SECTION-02 - BE-02

APTITUDE TEST (MATHEMATICS & SOFT SKILL)

MATHEMATICS

1. Determinant and Matrices
2. Trigonometry
3. Vectors
4. Co-ordinate Geometry
5. Function & Limit
6. Differentiation and its Applications
7. Integration
8. Logarithm
9. Statistics

ENGLISH

10. Comprehension of Unseen Passage
11. Theory of Communication
12. Grammar
13. Correction of Incorrect Words and Sentences

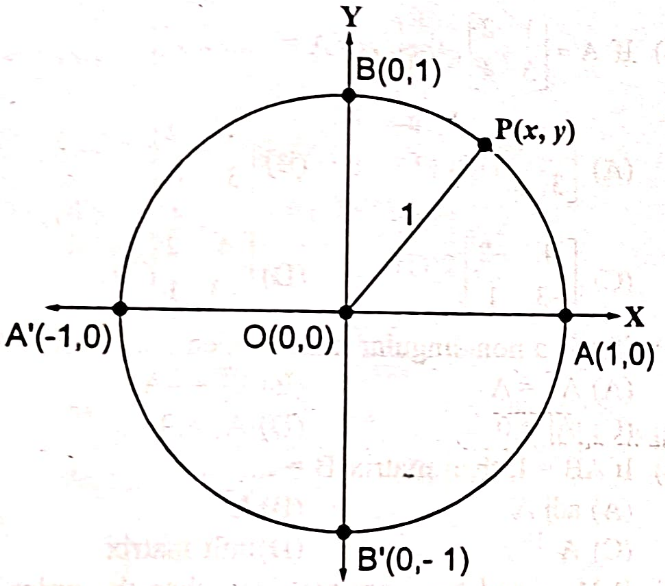
2. Trigonometry

[1] Introduction :

The word 'Trigonometry' is derived from two Greek words 'Trigono' and 'Metron'. The word 'Trigono' means a 'Triangle' and the word 'Metron' means 'To measure'. Hence the word 'Trigonometry' means 'Measurement of Triangle.'

[2] Unit circle :

Definition : In coordinate plane, a circle whose centre is origin and radius is one unit is called unit circle.



→ **Equation of unit circle :** Let $P(x, y)$ be any point on unit circle.

$\therefore$ $OP = 1$

$\therefore$ $OP^2 = 1$

$\therefore$ $(x - 0)^2 + (y - 0)^2 = 1$

$\therefore$ $x^2 + y^2 = 1$

This equation is known as the equation of unit circle.

(**Note :** In standard 10, you have studied the distance formula $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ for finding the distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ which is used in the above equation.)

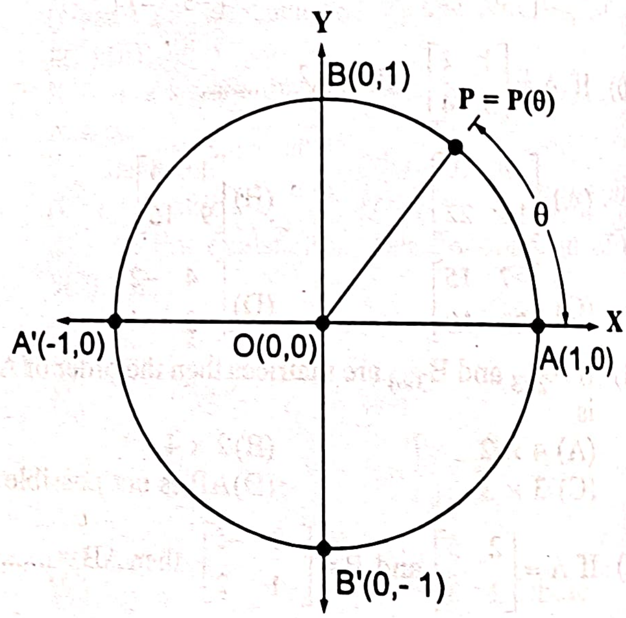
→ Circumference of unit circle $= 2\pi r = 2\pi$ ($\because r = 1$)

[3] Trigonometric point :

We shall denote the length $l(\overset{\frown}{AP})$ of the arc $\overset{\frown}{AP}$ from a point A to any point P on the unit circle in anticlockwise direction as θ. Now we can obtain a unique point on the unit circle corresponding to every real number θ as follows.

If $\theta = 0$, we take the point corresponding to θ as $A(1, 0)$ and call it $P(0)$.

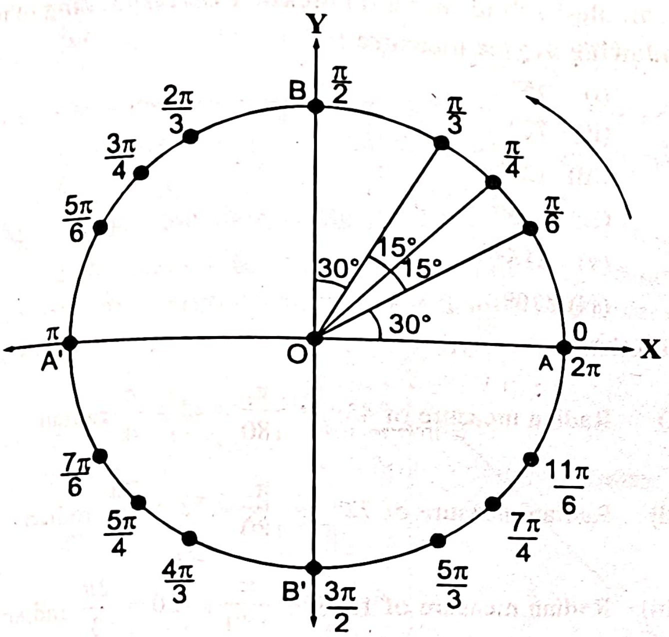
If $0 < \theta < 2\pi$ then a unique point P exists on the unit circle in the anticlockwise direction from point A such that $l(\overset{\frown}{AP}) = \theta$. We call this point P as $P(\theta)$.



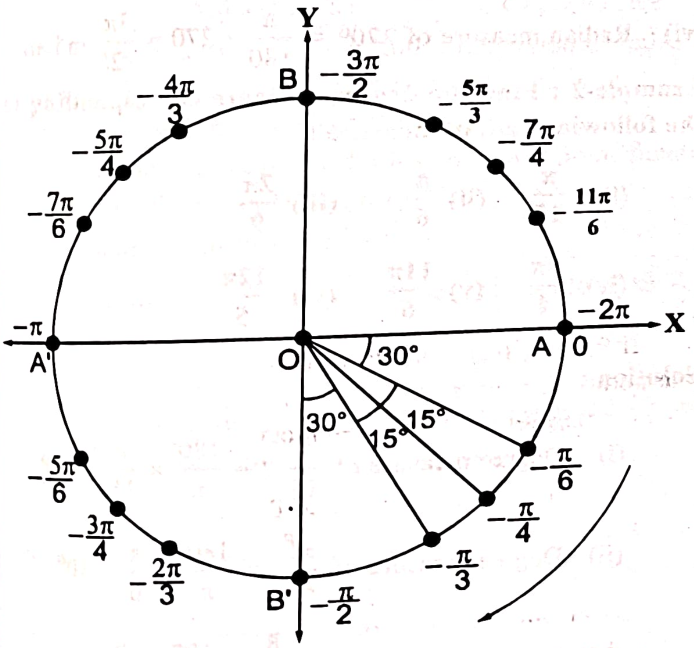
If $\theta \geq 2\pi$, then complete one cycle counterclockwise from point A on the unit circle and continue moving in the same direction and if $\theta < 0$, then move clockwise from point A on the unit circle in the same way. Thus the point obtained on the unit circle corresponding to θ is called $P(\theta)$.

Hence for every $\theta \in R$, we have a unique point $P(\theta)$ on the unit circle.

Trigonometric points on the unit circle, corresponding to the different values of θ from 0 to 2π are shown in the following figure.



Trigonometric points on the unit circle, corresponding to the different values of θ from -2π to 0 are shown in the following figure.



For example :

$p(0) = p(2\pi) = p(-2\pi) =$ Point A

$p\left(\frac{\pi}{2}\right) = p\left(-\frac{3\pi}{2}\right) =$ Point B

$P(\pi) = P(-\pi) =$ Point A'

$P\left(\frac{3\pi}{2}\right) = P\left(-\frac{\pi}{2}\right) =$ Point B'

If we denote the set of points on the unit circle by C, then the function $f : R \rightarrow C$, $f(\theta) = P(\theta) = P(x, y)$ is called trigonometric point (or T-point) function.

[4] Measurement of an angle :

Now we will learn the two main units of measurement of angles : (1) Degree measure (2) Radian measure

(1) Degree measurement :

In this method a right angle is divided into 90 congruent parts and measure of each such part is called 1 degree. It is denoted by 1°. Thus one degree is the 90^{th} part of the measure of right angle.

If 1 degree is divided into 60 congruent parts, then measure of each such part is called 1 minute. It is denoted by 1'.

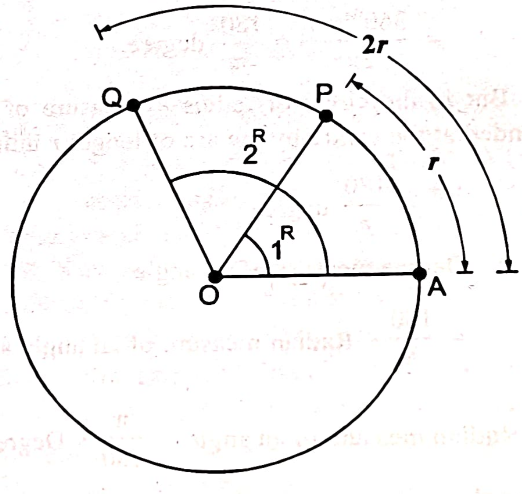
Thus $1^{\circ} = 60' = 60$ minute

If 1 minute is divided into 60 congruent parts, then measure of each such part is called 1 second. It is denoted by 1".

Thus $1' = 60'' = 60$ second

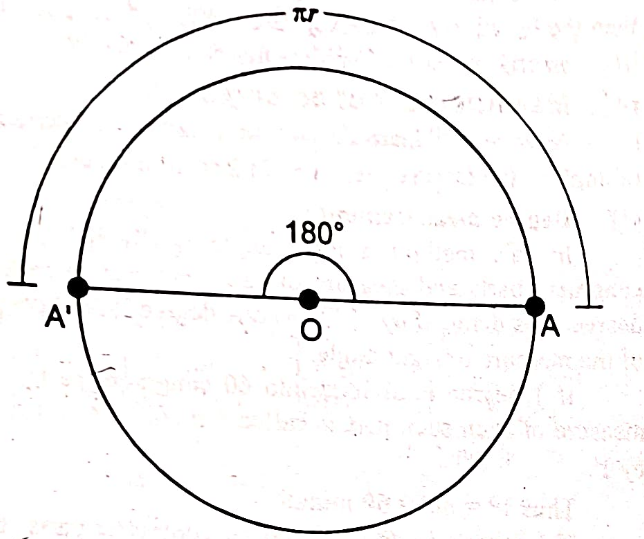
(2) Radian measurement :

The measure of the angle subtended at the centre of a circle by an arc of length equal to the radius of the circle is called 1 radian. It is denoted by 1^C or 1^R.



In the circle of radius r having centre O as shown in the above figure $l(\widehat{AP}) = r$. Then the measure of $\angle$ AOP is 1^R. Similarly $l(\widehat{AQ}) = 2r$, so the measure of $\angle AOQ$ is 2^R.

If we consider $r = 1$ unit, then the measure of the angle subtended by the arc of length 1 unit in the unit circle is called 1 radian. Thus the length of the arc of the unit circle is equal to the radian measure of the angle subtended at the centre of the circle.



Length of the circumference of a circle of radius r is $2\pi r$. Thus

Degree measure of the angle subtended at the centre of the circle by an arc of legnth $2\pi r$ unit $= 360^o$.

$\therefore$ Degree measure of the angle subtended at the centre of the circle by an arc of length r unit

$$= \frac{360^o \times r}{2\pi r} = \frac{180}{\pi} \text{ degree.}$$

But in the circle of radius r, measure of the angle subtended at the centre by the arc of length r unit $= 1^R$.

$$\therefore \quad 1^R = \frac{180}{\pi} \text{ degree}$$

$\therefore$ Degree measure of an angle

$$= \frac{180}{\pi} \times \text{Radian measure of an angle and}$$

Radian measure of an angle $= \frac{\pi}{180} \times$ Degree measure of an angle.

Using the above formula, the degree measure of an angle can be converted to the radian measure or the radian measure can be converted to the degree measure. Let's understand this by the following examples.

Example-1 : Find the radin measure corresponding to the following degree measure :

(i) 45^o

(ii) 72^o

(iii) 120^o

(iv) 135^o

(v) 216^o

(vi) 270^o

Solution :

(i) Radian measure of $45^o = \frac{\pi}{180} \times 45 = \frac{\pi}{4}$ radian

(ii) Radian measure of $72^o = \frac{\pi}{180} \times 72 = \frac{2\pi}{5}$ radian

(iii) Radian measure of $120^o = \frac{\pi}{180} \times 120 = \frac{2\pi}{3}$ radian

(iv) Radian measure of $135^o = \frac{\pi}{180} \times 135 = \frac{3\pi}{4}$ radian

(v) Radian measure of $216^o = \frac{\pi}{180} \times 216 = \frac{6\pi}{5}$ radian

(vi) Radian measure of $270^o = \frac{\pi}{180} \times 270 = \frac{3\pi}{2}$ radian

Example-2 : Find the degree measure corresponding to the following radian measure :

(i) $\frac{\pi}{15}$ (ii) $\frac{\pi}{6}$ (iii) $\frac{2\pi}{9}$

(iv) $\frac{7\pi}{4}$ (v) $\frac{11\pi}{6}$ (vi) $\frac{12\pi}{5}$

Solution :

(i) Degree measure of $\frac{\pi}{15}^R = \frac{180}{\pi} \times \frac{\pi}{15} = 12^0$

(ii) Degree measure of $\frac{\pi}{6}^R = \frac{180}{\pi} \times \frac{\pi}{6} = 30^0$

(iii) Degree measure of $\frac{2\pi}{9}^R = \frac{180}{\pi} \times \frac{2\pi}{9} = 40^0$

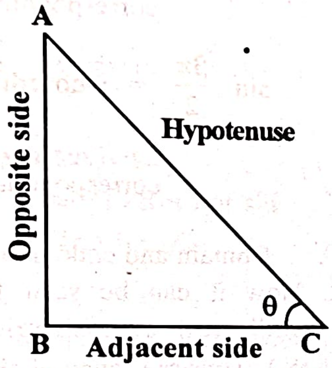
(iv) Degree measure of $\frac{7\pi}{4}^R = \frac{180}{\pi} \times \frac{7\pi}{4} = 315^0$

(v) Degree measure of $\frac{11\pi}{6}^R = \frac{180}{\pi} \times \frac{11\pi}{6} = 330^0$

(vi) Degree measure of $\frac{12\pi}{5}^R = \frac{180}{\pi} \times \frac{12\pi}{5} = 432^0$

[5] Trigonometric ratios :

In standard 9 and 10, we have defined trigonometric ratios in the context of right angle triangle, which were as follows.



$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$

$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$

$\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}}$

$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Opposite side}}$

$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$ $\cot\theta = \frac{\text{Adjacent side}}{\text{Opposite side}}$

Now we will define this trigonometric ratios as functions in the context of unit circle.

[6] Trigonometric functions :

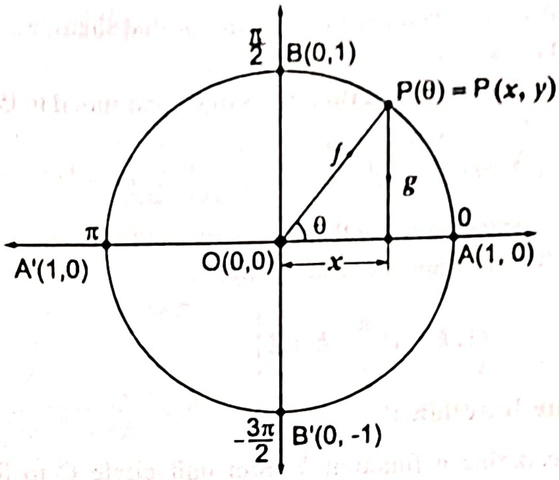
We have introduced the trigonomeric point function $f: R \to C, f(\theta) = P(\theta) = P(x, y)$

(1) cosine function :

We define a function g from unit circle C to R as follows.

$g: C \to R, g(x, y) = x$

This function g correlates every point on the unit circle with its x-coordinate.



The composite function $gof: R \to R$, of the T-point function $f: R \to C$, $f(\theta) = P(\theta) = (x, y)$ and the function $g: C \to R, g(x, y) = x$ is called cosine function and written in short as cos function.

Thus cos : R → R, $\cos\theta = (gof)(\theta)$
$= g(f(\theta))$
$= g(x, y)$
$= x$

Thus, the values of cos θ is the X-coordinate of the point p (θ) corresponding to θ on the unit circle.

- **For example :**

$\cos 0 =$ X-coordinate of the point A(1, 0) corresponding to 0 on the unit circle = 1.

$\cos\frac{\pi}{2} =$ X-coordinate of the point B(0, 1) corresponding to $\frac{\pi}{2}$ on the unit circle = 0.

$\cos\pi =$ X-coordinate of the point A'(−1, 0) corresponding to π on the unit circle = −1

Domain and codomain of the function cos : R → R is R. Now it can be seen from the figure 2.8 that the X-coordinate of trigonometric point takes all the values from -1 to 1. Thus the range of cos function is [−1, 1].

(**Note :** For any real function $f: A \to B$, the set $\{x \mid f(x) = 0, x \in A\}$ is called the set of zeroes of function f.

Thus the set of all those values of x for which $f(x) = 0$ is called the set of zeros of function f and such values of x are called the zeroes of function f.

To find the set of zeroes of cos function the x-coordinate of P(θ) should be zero.

∴ P(θ) lies on Y-axis.

But it can be seen from the figure that there are only two such points B(0, 1) and B'(0, −1).

∴ We know that the values of θ correspond to B and B' are $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, ...$ Thus, these value of θ are the zeroes of cos function.

∴ Set of zeroes of cos function

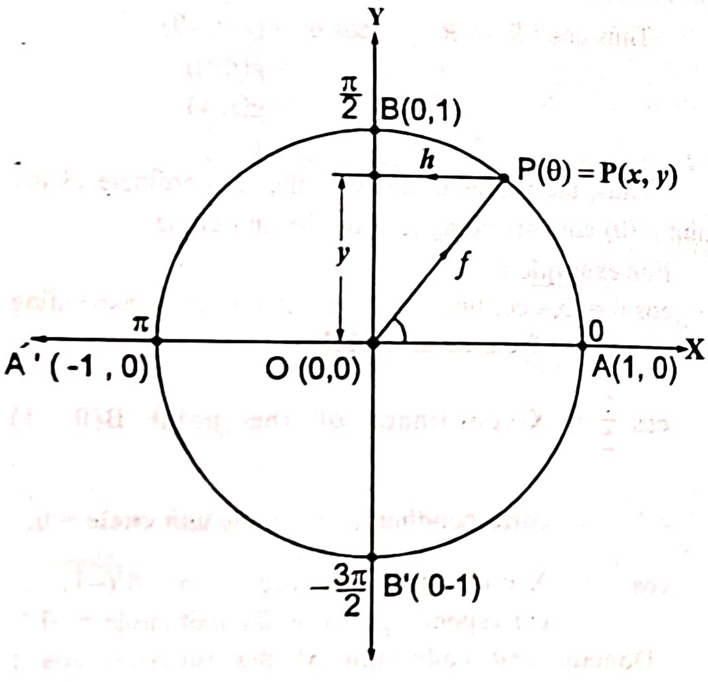
$$= \left\{(2k+1)\frac{\pi}{2} \mid k \in Z\right\}.$$

(2) sine function :

We define a function h from unit circle C to R as follows .

$h : C \to R, \; h(x, y) = y$

This function h correlates every point on the unit circle with its Y-coordinate.



The composite function $hof : R \to R$ of the T-point function $f : R \to C, \; f(\theta) = P(\theta) = (x, y)$ and the function $h : C \to R, h(x, y)) = y$ is called sine function and written in short as sin function.

Thus sin : R → R,

$$\begin{aligned}\sin\theta &= (hof)(\theta)\\ &= h(f(\theta))\\ &= h(x, y)\\ &= y\end{aligned}$$

Thus the value of sin θ is the y-coordinate of the point P(θ) corresponding to θ on the unit circle.

- **For exampe :**

sin 0 = y-coordinate of the point A(1, 0) coresonding to 0 on the unit circle = 0.

$\sin\frac{\pi}{2}$ = y-coordinate of the point B(0, 1) corresponding to $\frac{\pi}{2}$ on the unit circle = 1

sin π = y-coordinate of the point A'(−1, 0) corresponding to π on the unit circle = 0

$\sin\frac{3\pi}{2}$ = y-coordinate of the point B'(0, −1) corresponding to $\frac{3\pi}{2}$ on the unit circle = −1

Domain and codomain of the function sin : R → R is R. Now it can be seen from the figure 2.9 that the y-coordinate of trigonometric point takes all the values from −1 to 1. Thus the range of sin function is [−1, 1].

To find the set of zeroes of sin function, the y-coordinate of P(θ) should be zero.

∴ P(θ) lies on X-axis

But it can be seen from the figure 2.9 that there are only two such points A(1, 0) and A'(−1, 0).

We know that the values of θ correspond to A and A' are 0, ± π, ± 2π, ± 3π, ... Thus these values of θ are the zeroes of sin function.

∴ set of zeroes of sin function = $\{k\pi \mid k \in Z\}$

(**Note** : Thus the coordinates of the point P(θ) corresponding to θ on the unit circle are (x, y) = (cos θ, sin θ)).

(3) tangent function :

(**Note :** If $f : A \to R, g : B \to R$ are functions where $A \subset R, B \subset R$ and $(A \cap B) - \{x/g(x) = 0\} \neq \phi$ then $\frac{f}{g} : A \cap B - \{x/g(x) = 0\} \to R$ is called the quotient of two functions f and g. It is defined as $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$.)

The quotient function of sin : R → R and cos : R → R is called tangent function and is written in short as tan.

Thus, $\tan = \frac{\sin}{\cos} : R - \left\{(2k+1)\frac{\pi}{2} \mid k \in Z\right\} \to R$

where $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

Domain and codomain of tan function are respectively $R - \left\{(2k+1)\frac{\pi}{2} \mid k \in z\right\}$ and R. We will accept that the range of tan function is R without proof.

Now to find the set of zeroes of tan function we should have $\tan\theta = 0$.

$\therefore \quad \dfrac{\sin\theta}{\cos\theta} = 0$

$\therefore \quad \sin\theta = 0$

Thus $\tan\theta = 0$ if and only if $\sin\theta = 0$.

$\therefore$ set of zeroes of tan $=$ set of zeroes of sin $= \{k\pi \mid k \in Z\}$

(4) cotangent function :

Domain and codomain of cot function are respectively $R - \{k\pi \mid k \in z\}$ and R. We will accept that the range of the cot function is R without proof.

Now to find the set of zeroes of cot function we should have $\cot\theta = 0$.

Thus $\cot = \dfrac{\cos}{\sin} : R - \{k\pi \mid k \in Z\} \rightarrow R$,

Where $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$.

The quotient function of cos : R → R and sin : R → R is called cotangent function and is written in short as cot.

$\therefore \quad \dfrac{\cos\theta}{\sin\theta} = 0$

$\therefore \quad \cos\theta = 0$

Thus $\cot\theta = 0$ if and only if $\cos\theta = 0$

$\therefore$ Set of zeroes of cot $=$ set of zeroes of cos $= \left\{(2k+1)\frac{\pi}{2} \mid k \in Z\right\}$

(5) secant function :

The quotient function of the function $f : R \rightarrow R$, $f(x) = 1$ and cos : R → R is called secant function and is written in short as sec.

Thus,

$$\sec = \frac{1}{\cos} : R - \left\{(2k+1)\frac{\pi}{2} \mid k \in Z\right\} \rightarrow R$$

where $\sec\theta = \dfrac{1}{\cos\theta}$.

Domain and codomain of sec function are respectively $R - \left\{(2k+1)\frac{\pi}{2} \mid k \in Z\right\}$ and R. We will accept without proof that the range of the sec function is $R - (-1, 1)$ and its set of zeroes is ϕ.

(6) cosecant function :

The quotient function of the functions $f: R \rightarrow R$, $f(x) = 1$ and sin : R → R is called cosecant function and is written in short as cosec.

Thus $\operatorname{cosec} = \dfrac{1}{\sin} : R - \{k\pi \mid k \in Z\} \rightarrow R$

Where $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$.

Domain and codomain of cosec function are $R - \{k\pi / k \in z\}$ and R respectively. We will accept without proof that the range of cosec function is $R - (-1, 1)$ and its set of zeroes is ϕ.

→ The following table shows domain, codomain, range and set of zeroes of trigonometric functions.

Function	Domain	Codomain	Range	Set of zeroes
cos	R	R	[–1, 1]	$\left\{(2k+1)\frac{\pi}{2} \mid k \in Z\right\}$
sin	R	R	[–1, 1]	$\{k\pi / k \in Z\}$
tan	$R - \left\{(2k+1)\frac{\pi}{2} / k \in Z\right\}$	R	R	$\{k\pi / k \in Z\}$
cot	$R - \{k\pi / k \in Z\}$	R	R	$\left\{(2k+1)\frac{\pi}{2} / k \in Z\right\}$
sec	$R - \{(2k+1)\frac{\pi}{2} / k \in Z\}$	R	$R - (-1, 1)$	ϕ
cosec	$R - \{k\pi / k \in Z\}$	R	$R - (-1, 1)$	ϕ

The following table shows the values of trigonometric function for the different values of θ.

θ → / T-Function ↓	0 0°	$\frac{\pi}{6}$ 30°	$\frac{\pi}{4}$ 45°	$\frac{\pi}{3}$ 60°	$\frac{\pi}{2}$ 90°	π 180°	$\frac{3\pi}{2}$ 270°	2π 360°
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	–1	0	1
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	–1	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	–	0	–	0
cot	–	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	–	0	–
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	–	–1	–	1
cosec	–	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	–	–1	–

[7] Relations between Trigonometric functions :

(1) $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ $\quad\therefore\ \sin\theta \cdot \operatorname{cosec}\theta = 1$

(2) $\sec\theta = \dfrac{1}{\cos\theta}$ $\quad\therefore\ \cos\theta \cdot \sec\theta = 1$

(3) $\tan\theta = \dfrac{1}{\cot\theta}$ $\quad\therefore\ \tan\theta \cdot \cot\theta = 1$

(4) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\quad\therefore\ \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

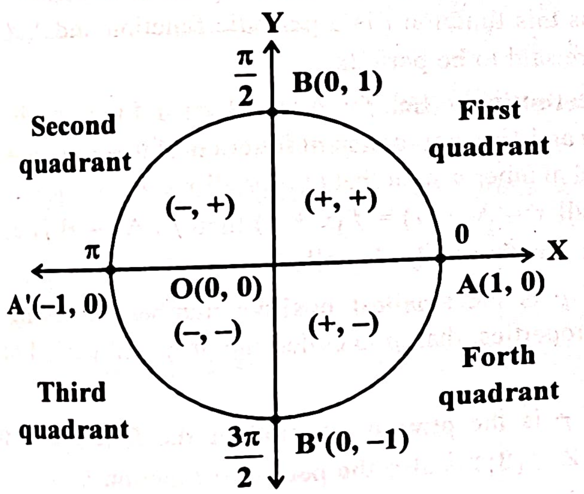
(5) $\sin^2\theta + \cos^2\theta = 1$

(6) $\sec^2\theta - \tan^2\theta = 1$

(7) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

[8] Sign of Trigonometric functions :

If $P(\theta) = P(x, y)$ is a point on the unit circle corresponding to θ, then by definition of cos and sin functions, $x = \cos\theta$ and $y = \sin\theta$.



From the above figure it can be seen that

If $P(\theta)$ lies in first quadrant, then $x > 0$ and $y > 0$

$\therefore\ \cos\theta > 0$ and $\sin\theta > 0$

If $P(\theta)$ lies in second quadrant, than $x < 0$ and $y > 0$

$\therefore\ \cos\theta < 0$ and $\sin\theta > 0$

If $P(\theta)$ lies in Third quadrant, then $x < 0$ and $y < 0$

$\therefore\ \cos\theta < 0$ and $\sin\theta < 0$

and if $P(\theta)$ lies in fourth quadrant, then $x > 0$ and $y < 0$

$\therefore\ \cos\theta > 0$ and $\sin\theta < 0$

The signs of the remaining trigonometric function can be obtained from the signs of cos and sin functions which are shown in the following table.

Quadrant → / T-Functions ↓	First $\left(0, \frac{\pi}{2}\right)$	Second $\left(\frac{\pi}{2}, \pi\right)$	Third $\left(\pi, \frac{3\pi}{2}\right)$	Fourth $\left(\frac{3\pi}{2}, 2\pi\right)$
cos	+	–	–	+
sin	+	+	–	–
tan	+	–	+	–
cot	+	–	+	–
sec	+	–	–	+
cosec	+	+	–	–

To remember, the sign of which trigonometric functions is positive in each quadrant, remember ALL, S, T, C shown in the figure which are interpreted as follows :

ALL → All trigonometric functions are positive.

S → sin function is positive and so cosec function is also positive, the rest of the T-functions are negative.

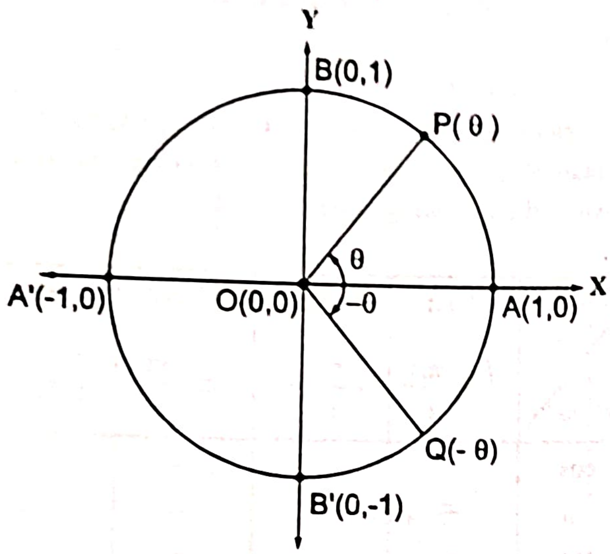
S	ALL
T	C

T → tan function is positive and so cot function is also positive, the rest of the T-functions are negative.

C → cos function is positive and so sec function is also positive, the rest of the T-functions are negative.

[9] Trigonometric functions for negative angles :

If $P(\theta) = P(x, y)$ is a point on the unit circle corresponding to θ, then $Q(-\theta) = Q(x, -y)$ is a point on the unit circle corresponding to $-\theta$, which is shown in the following figure.



$\therefore$ According to the definition of cos function,

$\cos\theta = x$ and $\cos(-\theta) = x$ $\quad \therefore \cos(-\theta) = \cos\theta$

Similarly, according to the definition of sin function

$\sin\theta = y$ and $\sin(-\theta) = -y$ $\quad \therefore \sin(-\theta) = -\sin\theta$

$$\therefore \quad \tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta,$$

$$\cot(-\theta) = \frac{\cos(-\theta)}{\sin(-\theta)} = \frac{\cos\theta}{-\sin\theta} = -\cot\theta,$$

$$\sec(-\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos\theta} = \sec\theta \text{ and}$$

$$\operatorname{cosec}(-\theta) = \frac{1}{\sin(-\theta)} = \frac{1}{-\sin\theta} = -\operatorname{cosec}\theta$$

Example-3 : Evaluate :

(i) $\sin\frac{\pi}{6}\sin\frac{\pi}{3} + \sin\frac{\pi}{2}\sin\pi$

(ii) $\sin 90^\circ \cdot \sin 60^\circ \cdot \sin 45^\circ \cdot \sin 0^\circ$

(i) $\sin\frac{\pi}{6}\sin\frac{\pi}{3} + \sin\frac{\pi}{2}\sin\pi$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + (1)(0) = \frac{\sqrt{3}}{4}$$

(ii) $\sin 90^\circ \cdot \sin 60^\circ \cdot \sin 45^\circ \cdot \sin 0^\circ$

$$= (1)\cdot\left(\frac{\sqrt{3}}{2}\right)\cdot\left(\frac{1}{\sqrt{2}}\right)\cdot 0 = 0$$

[10] Period of Trigonometric functions :

There are many phenomena in practice and in science that recur over a period of time. For example the earth takes 24 hours to complete one rotation on its axis and after 24 hours it repeats the previous action again. The minute hand of the clock keeps changing its position during the first 1 hour. But after 1 hour again it repeats the previous action. Events that recur at certain times are called periodic events and that certain time is called its period. In the above examples the period of the rotation of the earth is 24 hours and the period of the minute hand of the clock is 1 hour.

Similarly in mathematics, the values of many functions are repeated over a period of time. For example, the function $f: N \to N \cup \{0\}$, $f(n)$ = the remainder of n divided by 4. Now the values of $f(n)$ for different values of n are shown in the following table.

n	1	2	3	4	5	6	7	8	9	10	11	12
$f(n)$	1	2	3	0	1	2	3	0	1	2	3	0

It can be seen from the above table that the values 1, 2, 3 and 0 of the function f are repeated in the same order with period 4. Also it can be said that it repeats with period 8 and 12. Thus this function f is a periodic function and 4, 8, 12, ...etc. are said to be periods.

Definition : Let $f : A \to B$ be a function of real variable and f is a non-constant function. If there exists a non zero real number p such that (1) for all $x \in A$, $x + p \in A$ and (2) for all $x \in A$, $f(x) = f(x+p)$ then $f : A \to B$ is called periodic function and p is called a period of f.

If p is the smallest positive number satisfying the above properties, than p is called the principal period of $f : A \to B$.

If p is the principal period of the function f, then np $(n \in Z - \{0\})$ is also the period of function f.

Trigonometric functions are also periodic functions and their principal periods are shown in the table below, which we accept without proof.

T-Functions	cos	sin	tan	cot	sec	cosec
Principal period	2π	2π	π	π	2π	2π

Since the principal period of cos, sin, sec and cosec is 2π, $2k\pi$, $k \in z - \{0\}$ is also its period. Similarly the principal period of tan and cot is cot π, $k\pi$, $k \in z - \{0\}$ is also its period.

$\therefore$ for all $k \in z$

$\cos(2k\pi + \theta) = \cos\theta \qquad \sec(2k\pi + \theta) = \sec\theta$

$\sin(2k\pi + \theta) = \sin\theta$ $\qquad$ $\operatorname{cosec}(2k\pi + \theta) = \operatorname{cosec}\theta$

$\tan(k\pi + \theta) = \tan\theta$ $\qquad$ $\cot(k\pi + \theta) = \cot\theta$

The formulae for finding the principal periods of trigonometric functions are given in the following table, where $a, b, c, d, \in R$.

T-Functions	Principal period
$a\cos(bx + c) + d$	$\frac{2\pi}{\lvert b\rvert}$
$a\sin(bx + c) + d$	$\frac{2\pi}{\lvert b\rvert}$
$a\sec(bx + c) + d$	$\frac{2\pi}{\lvert b\rvert}$
$a\operatorname{cosec}(bx + c) + d$	$\frac{2\pi}{\lvert b\rvert}$
$a\tan(bx + c) + d$	$\frac{\pi}{\lvert b\rvert}$
$a\cot(bx + c) + d$	$\frac{\pi}{\lvert b\rvert}$

It can be seen from the above table that a, c and d have no effect on the principal period.

Example 4 : Find the principal periods of the following T-functions.

(i) $f(x) = 3\cos 2x$

(ii) $f(x) = \sin(2x + 7)$

(iii) $f(x) = \cos(3x + 5)$

(iv) $f(x) = \cot\frac{x}{6}$

Solution :

(i) $f(x) = 3\cos 2x$

$\therefore$ b = coefficient of $x = 2$

$\therefore$ Principal period of the function

$$f = \frac{2\pi}{|b|} = \frac{2\pi}{|2|} = \pi$$

(ii) $f(x) = \sin(2x + 7)$

$\therefore$ b = coefficient of $x = 2$

$\therefore$ Principal period of the function

$$f = \frac{2\pi}{|b|} = \frac{2\pi}{|2|} = \pi$$

(iii) $f(x) = \cos(3x + 5)$

$\therefore$ b = coefficient of $x = 3$

$\therefore$ Principal period of the function

$$= \frac{2\pi}{|b|} = \frac{2\pi}{|3|} = \frac{2\pi}{3}$$

(iv) $f(x) = \cot\frac{x}{6}$

$\therefore$ b = coefficient of $x = \frac{1}{6}$

$\therefore$ Principal period of the function f

$$= \frac{\pi}{|b|} = \frac{\pi}{\left|\frac{1}{6}\right|} = 6\pi$$

[11] Addition formulae :

Now we will see that how to express the values of trigonometric functions for the numbers $\alpha + \beta$ and $\alpha - \beta$ obtained by adding and subtracting two real numbers α and β using the values of trigonometric functions for α and β. The formulae for this are called addition formulae.

(1) $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha + \sin\beta, \alpha, \beta \in R$

We will accept the above result without proof.

(2) $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta, \alpha, \beta \in R$

Proof :

$\cos(\alpha + \beta) = \cos(\alpha - (-\beta))$

$= \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta)$ (from (1))

$= \cos\alpha\cos\beta + \sin\alpha(-\sin\beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

(3) (i) $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta,$

(ii) $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta, \quad \theta \in R$

Proof :

(i) $\cos\left(\frac{\pi}{2} - \theta\right) = \cos\frac{\pi}{2}\cos\theta + \sin\frac{\pi}{2}\sin\theta$ (from (1))

$= 0 \cdot \cos\theta + 1 \cdot \sin\theta$

$= \sin\theta$

(ii) Taking $\frac{\pi}{2} - \theta$ instead of θ in the above result

$$\cos\left(\frac{\pi}{2} - \left(\frac{\pi}{2} - \theta\right)\right) = \sin\left(\frac{\pi}{2} - \theta\right)$$

$\therefore \quad \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

$\therefore \quad \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

Remember that in standard-10, we learned the formulae $\cos(90^\circ - \theta) = \sin\theta$ and $\sin(90^\circ - \theta) = \cos\theta$.

(4) $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta,\ \alpha, \beta \in R$

Proof :

$\sin(\alpha - \beta) = \cos\left(\frac{\pi}{2} - (\alpha - \beta)\right)$ (from (3) (i))

$= \cos\left(\left(\frac{\pi}{2} - \alpha\right) + \beta\right)$

$= \cos\left(\frac{\pi}{2} - \alpha\right)\cos\beta - \sin\left(\frac{\pi}{2} - \alpha\right)\sin\beta$ (from (2))

$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$ (from (3))

(5) $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta,\ \alpha, \beta \in R$

Proof :

$\sin(\alpha + \beta) = \sin(\alpha - (-\beta))$

$= \sin\alpha\cos(-\beta) - \cos\alpha\sin(-\beta)$ ((4) પરથી)

$= \sin\alpha\cos\beta - \cos\alpha(-\sin\beta)$

$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$

[12] Some important results :

(1) $\sin(\alpha + \beta)\cdot\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha$

Proof :

$\sin(\alpha + \beta)\cdot\sin(\alpha - \beta)$

$= (\sin\alpha\cos\beta + \cos\alpha\sin\beta)(\sin\alpha\cos\beta - \cos\alpha\sin\beta)$

$= \sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta$

$= \sin^2\alpha(1 - \sin^2\beta) - (1 - \sin^2\alpha)\sin^2\beta$

$= \sin^2\alpha - \sin^2\alpha\sin^2\beta - \sin^2\beta + \sin^2\alpha\sin^2\beta$

$= \sin^2\alpha - \sin^2\beta$

$= (1 - \cos^2\alpha) - (1 - \cos^2\beta)$

$= 1 - \cos^2\alpha - 1 + \cos^2\beta$

$= \cos^2\beta - \sin^2\alpha$

(2) $\cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta = \cos^2\beta - \sin^2\alpha$

Proof :

$\cos(\alpha + \beta)\cdot\cos(\alpha - \beta)$

$= (\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$= \cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta$

$= \cos^2\alpha(1 - \sin^2\beta) - (1 - \cos^2\alpha)\sin^2\beta$

$= \cos^2\alpha - \cos^2\alpha\sin^2\beta - \sin^2\beta + \cos^2\alpha\sin^2\beta$

$= \cos^2\alpha - \sin^2\beta$

$= (1 - \sin^2\alpha) - (1 - \cos^2\beta)$

$= 1 - \sin^2\alpha - 1 + \cos^2\beta$

$= \cos^2\beta - \sin^2\alpha$

(3) The values of sin and cos functions for $\frac{\pi}{12}(15^\circ)$ and $\frac{5\pi}{12}(75^\circ)$:

$\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$ $(\because 60^\circ - 45^\circ = 15^\circ)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{2}\cdot\frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6} - \sqrt{2}}{4}$

Similarly,

$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

Now, $\sin\frac{5\pi}{12} = \sin\left(\frac{6\pi-\pi}{12}\right) = \sin\left(\frac{\pi}{2}-\frac{\pi}{12}\right)$

$$= \cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

and $\cos\frac{5\pi}{12} = \cos\left(\frac{6\pi-\pi}{12}\right) = \cos\left(\frac{\pi}{2}-\frac{\pi}{12}\right)$

$$= \sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

The values of remaining trigonometric functions can be easily derived from the values of sin and cos functions.

[13] Formulae for allied angles :

We learned the following four results in 2.12.

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ ... (i)

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ ... (ii)

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ ... (iii)

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ ... (iv)

We have proved in point 11 (3) that

$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$ and $\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$

$$\therefore \tan\left(\frac{\pi}{2}-\theta\right) = \frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\cos\left(\frac{\pi}{2}-\theta\right)} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

$$\text{and } \sec\left(\frac{\pi}{2}-\theta\right) = \frac{1}{\cos\left(\frac{\pi}{2}-\theta\right)} = \frac{1}{\sin\theta} = \operatorname{cosec}\theta$$

Similarly $\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right) = \sec\theta$

Now substitute $\alpha = \frac{\pi}{2}$ and $\beta = \theta$ in (ii) and (iv)

$$\cos\left(\frac{\pi}{2}+\theta\right) = \cos\frac{\pi}{2}\cos\theta - \sin\frac{\pi}{2}\sin\theta$$
$$= 0\cdot\cos\theta - 1\cdot\sin\theta$$
$$= -\sin\theta$$

and $\sin\left(\frac{\pi}{2}+\theta\right) = \sin\frac{\pi}{2}\cos\theta + \cos\frac{\pi}{2}\sin\theta$

$$= 1\cdot\cos\theta + 0\cdot\sin\theta$$
$$= \cos\theta$$

$\therefore$ $\tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$, $\cot\left(\frac{\pi}{2}+\theta\right) = -\tan\theta$

$\sec\left(\frac{\pi}{2}+\theta\right) = -\operatorname{cosec}\theta$ and $\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right) = \sec\theta$

Similarly, taking $\alpha = \pi$ and $\beta = \theta$ in (i) and (iii),

$\cos(\pi-\theta) = -\cos\theta$, $\sin(\pi-\theta) = \sin\theta$,

$\therefore$ $\tan(\pi-\theta) = -\tan\theta$, $\cot(\pi-\theta) = -\cot\theta$

$\sec(\pi-\theta) = -\sec\theta$, $\operatorname{cosec}(\pi-\theta) = \operatorname{cosec}\theta$,

and taking $\alpha = \pi$ and $\beta = \theta$ in (ii) and (iv),

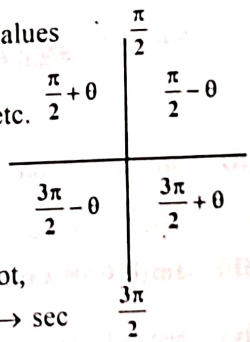
$\cos(\pi+\theta) = -\cos\theta$, $\sin(\pi+\theta) = -\sin\theta$,

$\therefore$ $\tan(\pi+\theta) = \tan\theta$, $\cot(\pi+\theta) = \cot\theta$

$\sec(\pi+\theta) = -\sec\theta$, $\operatorname{cosec}(\pi+\theta) = -\operatorname{cosec}\theta$,

Similarly the results for $\frac{3\pi}{2} \pm \theta$, $\frac{5\pi}{2} \pm \theta$,... etc. as well as $2\pi \pm \theta$, $3\pi \pm \theta$ ... etc. can also be derived. But how to remember all these results ? Let us consider the following trick.

→ As shown in the figure for the values $\frac{\pi}{2} \pm \theta$, $\frac{3\pi}{2} \pm \theta$, $\frac{5\pi}{2} \pm \theta$, ... etc.



first of all change the function as follows.

sin → cos, cos → sin, tan → cot,

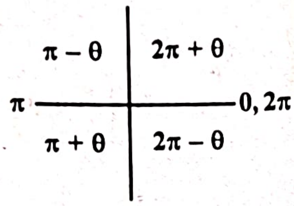
cot → tan, sec → cosec, cosec → sec

After that keep the sign of resulting function same as the sign of the original function according to the quadrant.

For example, for $\cos\left(\frac{3\pi}{2}-\theta\right)$, change the function cos → sin . Now $\frac{3\pi}{2}-\theta$ is in third quadrant and in the third quadrant, cos is negative.

$\therefore \quad \cos\left(\frac{3\pi}{2} - \theta\right) = -\sin\theta$ થાય.

→ As shown in the figure, for the values $\pi \pm \theta$, $2\pi \pm \theta$, $3\pi \pm \theta$ ··· etc. keep the function as it is and keep the sign of the resulting function same as the sign of the original function according to the quadrant.



For example, for $\sin(2\pi - \theta)$, keep the sin function as it is . Now $2\pi - \theta$ is in fourth quadrant and in the fourth quadrant sin function is neagtive.

$\therefore \quad \sin(2\pi - \theta) = -\sin\theta$

Example-5 :

(i) $\sin 120^\circ$

(ii) $\sin 135^\circ$

(iii) $\sin 150^\circ$

(iv) $\sec(-1305^\circ)$

(v) $\cot 225^\circ$

(vi) $\sin\left(-\frac{17\pi}{4}\right)$ **(vii)** $\tan\left(-\frac{15\pi}{4}\right)$

(viii) $\text{cosec}\left(\frac{17\pi}{6}\right)$

Solution :

(i) $\sin 120^\circ = \sin(180^\circ - 60^\circ)$ $(180^\circ = \pi^R)$

$= \sin 60^\circ = \frac{\sqrt{3}}{2}$

(ii) $\sin 135^\circ = \sin(180^\circ - 45^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$

(iii) $\sin 150^\circ = \sin(180^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2}$

(iv) $\sec(-1305^\circ) = \sec 1305^\circ$ $(\because \sec(-\theta) = \sec\theta)$

$= \sec(15 \times 90^\circ - 45^\circ)$

$\left(15 \times 90^\circ = \frac{15\pi^R}{2}\right)$

$= -\text{cosec}\, 45^\circ$

$= -\sqrt{2}$

(v) $\cot 225^\circ = \cot(180^\circ + 45^\circ) = \cot 45^\circ = 1$

(vi) $\sin\left(-\frac{17\pi}{4}\right) = -\sin\frac{17\pi}{4}$

$= -\sin\left(\frac{16\pi + \pi}{4}\right)$

$= -\sin\left(4\pi + \frac{\pi}{4}\right)$

$= -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

(vii) $\tan\left(-\frac{15\pi}{4}\right) = -\tan\left(\frac{15\pi}{4}\right)$

$= -\tan\left(\frac{16\pi - \pi}{4}\right)$

$= -\tan\left(4\pi - \frac{\pi}{4}\right)$

$= -\left(-\tan\frac{\pi}{4}\right)$

$= \tan\frac{\pi}{4} = 1$

(viii) $\text{cosec}\left(\frac{17\pi}{6}\right) = \text{cosec}\left(\frac{18\pi - \pi}{6}\right)$

$= \text{cosec}\left(3\pi - \frac{\pi}{6}\right)$

$= \text{cosec}\,\frac{\pi}{6}$

$= 2$

[14] Addition formulae for tan and cot :

(1) $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$

Proof :

$\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)}$

$= \frac{\sin\alpha \cdot \cos\beta + \cos\alpha \sin\beta}{\cos\alpha \cos\beta - \sin\alpha \sin\beta}$

$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$

(Dividing the numerator and denominator by $\cos\alpha \cdot \cos\beta$)

Now we will accept the following results wihtout proof.

(2) $$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \cdot \tan\beta}$$

(3) $$\cot(\alpha + \beta) = \frac{\cot\alpha \cdot \cot\beta - 1}{\cot\beta + \cot\alpha}$$

(4) $$\cot(\alpha - \beta) = \frac{\cot\alpha \cdot \cot\beta + 1}{\cot\beta - \cot\alpha}$$

(5) **The values of tan and cot functions for $\frac{\pi}{12}$ (15°) and $\frac{5\pi}{12}$ (75°) :**

$$\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3} \cdot \tan\frac{\pi}{4}}$$

$$= \frac{\sqrt{3} - 1}{1 + (\sqrt{3})(1)}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$= \frac{3 - 2\sqrt{3} + 1}{3 - 1}$$

$$= \frac{4 - 2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3}$$

and $$\cot\frac{\pi}{12} = \frac{1}{\tan\frac{\pi}{12}} = \frac{1}{2 - \sqrt{3}}$$

$$= \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{4 - 3}$$

$$\therefore \quad \cot\frac{\pi}{12} = 2 + \sqrt{3}$$

Now, $$\tan\frac{5\pi}{12} = \tan\left(\frac{6\pi - \pi}{12}\right) = \tan\left(\frac{\pi}{2} - \frac{\pi}{12}\right)$$

$$= \cot\frac{\pi}{12} = 2 + \sqrt{3}$$

$$\cot\frac{5\pi}{12} = \cot\left(\frac{6\pi - \pi}{12}\right) = \cot\left(\frac{\pi}{2} - \frac{\pi}{12}\right)$$

$$= \tan\frac{\pi}{12} = 2 - \sqrt{3}$$

Example-6 : Prove that :

$$\tan 55^\circ = \frac{\cos 10^\circ + \sin 10^\circ}{\cos 10^\circ - \sin 10^\circ}$$

Solution :

L.H.S. $= \tan 55^\circ = \tan(45^\circ + 10^\circ)$

$$= \frac{\tan 45^\circ + \tan 10^\circ}{1 - \tan 45^\circ \cdot \tan 10^\circ}$$

$$= \frac{1 + \tan 10^\circ}{1 - \tan 10^\circ} \qquad (\because \tan 45^\circ = 1)$$

$$= \frac{1 + \dfrac{\sin 10^\circ}{\cos 10^\circ}}{1 - \dfrac{\sin 10^\circ}{\cos 10^\circ}}$$

$$= \frac{\cos 10^\circ + \sin 10^\circ}{\cos 10^\circ - \sin 10^\circ}$$

= R.H.S.

[15] Expression of product in the form of sum or difference :

In 2.12, we have derived the following formulae for $\alpha, \beta \in R$.

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ ··· (i)

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ ··· (ii)

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ ··· (iii)

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ ··· (iv)

By adding results (i) and (ii),

$2\sin\alpha\cos\beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$

By subtracting result (ii) from (i),

$2\cos\alpha\sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$

By adding result (iii) and (iv),

$2\cos\alpha\cos\beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$

By subtracting results (iv) from (iii),

$-2\sin\alpha\sin\beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$

OR

$2 \sin\alpha \sin\beta = -\cos(\alpha+\beta) + \cos(\alpha-\beta)$

The above formulae can be remembered briefly as follows :

2SC = S + S

2CS = S − S

2CC = C + C

−2SS = C − C

Example-7 : Express the following in the form of sum or difference :

(i) $2 \sin 5\theta \cdot \cos 2\theta$

(ii) $2\cos\frac{5\theta}{2} \cdot \sin\frac{3\theta}{2}$

(iii) $2\cos 3\theta \cdot \cos\frac{\theta}{2}$

(iv) $2\sin\frac{3\theta}{2} \cdot \sin 2\theta$

Solution :

(i) $2\sin 5\theta \cdot \cos 2\theta = \sin(5\theta + 2\theta) + \sin(5\theta - 2\theta)$

$= \sin 7\theta + \sin 3\theta$

(ii) $2\cos\frac{5\theta}{2} \cdot \sin\frac{3\theta}{2} = \sin\left(\frac{5\theta}{2} + \frac{3\theta}{2}\right) - \sin\left(\frac{5\theta}{2} - \frac{3\theta}{2}\right)$

$= \sin\left(\frac{8\theta}{2}\right) - \sin\left(\frac{2\theta}{2}\right)$

$= \sin 4\theta - \sin\theta$

(iii) $2\cos 3\theta \cdot \cos\frac{\theta}{2} = \cos\left(3\theta + \frac{\theta}{2}\right) + \cos\left(3\theta - \frac{\theta}{2}\right)$

$= \cos\frac{7\theta}{2} + \cos\frac{5\theta}{2}$

(iv) $2\sin\frac{3\theta}{2} \cdot \sin 2\theta = 2\sin 2\theta \cdot \sin\frac{3\theta}{2}$

$= -\cos\left(2\theta + \frac{3\theta}{2}\right) + \cos\left(2\theta - \frac{3\theta}{2}\right)$

$= -\cos\frac{7\theta}{2} + \cos\frac{\theta}{2}$

[16] Expression of sum or difference in the form of product :

We have derived the following results in 2.16.

$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

$2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$

$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

$-2\sin\alpha\sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

By substituting $\alpha + \beta = A$ and $\alpha - \beta = B$ in the above results,

$\alpha = \frac{A+B}{2}$ and $\beta = \frac{A-B}{2}$.

By substituting these values of α and β in the above results,

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$$

Example-8 : Express the following in the form of produc

(i) $\sin 7\theta + \sin 5\theta$ (ii) $\sin 6\theta - \sin 2\theta$

Solution :

(i) $\sin 7\theta + \sin 5\theta = 2\sin\left(\frac{7\theta + 5\theta}{2}\right) \cdot \cos\left(\frac{7\theta - 5\theta}{2}\right)$

$= 2\sin\left(\frac{12\theta}{2}\right) \cdot \cos\left(\frac{2\theta}{2}\right)$

$= 2\sin 6\theta \cdot \cos\theta$

(ii) $\sin 6\theta - \sin 2\theta$

$= 2\cos\left(\frac{6\theta + 2\theta}{2}\right) \cdot \sin\left(\frac{6\theta - 2\theta}{2}\right)$

$= 2\cos\left(\frac{8\theta}{2}\right) \cdot \sin\left(\frac{4\theta}{2}\right)$

$= 2\cos 4\theta \cdot \sin 2\theta$

[17] Formulae for Multiples and Sub–Multiples Angles :

1. **Formulae for $2\alpha \to \alpha$:**

(i) Substituting $\beta = \alpha$ in $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$,

$\sin(\alpha + \alpha) = \sin\alpha\cos\alpha + \cos\alpha\sin\alpha$

$\therefore$ $\mathbf{\sin 2\alpha = 2\sin\alpha\cos\alpha}$ ... (a)

$$= \frac{2\sin\alpha\cos\alpha}{\cos^2\alpha + \sin^2\alpha}$$

$\therefore$ $\sin 2\alpha = \dfrac{2\tan\alpha}{1 + \tan^2\alpha}$ ... (b)

(Dividing numerator and denominator by $\cos^2\alpha$)

(ii) Substituting $\beta = \alpha$ in $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$,

$\cos(\alpha + \alpha) = \cos\alpha \cdot \cos\alpha - \sin\alpha \cdot \sin\alpha$

$\therefore$ $\mathbf{\cos 2\alpha = \cos^2\alpha - \sin^2\alpha}$ ... (c)

$= 1 - \sin^2\alpha - \sin^2\alpha$

$\therefore$ $\mathbf{\cos 2\alpha = 1 - 2\sin^2\alpha}$ ... (d)

$= 1 - 2(1 - \cos^2\alpha)$

$= 1 - 2 + 2\cos^2\alpha$

$\mathbf{\cos 2\alpha = 2\cos^2\alpha - 1}$ .... (e)

Now from (c),

$$\cos 2\alpha = \frac{\cos^2\alpha - \sin^2\alpha}{\cos^2\alpha + \sin^2\alpha}$$

$\therefore$ $\cos 2\alpha = \dfrac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$ ... (f)

(Dividing numerator and denominator by $\cos^2\alpha$)

(iii) Substituting $\beta = \alpha$ in

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta},$$

$$\tan(\alpha + \alpha) = \frac{\tan\alpha + \tan\alpha}{1 - \tan\alpha \cdot \tan\alpha}$$

$\therefore$ $\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$ ... (g)

2. **Formulae for $3\alpha \to \alpha$:**

(i) $\sin 3\alpha = \sin(2\alpha + \alpha)$

$= \sin 2\alpha\cos\alpha + \cos 2\alpha\sin\alpha$

$= 2\sin\alpha\cos\alpha \cdot \cos\alpha + (1 - 2\sin^2\alpha) \cdot \sin\alpha$

$= 2\sin\alpha \cdot \cos^2\alpha + \sin\alpha - 2\sin^3\alpha$

$= 2\sin\alpha(1 - \sin^2\alpha) + \sin\alpha - 2\sin^3\alpha$

$= 2\sin\alpha - 2\sin^3\alpha + \sin\alpha - 2\sin^3\alpha$

$= 3\sin\alpha - 4\sin^3\alpha$

$\therefore$ $\mathbf{\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha}$ ... (h)

(ii) $\cos 3\alpha = \cos(2\alpha + \alpha)$

$= \cos 2\alpha\cos\alpha - \sin 2\alpha\sin\alpha$

$= (2\cos^2\alpha - 1)\cos\alpha - 2\sin\alpha \cdot \cos\alpha \cdot \sin\alpha$

$= 2\cos^3\alpha - \cos\alpha - 2\sin^2\alpha \cdot \cos\alpha$

$= 2\cos^3\alpha - \cos\alpha - 2(1 - \cos^2\alpha) \cdot \cos\alpha$

$= 2\cos^3\alpha - \cos\alpha - 2\cos\alpha + 2\cos^3\alpha$

$= 4\cos^3\alpha - 3\cos\alpha$

$\therefore$ $\mathbf{\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha}$ ... (i)

(iii) $\tan 3\alpha = \tan(2\alpha + \alpha)$

$$= \frac{\tan 2\alpha + \tan\alpha}{1 - \tan 2\alpha \cdot \tan\alpha}$$

$$= \frac{\left(\dfrac{2\tan\alpha}{1 - \tan^2\alpha}\right) + \tan\alpha}{1 - \left(\dfrac{2\tan\alpha}{1 - \tan^2\alpha}\right) \cdot \tan\alpha}$$

$$= \frac{2\tan\alpha + \tan\alpha - \tan^3\alpha}{1 - \tan^2\alpha - 2\tan^2\alpha}$$

$\therefore$ $\tan 3\alpha = \dfrac{3\tan\alpha - \tan^3\alpha}{3\tan^2\alpha}$ ... (j)

3. **Formulae for $\dfrac{\alpha}{2} \to \alpha$:**

(i) Replace 2α by α in $\cos 2\alpha = 2\cos^2\alpha - 1$,

$$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1$$

$\therefore$ $2\cos^2\dfrac{\alpha}{2} = 1 + \cos\alpha$

$\therefore \quad \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$ ... (k)

(ii) Replace 2α by α in $\cos 2\alpha = 1 - 2\sin^2\alpha$,

$\cos\alpha = 1 - 2\sin^2\frac{\alpha}{2}$

$\therefore \quad 2\sin^2\frac{\alpha}{2} = 1 - \cos\alpha$

$\therefore \quad \sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$ ... (l)

(iii) $\tan^2\frac{\alpha}{2} = \dfrac{\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}} = \dfrac{\frac{1-\cos\alpha}{2}}{\frac{1+\cos\alpha}{2}}$

$\therefore \quad \tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha}$ ... (m)

[18] Values of trigonometric functions for some special numbers :

1. **Values for $18^\circ \left(\frac{\pi}{10}\right)$:**

(i) Value of sin 18° :

Let $\theta = 18^\circ$

$\therefore \quad 5\theta = 90^\circ$

$\therefore \quad 3\theta + 2\theta = 90^\circ$

$\therefore \quad 2\theta = 90^\circ - 3\theta$

$\therefore \quad \sin 2\theta = \sin(90^\circ - 3\theta)$

$\therefore \quad \sin 2\theta = \cos 3\theta$

$\therefore \quad 2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$\therefore \quad 2\sin\theta = 4\cos^2\theta - 3$ ($\because \cos\theta \neq 0$, where $\theta = 18^\circ$)

$\therefore \quad 2\sin\theta = 4 - 4\sin^2\theta - 3$

$\therefore \quad 4\sin^2\theta + 2\sin\theta - 1 = 0$

$\therefore \quad \sin\theta = \frac{-2 \pm \sqrt{(2)^2 - 4(4)(-1)}}{2(4)}$

($\because a = 4, b = 2, c = -1$)

$= \frac{-2 \pm \sqrt{4+16}}{8}$

$= \frac{-2 \pm \sqrt{20}}{8} = \frac{-2 \pm 2\sqrt{5}}{8}$

$\therefore \quad \sin\theta = \frac{-1 \pm \sqrt{5}}{4}$

Since $\theta = 18^\circ$ is in first quadrant, $\sin\theta$ is positive.

$\therefore \quad \sin 18^\circ = \frac{\sqrt{5}-1}{4}$

(ii) Value of cos 18° :

$\cos^2 18^\circ = 1 - \sin^2 18^\circ$

$= 1 - \left(\frac{\sqrt{5}-1}{4}\right)^2 = 1 - \frac{5 - 2\sqrt{5} + 1}{16}$

$= 1 - \frac{6 - 2\sqrt{5}}{16}$

$= \frac{16 - 6 + 2\sqrt{5}}{16}$

$= \frac{10 + 2\sqrt{5}}{16}$

$\therefore \quad \cos 18^\circ = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$

2. **Values for $36^\circ \left(\frac{\pi}{5}\right)$:**

(i) Value of cos 36° :

$\cos 36^\circ = 1 - 2\sin^2 18^\circ$

$= 1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2 = 1 - 2\cdot\frac{(5 - 2\sqrt{5} + 1)}{16}$

$= 1 - \frac{6 - 2\sqrt{5}}{8}$

$= 1 - \frac{3 - \sqrt{5}}{4} = \frac{4 - 3 + \sqrt{5}}{4} = \frac{1 + \sqrt{5}}{4}$

$\therefore \quad \cos 36^\circ = \frac{\sqrt{5}+1}{4}$

(ii) Value of $\sin 36^o$:

$\sin^2 36^o = 1 - \cos^2 36^o$

$$= 1 - \left(\frac{\sqrt{5}+1}{4}\right)^2 = 1 - \frac{(5 + 2\sqrt{5} + 1)}{16}$$

$$= \frac{16 - 6 - 2\sqrt{5}}{16} = \frac{10 - 2\sqrt{5}}{16}$$

$$\therefore \quad \sin 36^o = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$$

3. **Values for $54^o \left(\frac{3\pi}{10}\right)$:**

$$\sin 54^o = \sin(90^o - 36^o) = \cos 36^o = \frac{\sqrt{5}+1}{4}$$

$$\cos 54^o = \cos(90^o - 36^o) = \sin 36^o = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$$

4. **Values for $72^o \left(\frac{2\pi}{5}\right)$:**

$$\sin 72^o = \sin(90^o - 18^o) = \cos 18^o = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$$

$$\cos 72^o = \cos(90^o - 18^o) = \sin 18^o = \frac{\sqrt{5}-1}{4}$$

5. **Values for $22\frac{1}{2}^o \left(\frac{\pi}{8}\right)$:**

(i) Value of $\sin 22\frac{1}{2}^o$:

$$\sin^2 22\frac{1}{2}^o = \sin^2 \frac{45^o}{2} = \frac{1 - \cos 45^o}{2}$$

$$= \frac{1 - \frac{1}{\sqrt{2}}}{2} = \frac{\sqrt{2}-1}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2-\sqrt{2}}{4}$$

$$\therefore \quad \sin 22\frac{1^o}{2} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

(ii) Value of $\cos 22\frac{1}{2}^o$:

$$\cos^2 22\frac{1^o}{2} = \cos^2 \frac{45^o}{2} = \frac{1 + \cos 45^o}{2}$$

$$= \frac{1 + \frac{1}{\sqrt{2}}}{2} = \frac{\sqrt{2}+1}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2-\sqrt{2}}{4}$$

$$\therefore \quad \cos 22\frac{1^o}{2} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

(iii) Value of $\tan 22\frac{1}{2}^o$:

$$\tan^2 22\frac{1^o}{2} = \tan^2 \frac{45^o}{2} = \frac{1 - \cos 45^o}{1 + \cos 45^o}$$

$$= \frac{1 - \frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}} = \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$= \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(\sqrt{2}-1)^2}{2-1} = (\sqrt{2}-1)^2$$

$$\therefore \quad \tan 22\frac{1^o}{2} = \sqrt{2}-1$$

Multiple Choice Questions (MCQs) (Solution with Explanation)

1. 135° = ... radian.

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{5\pi}{6}$

Ans. : (B)

Explanation : Radian measure of 135° $= \frac{\pi}{180} \times 135$

$= \frac{3\pi}{4}$ radian.

2. 270° = ... radian.

(A) $\frac{4\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{3\pi}{2}$ (D) $\frac{\pi}{9}$

Ans. : (C)

Explanation : Radian measure of 270° $= \frac{\pi}{180} \times 270$

$= \frac{3\pi}{2}$ radian.

3. 120° = ... radian.

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{2}$ (C) $\frac{\pi}{12}$ (D) 2π

Ans. : (A)

Explanation : Radian measure of 120° $= \frac{\pi}{180} \times 120$

$= \frac{2\pi}{3}$ radian.

4. 72° = ... radian.

(A) $\frac{2\pi}{5}$ (B) $\frac{12\pi}{7}$ (C) $\frac{4\pi}{5}$ (D) $\frac{3\pi}{5}$

Ans. : (A)

Explanation : Radian measure of 72° $= \frac{\pi}{180} \times 72$

$= \frac{2\pi}{5}$ radian.

5. 216° = ... radian.

(A) $\frac{5\pi}{6}$ (B) $\frac{5\pi}{3}$ (C) $\frac{6\pi}{5}$ (D) $\frac{3\pi}{5}$

Ans. : (C)

Explanation : Radian measure of 216° $= \frac{\pi}{180} \times 216$

$= \frac{6\pi}{5}$ radian.

6. $\frac{2\pi}{9}$ radian = ... degree.

(A) 40 (B) 80 (C) 20 (D) 10

Ans. : (A)

Explanation : Degree measure of $\frac{2\pi}{9}^R = \frac{180}{\pi} \times \frac{2\pi}{9} = 4$

7. $\frac{12\pi}{5}$ radian = ... degree.

(A) 60 (B) 432 (C) 234 (D) 90

Ans. : (B)

Explanation : Degree measure of $\frac{12\pi}{5}^R$

$= \frac{180}{\pi} \times \frac{12\pi}{5} = 432^\circ$

8. Principal period of $\cos\left(\frac{2x}{3} + 5\right)$ = ...

(A) $\frac{2\pi}{3}$ (B) 2π (C) $\frac{3\pi}{2}$ (D) 3π

Ans. : (D)

Explanation : $f(x) = \cos\left(\frac{2x}{3} + 5\right)$

$\therefore$ b = coefficient of $x = \frac{2}{3}$

$\therefore$ Principal period of function $f = \frac{2\pi}{|b|} = \frac{2\pi}{\left|\frac{2}{3}\right|} =$

9. Principal period of $\cos(3x + 5)$ = ...

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{5}$ (D) $\frac{2\pi}{5}$

Ans. : (B)

Explanation : $f(x) = \cos(3x + 5)$

$\therefore$ b = coefficient of $x = 3$

$\therefore$ Principal period of function $f = \frac{2\pi}{|b|} = \frac{2\pi}{|3|} = \frac{2\pi}{3}$

10. Principal period of $\sin(2x + 7)$ = ...

(A) 2π (B) $2\pi + 7$

(C) π (D) 4π

Ans. : (C)

Explanation : $f(x) = \sin(2x + 7)$

$\therefore$ b = coefficient of $x = 2$

$\therefore$ Principal period of function $f = \frac{2\pi}{|b|} = \frac{2\pi}{|2|} = \pi$

11. Principal period of $\sin^2 39^\circ + \cos^2 39^\circ$ = ...

(A) 39° (B) 1

(C) 2π (D) does not exist

Ans. : (D)

Explanation : $f(x) = \sin^2 39^\circ + \cos^2 39^\circ$

$= 1$ $(\because \cos^2 x + \sin^2 x = 1)$

Since f is a constant function, according to the definition, f is not a periodic function so period of f does not exist.

12. Principal period of $\cot \frac{x}{6}$ = ...

(A) 6π (B) 12π (C) 8π (D) $\frac{\pi}{6}$

Ans. : (B)

Explanation : $f(x) = \cot \frac{x}{6}$

$\therefore$ b = coefficient of $x = \frac{1}{6}$

$\therefore$ Principal period of function $f = \frac{\pi}{|b|} = \frac{\pi}{\left|\frac{1}{6}\right|} = 6\pi$

13. Principal period of $3\cos 2x$ = ...

(A) 0 (B) π (C) 2π (D) 3π

Ans. : (B)

Explanation : $f(x) = 3\cos 2x$

$\therefore$ b = coefficient of $x = 2$

$\therefore$ Principal period of function $f = \frac{2\pi}{|b|} = \frac{2\pi}{|2|} = \pi$

14. $\sin\left(\frac{\pi}{2} - \theta\right)$ = ...

(A) $-\cos\theta$ (B) $\cos\theta$ (C) $\sin\theta$ (D) $-\sin\theta$

Ans. : (B)

Explanation : $\sin\left(\frac{\pi}{2} - \theta\right) = \sin\frac{\pi}{2}\cos\theta - \cos\frac{\pi}{2}\sin\theta$

$= (1)\cdot\cos\theta - (0)\cdot\sin\theta$

$= \cos\theta$

15. $\sin 135^\circ$ = ...

(A) $\frac{1}{\sqrt{2}}$ (B) $-\frac{1}{\sqrt{2}}$

(C) $\sqrt{2}$ (D) $-\sqrt{2}$

Ans. : (A)

Explanation : $\sin 135^\circ = \sin(180^\circ - 45^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$

16. $\sin 150^\circ$ = ...

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $-\frac{\sqrt{3}}{2}$

Ans. : (B)

Explanation : $\sin 150^\circ = \sin(180^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2}$

17. $\sin 120^\circ$ = ...

(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$

Ans. : (C)

Explanation : $\sin 120^\circ = \sin(180^\circ - 60^\circ)$ $(180^\circ = \pi^R)$

$= \sin 60^\circ = \frac{\sqrt{3}}{2}$

18. $\sin\frac{\pi}{8} + \sin\frac{9\pi}{8}$ = ...

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) 0 (D) $-\frac{1}{\sqrt{2}}$

Ans. : (C)

Explanation : $\sin\frac{\pi}{8} + \sin\frac{9\pi}{8} = \sin\frac{\pi}{8} + \sin\left(\frac{8\pi+\pi}{8}\right)$

$= \sin\frac{\pi}{8} + \sin\left(\pi + \frac{\pi}{8}\right)$

$= \sin\frac{\pi}{8} - \sin\frac{\pi}{8}$

$= 0$

19. $\sin^2 40^\circ + \sin^2 50^\circ = \dots\dots\dots$

(A) 1 (B) –1

(C) 0 (D) None of these

Ans. : (A)

Explanation : $\sin^2 40^\circ + \sin^2 50^\circ = \sin^2 40^\circ + \sin^2(90^\circ - 40^\circ)$

$= \sin^2 40^\circ + \cos^2 40^\circ$

$= 1$

20. In Δ ABC, $\sec\left(\frac{B+C}{2}\right) = \dots\dots\dots$

(A) $\operatorname{cosec}\frac{A}{2}$ (B) cosec A

(C) $\sec\frac{A}{2}$ (D) sec A

Ans. : (A)

Explanation : In Δ ABC, $A + B + C = \pi$

$\therefore\ B + C = \pi - A$

$\therefore\ \sec\left(\frac{B+C}{2}\right) = \sec\left(\frac{\pi - A}{2}\right)$

$= \sec\left(\frac{\pi}{2} - \frac{A}{2}\right) = \operatorname{cosec}\frac{A}{2}$

21. $\sin(A+B)\cdot\sin(A-B) = \dots\dots\dots$

(A) $\sin^2 A - \cos^2 B$ (B) $\sin^2 A - \sin^2 B$

(C) $\cos^2 A - \cos^2 B$ (D) $\cos^2 A - \sin^2 B$

Ans. : (B)

22. If $\cos\theta + \sin\theta = \sqrt{2}$, then $\sin 2\theta = \dots\dots\dots$.

(A) 1 (B) 2 (C) 3 (D) 4

Ans. : (A)

Explanation : $\cos\theta + \sin\theta = \sqrt{2}$

$\therefore\ (\cos\theta + \sin\theta)^2 = \left(\sqrt{2}\right)^2$

$\therefore\ \cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta = 2$

$\therefore\ \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 2$

$\therefore\ 1 + \sin 2\theta = 2$

$\therefore\ \sin 2\theta = 1$

23. $\sin 3A = \dots\dots\dots$.

(A) $3\sin A - 4\sin^3 A$

(B) $4\sin^3 A - 3\sin A$

(C) $4\sin A - 3\sin^4 A$

(D) $3\sin^4 A - 4\sin A$

Ans. : (A)

24. $\sin 2x = \dots\dots\dots$.

(A) $\frac{1-\tan^2 x}{1+\tan^2 x}$ (B) $\frac{2\tan x}{1+\tan^2 x}$

(C) $\frac{1+\tan^2 x}{1-\tan^2 x}$ (D) $\frac{2\tan x}{1-\tan^2 x}$

Ans. : (B)

25. $\sin 40^\circ + \sin 20^\circ = \dots\dots\dots$.

(A) $\cos 10^\circ$ (B) $-\cos 10^\circ$

(C) $\cos 20^\circ$ (D) $-\cos 20^\circ$

Ans. : (A)

Explanation : $\sin 40^\circ + \sin 20^\circ$

$= 2\sin\left(\frac{40^\circ + 20^\circ}{2}\right)\cdot\cos\left(\frac{40^\circ - 20^\circ}{2}\right)$

$= 2\sin 30^\circ\cdot\cos 10^\circ$

$= 2\left(\frac{1}{2}\right)\cdot\cos 10^\circ$

$= \cos 10^\circ$

26. $\sin\frac{\pi}{8} = \dots\dots\dots$.

(A) $\frac{\sqrt{2-\sqrt{2}}}{4}$ (B) $\sqrt{\frac{2+\sqrt{2}}{2}}$

(C) $\frac{2-\sqrt{2}}{4}$ (D) $\frac{2+\sqrt{2}}{4}$

Ans. : (A)

Explanation :

$$\sin^2 22\frac{1^\circ}{2} = \sin^2 \frac{45^\circ}{2} = \frac{1-\cos 45^\circ}{2}$$

$$= \frac{1-\frac{1}{\sqrt{2}}}{2} = \frac{\sqrt{2}-1}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2-\sqrt{2}}{4}$$

$$\therefore \quad \sin 22\frac{1^\circ}{2} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

27. $\sin^{-1}x + \cos^{-1}x = \ldots\ldots\ldots$.

(A) 0 (B) 1 (C) π (D) $\frac{\pi}{2}$

Ans. : (A)

28. $\tan^{-1}x + \cot^{-1}x = \ldots\ldots\ldots$.

(A) 1 (B) 0 (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

Ans. : (C)

29. $\sin^{-1}\left(\cos\frac{\pi}{3}\right) = \ldots\ldots\ldots$

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

Ans. : (C)

Explanation : $\sin^{-1}\left(\cos\frac{\pi}{3}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{2}-\frac{\pi}{3}\right)\right)$

$$= \sin^{-1}\left(\sin\frac{\pi}{6}\right)$$

$$= \frac{\pi}{6} \qquad \left(\because \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right)$$

30. If $xy = 1$, then $\tan^{-1}x + \tan^{-1}y = \ldots\ldots\ldots$.

(A) $\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ (B) $\tan^{-1}\left(\frac{x-y}{1+xy}\right)$

(C) $\frac{\pi}{2}$ (D) π

Ans. : (C)

31. $\sin^{-1}\left(\cos\frac{\pi}{6}\right) = \ldots\ldots\ldots$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) π (D) 2π

Ans. : (B)

Explanation : $\sin^{-1}\left(\cos\frac{\pi}{6}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{2}-\frac{\pi}{6}\right)\right)$

$$= \sin^{-1}\left(\sin\frac{\pi}{3}\right)$$

$$= \frac{\pi}{3} \qquad \left(\because \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right)$$

32. If $f(x) = \cos x$ then $f\left(\frac{3\pi}{2}-x\right) = \ldots\ldots\ldots$

(A) $-\sin x$ (B) $\sin x$

(C) $\cos x$ (D) $-\cos x$

Ans. : (A)

Explanation : $f(x) = \cos x$

$$\therefore \quad f\left(\frac{3\pi}{2}-x\right) = \cos\left(\frac{3\pi}{2}-x\right)$$

$$= -\sin x$$

33. If $f(x) = \log(\tan x)$ હોય તો $f\left(\frac{\pi}{4}\right) = \ldots\ldots\ldots$

(A) 1 (B) e

(C) 0 (D) π

Ans. : (C)

Explanation : $f(x) = \log(\tan x)$

$$\therefore \quad f\left(\frac{\pi}{4}\right) = \log\left(\tan\frac{\pi}{4}\right) = \log 1 = 0$$

Multiple Choice Questions (MCQ's) with (Final Answers)

1. $\frac{4\pi}{9}$ radian .... degree.

(A) 40 (B) 80 (C) 20 (D) 10

2. $\frac{6\pi}{5}$ radian .... degree.

(A) 210 (B) 225 (C) 240 (D) 216

3. $\frac{5\pi}{6}$ radian = ... Degree.

(A) 150 (B) 250 (C) 196 (D) 180

4. Principal period of $\sin 2x$ = ...

(A) 2π (B) $\frac{\pi}{2}$ (C) π (D) 2

5. Principal period of $\sin 3x$ = ...

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$ (C) π (D) 2π

6. Principal period of $3 \sin 2x$ = ...

(A) 2π (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{3}$ (D) π

7. Principal period of $\sin (2x + 7)$ = ...

(A) 2π (B) π (C) $2\pi + 7$ (D) 4π

8. Principal period of $\sin (2x + 3)$ = ...

(A) $\frac{2\pi}{3}$ (B) 2π (C) π (D) $\frac{\pi}{2}$

9. Principal period of $2 \sin \theta \cdot \cos \theta$ = ...

(A) π (B) 2π (C) 0 (D) $\frac{\pi}{2}$

(**Hint :** $2 \sin \theta \cdot \cos \theta = \sin 2\theta$)

10. Principal period of $4 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$ = ...

(A) π (B) 2π (C) 0 (D) $\frac{\pi}{2}$

(**Hint :** $4 \sin \frac{\theta}{2} \cos \frac{\theta}{2} = 2 \sin \theta$)

11. Principal period of $\sin (2x + 5)$ = ...

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) 2π (D) π

12. Principal period of $\sin^2 36^\circ + \cos^2 36^\circ$ = ...

(A) 36 (B) 1 (C) 2π (D) આપ્યા...

13. $\cos\left(\frac{3\pi}{2} + \theta\right)$ = ...

(A) $\sin \theta$ (B) $\cos \theta$

(C) $-\cos \theta$ (D) $-\sin \theta$

14. $\cos (\pi + \theta)$ = ...

(A) $\cos \theta$ (B) $\sin \theta$

(C) $-\cos \theta$ (D) $-\sin \theta$

15. $\sin\left(\frac{\pi}{2} + \theta\right)$ = ...

(A) $\cos \theta$ (B) $\sin \theta$ (C) $-\cos \theta$ (D) $-\sin \theta$

16. $\sin \frac{\pi}{6} \cdot \sin \frac{\pi}{3} + \sin \frac{\pi}{2} \cdot \sin \pi$ = ...

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{4}$

17. If $\theta = \frac{7\pi}{4}$, then θ is in the .... quadrant.

(A) First (B) Second

(C) Third (D) Fourth

18. If $\sec \theta = \frac{3}{2}$ $\left(0 < \theta < \frac{\pi}{2}\right)$, then $\tan \theta$ = ...

(A) $\frac{\sqrt{13}}{2}$ (B) $\frac{\sqrt{5}}{2}$ (C) $\frac{9}{4}$ (D) 2

19. If $\sin \theta = \frac{4}{5}$, then $\operatorname{cosec} \theta$ = ...

(A) $\frac{4}{5}$ (B) $\frac{5}{4}$ (C) $\frac{3}{5}$ (D) $\frac{3}{4}$

20. If $\tan \theta = \sqrt{2}$ and $\cos \theta = \frac{1}{\sqrt{3}}$, then θ is in the ...... quadrant.

(A) First (B) Second

(C) Third (D) Fourth

21. $\sin 90^\circ \cdot \sin 60^\circ \cdot \sin 45^\circ \cdot \sin 0^\circ =$ ....

(A) 0 (B) 1 (C) −1 (D) $\frac{1}{2}$

22. $\sin 27^\circ \cos 18^\circ + \cos 27^\circ \sin 18^\circ =$ ....

(A) 1 (B) −1

(C) $-\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{2}}$

23. $\sin 2\theta =$ ....

(A) $2 \sin \theta$ (B) $2 \sin \theta \cos \theta$

(C) $\sin^2\theta - \cos^2\theta$ (D) $\cos^2\theta - \sin^2\theta$

24. If $\tan \theta = \frac{3}{4}$, then $\tan 2\theta =$ ....

(A) $\frac{24}{25}$ (B) $\frac{12}{7}$

(C) $\frac{24}{7}$ (D) $\frac{12}{25}$

25. $3 \sin 20^\circ - 4 \sin^3 20^\circ =$ ....

(A) $\frac{1}{2}$ (B) −1

(C) $\frac{\sqrt{3}}{2}$ (D) $-\frac{1}{2}$

26. $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{4}{3} =$ ....

(A) $\frac{\pi}{2}$ (B) $\frac{3\pi}{2}$ (C) π (D) 0

27. $\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3} =$ ....

(A) $\frac{\pi}{3}$ (B) π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

28. $\tan^{-1}\left(\sqrt{3}\right) =$ ....

(A) 30° (B) 45° (C) 60° (D) 120°

29. If $f(x) = \cos x$, then $f(\pi - x) =$ ....

(A) $\cos x$ (B) $-\cos x$

(C) $\sin x$ (D) $-\sin x$

30. If $f(x) = \cos x$, then $f\left(\frac{\pi}{2} + x\right) =$ ....

(A) $\cos x$ (B) $\sin x$

(C) $-\cos x$ (D) $-\sin x$

Answers

(1) B	(2) D	(3) A	(4) C	(5) A
(6) D	(7) B	(8) C	(9) C	(10) B
(11) D	(12) D	(13) A	(14) C	(15) A
(16) D	(17) D	(18) B	(19) B	(20) A
(21) A	(22) D	(23) B	(24) C	(25) C
(26) A	(27) C	(28) C	(29) B	(30) D

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