A^{-1} (D) unit matrix

(29) If $M_{2\times3}$ and $N_{3\times1}$ are matrices, then the order of $(MN)^T$ is ....

(A) 2×1 (B) 3×1

(C) 1×2 (D) 3×3

(30) If $A_{3\times2}$ and $(BA)_{2\times2}$ are matrices, then the order of B is ....

(A) 3×2 (B) 2×3

(C) 2×2 (D) 3×3

Answers

(1) D	(2) D	(3) A	(4) C	(5) B
(6) A	(7) B	(8) B	(9) A	(10) C
(11) C	(12) B	(13) D	(14) C	(15) A
(16) C	(17) D	(18) C	(19) C	(20) A
(21) B	(22) C	(23) B	(24) A	(25) C
(26) C	(27) C	(28) C	(29) C	(30) B

★★★