SECTION-02 - BE-02

APTITUDE TEST (MATHEMATICS & SOFT SKILL)

MATHEMATICS

1. Determinant and Matrices
2. Trigonometry
3. Vectors
4. Co-ordinate Geometry
5. Function & Limit
6. Differentiation and its Applications
7. Integration
8. Logarithm
9. Statistics

ENGLISH

10. Comprehension of Unseen Passage
11. Theory of Communication
12. Grammar
13. Correction of Incorrect Words and Sentences

3. Vectors

[1] Introduction :

In the middle of 18th century, Hamilton divided physical quantities into two types : (1) Vector quantities and (2) Scalar quantities. Thus mathematics got a new branch, which included vector algebra and vector calculus, which proved to be very useful in the subjects of physics as well as engineering. So its study is very necessary for engineering students. In this chapter we will get a basic idea of vector algebra.

In Mathematics also there are many quantities with which magnitude and direction can be associated. Such quantities are called vector quantities. For example, let's take $R^2 = \{(x, y) \mid x, y, \in R\}$. The origin O is associated with a member (0, 0) of R^2. Suppose if a point P is associated with an another member (3, 4) of R^2 and if we have the length $\left(\sqrt{3^2 + 4^2} = 5\right)$ of $\overline{OP}$ as the magnitude of (3, 4) and the direction of $\overrightarrow{OP}$ as the direction of (3, 4), then (3, 4) is a vector. In the same way each member of set $R^3 = \{(x, y, z) \mid x, y, z \in R\}$ can be considered as a vector.

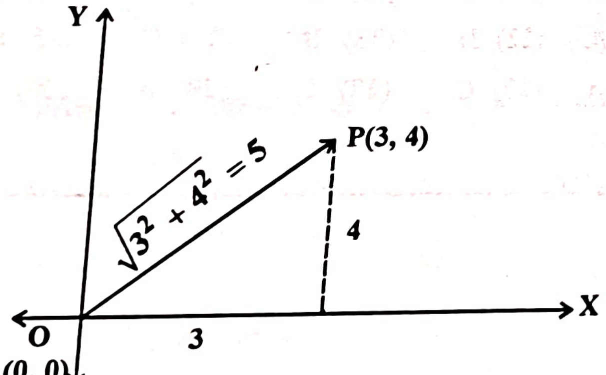

Fig. 3.1

Thus R^2 and R^3 are the sets of vectors. We will study the algebra of vectors by applying operations on such set. Later on we will see how the knowledge obtained in this way about vectors is used in physics.

[2] Vector :

Our study will be limited to the geometry of R^2 and R^3. We will denote the vector (x_1, x_2) of R^2 or the vector (x_1, x_2, x_3) of R^3 by $\overline{x}$ (read : 'x bar')

(1) Equality of vectors :

For vectors $\overline{x} = (x_1, x_2)$ and $\overline{y} = (y_1, y_2)$ of R^2,

$\overline{x} = \overline{y} \Leftrightarrow (x_1, x_2) = (y_1, y_2) \Leftrightarrow x_1 = y_1$ and $x_2 = y_2$

Similarly, for the vectors $\overline{x} = (x_1, x_2, x_3)$ and $\overline{y} = (y_1, y_2, y_3)$ of R^3,

$$\overline{x} = \overline{y} \Leftrightarrow (x_1, x_2, x_3) = (y_1, y_2, y_3)$$
$$\Leftrightarrow x = y_1, x_2 = y_2, x_3 = y_3.$$

(2) Addition of vectors :

We will defined the addition of two vectors as follows.

For two vectors $\overline{x} = (x_1, x_2)$ and $\overline{y} = (y_1, y_2)$ of R^2,

$$\overline{x} + \overline{y} = (x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$$

And for two vectors $\overline{x} = (x_1, x_2, x_3)$ and $\overline{y} = (y_1, y_2, y_3)$ of R^3,

$$\overline{x} + \overline{y} = (x_1, x_2, x_3) + (y_1, y_2, y_3)$$
$$= (x_1 + y_1, x_2 + y_2, x_3 + y_3)$$

(3) Multiplication of a vector by a scalar :

Multiplication of a vector $\overline{x}$ by a scalar $k \in R$ is defined as follows :

For the vector $\overline{x} = (x_1, x_2)$ of R^2 and $k \in R$,

$$k\overline{x} = k(x_1, x_2) = (kx_1, kx_2)$$

and for the vector $\overline{x} = (x_1, x_2, x_3)$ of R^3 and $k \in R$,

$$k\overline{x} = k(x_1, x_2, x_3) = (kx_1, kx_2, kx_3)$$

[3] Properties of Addition and Multiplication by a Scalar :

We will accept the following properties for the vectors of R^2 and R^3 without proof.

(i) Commutative law : $\overline{x} + \overline{y} = \overline{y} + \overline{x}$

(ii) Associative law : $\overline{x} + (\overline{y} + \overline{z}) = (\overline{x} + \overline{y}) + \overline{z}$

(iii) Identity element : For $\overline{0} = (0, 0)$ of R^2 and $\overline{0} = (0, 0, 0)$ of R^3,

$$\overline{x} + \overline{0} = \overline{0} + \overline{x} = \overline{x}$$

Here $\bar{0}$ is called zero vector.

(iv) Additive inverse : For the vector $\bar{x} = (x_1, x_2)$ of R^2 and the vector $\bar{x} = (x_1, x_2, x_3)$ of R^3, there exist $-\bar{x} = (-x_1, -x_2)$ and $-\bar{x} = (-x_1, -x_2, -x_3)$ respectively such that

$\bar{x} + (-\bar{x}) = (-\bar{x}) + \bar{x} = \bar{0}$.

Here $-\bar{x}$ is called additive inverse of $\bar{x}$.

(v) $k(\bar{x} + \bar{y}) = k\bar{x} + k\bar{y}, k \in R$

(vi) $(k + l)\,\bar{x} = k\bar{x} + l\bar{x}, \quad k, l \in R$

(vii) $(kl)\,\bar{x} = k\,(l\bar{x}), \quad k, l \in R$

(viii) $1\bar{x} = \bar{x}$

With all the above properties, the set R^2 (and R^3) is called a vector space over R and its element is called vector. Thus R^2 (and R^3) is a vector space and $\bar{x}$, $\bar{y}$ ... etc. are called vectors.

It can be easily seen that the following properties are satisfied in the sets of vectors R^2 and R^3.

(i) Zero vector and additive inverse of a vector is unique.

(ii) $(-1)\bar{x} = -\bar{x}$ (iii) $0\bar{x} = \bar{0}$

(iv) $k\bar{0} = \bar{0}$ (v) $\alpha\bar{x} = \bar{0} \Leftrightarrow \alpha = 0$ or $\bar{x} = \bar{0}$

(vi) $\bar{x} + \bar{y} = \bar{x} + \bar{z} \Leftrightarrow \bar{y} = \bar{z}$

(vii) $\bar{x} - \bar{y} = \bar{x} + (-\bar{y})$

[4] Geometric representation of vector :

Now we will talk about directed line segment. As shown in the figure 3.2, the directed line segment within direction of $\overrightarrow{OP}$ i.e. the direction from origin O to P is denoted by $\overrightarrow{OP}$. Thus $\overrightarrow{OP}$ is a vector.

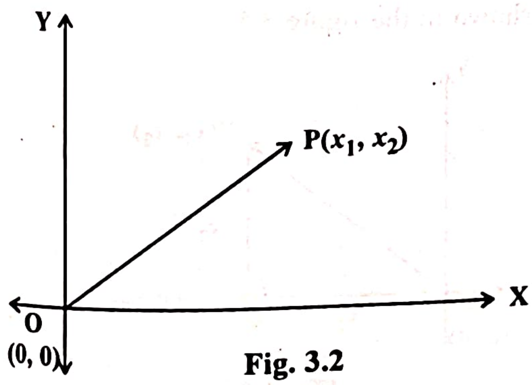

Fig. 3.2

If the coordinates of P are (x_1, x_2), then it is said that $\overrightarrow{OP} = (x_1, x_2)$ or the position vector of P is (x_1, x_2). The position vector of O is (0, 0).

Now if the position vectors of A, B and P are (x_1, x_2), (y_1, y_2) and $(y_1 - x_1, y_2 - x_2)$, then $\overrightarrow{AB}$ and $\overrightarrow{OP}$ are same as shown in the figure 3.3.

Also $OP = AB = \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2}$

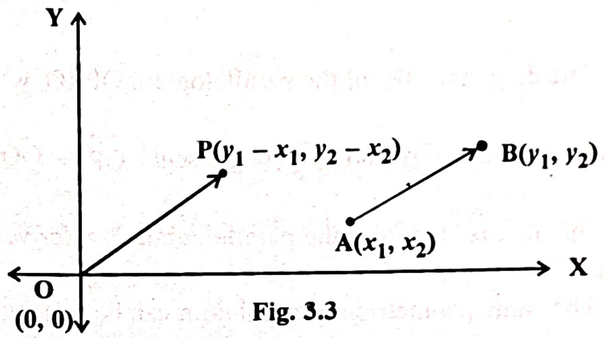

Fig. 3.3

Thus the vectors $\overrightarrow{AB}$ and $\overrightarrow{OP}$ are equal.

$\therefore \quad \overrightarrow{AB} = \overrightarrow{OP}$

$= (y_1 - x_1, y_2 - x_2)$

$= (y_1, y_2) - (x_1, x_2)$

= Position vector of B – Position vector of A

- **Geometric representation of addition of vectors :**

Suppose the position vectors of P and Q are $\bar{x} = (x_1, x_2)$ and $\bar{y} = (y_1, y_2)$ respectively.

$\therefore \quad \bar{x} + \bar{y} = (x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$

Suppose the position vector of R is $\bar{x} + \bar{y} = (x_1 + y_1, x_2 + y_2)$.

Mid point of $\overline{PQ} = \left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right)$ = Mid point of OR

As shown in the figure 3.4, PQRS is a parallelogram.

$\therefore \quad \overrightarrow{OP} + \overrightarrow{OQ} = \overrightarrow{OR}$

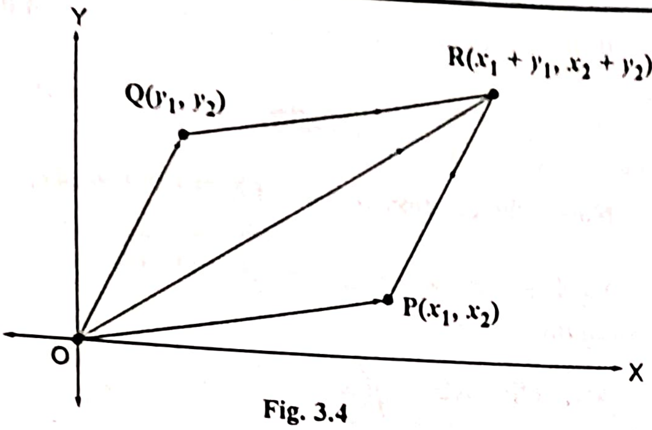

Fig. 3.4

The diagonal $\overrightarrow{OR}$ of the parallelogram OPRQ, whose adjacent sides are $\overline{OP}$ and $\overline{OQ}$ respresents $\overrightarrow{OP} + \overrightarrow{OQ} = \overrightarrow{OR}$. This fact is known as the parallelogram law for vector addition.

The same geometric representatoin can be understand for the vector of R^3.

Example-1 : If $\overline{x} = (-1, 2, 3)$ and $\overline{y} = (2, -5, 8)$, then find $2\overline{x} + 3\overline{y}$.

Solution : $2\overline{x} + 3\overline{y} = 2(-1, 2, 3) + 3(2, -5, 8)$
$= (-2, 4, 6) + (6, -15, 24)$
$= ((-2) + 6, 4 + (-15), 6 + 24)$
$= (4, -11, 30)$

[5] Inner product / Scalar product / Dot product of vectors in R^2 and R^3 :

If $\overline{x} = (x_1, x_2)$ and $\overline{y} = (y_1, y_2)$ are vectors of R^2, then their Inner product is denoted by $\overline{x} \cdot \overline{y}$ which is defined as follows.

$$\overline{x} \cdot \overline{y} = (x_1, x_2) \cdot (y_1, y_2) = x_1 y_1 + x_2 y_2$$

Similarly, for the vectors $\overline{x} = (x_1, x_2, x_3)$ and $\overline{y} = (y_1, y_2, y_3)$ of R^3,

$$\overline{x} \cdot \overline{y} = (x_1, x_2, x_3) \cdot (y_1, y_2, y_3)$$
$$= x_1 y_1 + x_2 y_2 + x_3 y_3$$

Let us note that $\overline{x}$ and $\overline{y}$ are vectors but their inner product $\overline{x} \cdot \overline{y}$ is a scalar. Also multiplication of a vector by a scalar $k\overline{x}$ and scalar product $\overline{x} \cdot \overline{y}$ are different. $k\overline{x}$ is a vector, while $\overline{x} \cdot \overline{y}$ is a scalar.

- **Properties of Inner product :**

If $\overline{x}, \overline{y}, \overline{z} \in R^2$ (or R^3) and $k \in R$, then

(i) $\overline{x} \cdot \overline{x} \geq 0$

(ii) $\overline{x} \cdot \overline{x} = 0 \Leftrightarrow \overline{x} = \overline{0}$

(iii) $\overline{x} \cdot (\overline{y} + \overline{z}) = \overline{x} \cdot \overline{y} + \overline{x} \cdot \overline{z}$

(iv) $\overline{x} \cdot (k\overline{y}) = (k\overline{x}) \cdot \overline{y} = k(\overline{x} \cdot \overline{y})$

(v) $\overline{x} \cdot \overline{y} = \overline{y} \cdot \overline{x}$

We will accept the above results without proof.

Example-2 : If $\overline{x} = (3, 2, 1)$ and $\overline{y} = (2, -1, -2)$, then find $\overline{x} \cdot \overline{y}$.

Solution : $\overline{x} \cdot \overline{y} = (3, 2, 1) . (2, -1, -2)$
$= (3)(2) + (2)(-1) + (1)(-2)$
$= 6 - 2 - 2$
$= 2$

[6] Magnitude of Vector :

Magnitude of vector $\overline{x}$, denoted by $|\overline{x}|$ is defined as follows.

$$|\overline{x}| = \sqrt{\overline{x} \cdot \overline{x}}$$

If $\overline{x} = (x_1, x_2) \in R^2$, then

$$|\overline{x}| = \sqrt{\overline{x} \cdot \overline{x}} = \sqrt{(x_1, x_2) \cdot (x_1, x_2)} = \sqrt{x_1^2 + x_2^2}$$

and if $\overline{x} = (x_1, x_2, x_3) \in R^3$, then

$$|\overline{x}| = \sqrt{\overline{x} \cdot \overline{x}} = \sqrt{(x_1, x_2, x_3) \cdot (x_1, x_2, x_3)}$$
$$= \sqrt{x_1^2 + x_2^2 + x_3^2}$$

For example, if $\overline{x} = (1, 2, -3)$, then

$$|\overline{x}| = \sqrt{(1)^2 + (2)^2 + (-3)^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$$

If the position vector of P is $\overline{x} = (x_1, x_2)$, then $\overrightarrow{OP} = (x_1, x_2)$ as shown in the figure 3.5

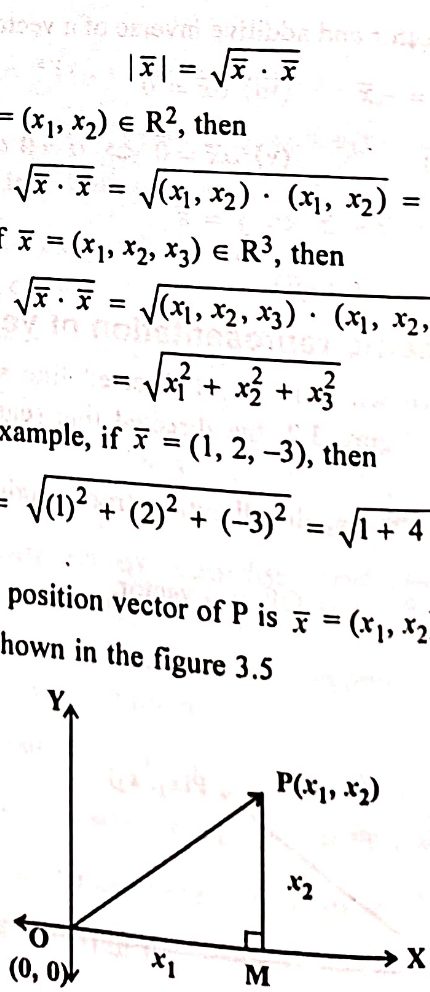

Fig. 3.5

Now in right angle triangle OMP

$OP^2 = OM^2 + PM^2 = x_1^2 + x_2^2$

$\therefore \quad OP = \sqrt{x_1^2 + x_2^2} = |\overline{x}|$

Thus geometrically, magnitude of a vector represents the length of that vector.

- **Properties of Magnitude of Vector :**

(i) $|\overline{x}| \geq 0$

(ii) $|\overline{x}| = 0 \Leftrightarrow \overline{x} = \overline{0}$

(iii) $|k\,\overline{x}| = |k|\,|\overline{x}|, \quad k \in R$

[7] Unit Vector :

A vector whose magnitude is one unit is called a unit vector. Thus for a vector $\overline{x}$ of R^2 or R^3, if $|\overline{x}| = 1$, then $\overline{x}$ is called a unit vector.

For example, vectors (1, 0), (0, 1), $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, (cos θ, sin θ)... etc. of R^2 and vectors (1, 0, 0), (0, 1, 0), (0, 0, 1), $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ ... etc. of R^3 are unit vectors.

Vector $\frac{\overline{x}}{|\overline{x}|}$ obtained by dividing a non zero vector $\overline{x}$ by its magnitude $|\overline{x}|$ is always a unit vector.

For example,

$\overline{x} = (1, 2, -3) \quad \therefore \ |\overline{x}| = \sqrt{1+4+9} = \sqrt{14}$

Thus it is not a unit vector as $|\overline{x}| = \sqrt{14}$

But for $\frac{\overline{x}}{|\overline{x}|} = \frac{(1, 2, -3)}{\sqrt{14}} = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}\right)$,

$$\left|\frac{\overline{x}}{|\overline{x}|}\right| = \sqrt{\frac{1}{14} + \frac{4}{14} + \frac{9}{14}} = \sqrt{\frac{14}{14}} = 1$$

$\therefore \ \frac{\overline{x}}{|\overline{x}|}$ is a unit vector.

Unit vectors (1, 0) and (0, 1) of R^2 in the positive direction of axis are denoted by $\overline{i}$ and $\overline{j}$ respectively.

Thus in R^2, $\overline{i} = (1, 0)$ and $\overline{j} = (0, 1)$

Similalry, unit vectors (1, 0, 0), (0, 1, 0) and (0, 0, 1) of R^3 in the positive direction of axis are denoted by $\overline{i}$, $\overline{j}$ and $\overline{k}$ respectively.

Thus in R^3, $\overline{i} = (1, 0, 0)$, $\overline{j} = (0, 1, 0)$, $\overline{k} = (0, 0, 1)$

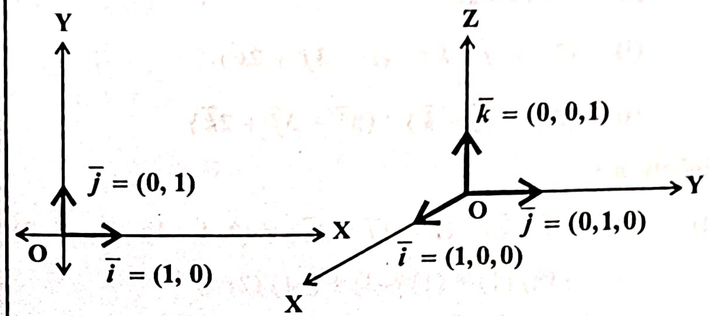

Fig. 3.6

[8] Representation of vectors in the form of $\overline{i}, \overline{j}$ and $\overline{k}$:

Let $\overline{x} = (x_1, x_2) \in R^2$

$\therefore \ \overline{x} = (x_1, x_2) = (x_1, 0) + (0, x_2)$
$= x_1(1, 0) + x_2(0, 1)$
$= x_1\overline{i} + x_2\overline{j}$

Thus every vector of R^2 can be represented as a unique linear combination of $\overline{i}$ and $\overline{j}$.

Similarly for $\overline{x} = (x_1, x_2, x_3) \in R^3$

$\therefore \ \overline{x} = (x_1, x_2, x_3) = (x_1, 0, 0) + (0, x_2, 0) + (0, 0, x_3)$
$= x_1(1, 0, 0) + x_2(0, 1, 0) + x_3(0, 0, 1)$
$= x_1\overline{i} + x_2\overline{j} + x_3\overline{k}$

Thus every vector of R^3 can be represented as a unique linear combination of $\overline{i}$, $\overline{j}$ and $\overline{k}$.

For example,

$(3, -2) = 3\overline{i} - 2\overline{j}, \quad (4, 5) = 4\overline{i} + 5\overline{j}$

$(-1, 2, 3) = -\overline{i} + 2\overline{j} + 3\overline{k}$,

$(2, 0, 3) = 2\overline{i} + 0\overline{j} + 3\overline{k} = 2\overline{i} + 3\overline{k}$

For the vectors $\overline{i} = (1, 0)$ and $\overline{j} = (0, 1)$ of R^2

(i) $|\overline{i}| = |\overline{j}| = 1$

(ii) $\overline{i} \cdot \overline{i} = \overline{j} \cdot \overline{j} = 1$

(iii) $\overline{i} \cdot \overline{j} = 0$

For the vectors $\overline{i} = (1, 0, 0)$, $\overline{j} = (0, 1, 0)$ and $\overline{k} = (0, 0, 1)$ of R^3

(i) $|\bar{i}| = |\bar{j}| = |\bar{k}| = 1$

(ii) $\bar{i} \cdot \bar{i} = \bar{j} \cdot \bar{j} = \bar{k} \cdot \bar{k} = 1$

(iii) $\bar{i} \cdot \bar{j} = \bar{j} \cdot \bar{k} = \bar{k} \cdot \bar{i} = 0$

Example-3 : Evaluate

(i) $(2\bar{i} + \bar{j} - \bar{k}) \cdot (\bar{i} - 3\bar{j} + 2\bar{k})$

(ii) $(3\bar{i} - 2\bar{j} + \bar{k}) \cdot (5\bar{i} + 3\bar{j} + 2\bar{k})$

Solution :

(i) $(2\bar{i} + \bar{j} - \bar{k}) \cdot (\bar{i} - 3\bar{j} + 2\bar{k}) = (2, 1, -1) \cdot (1, -3, 2)$

$= (2)(1) + (1)(-3) + (-1)(2)$

$= 2 - 3 - 2$

$= -3$

(ii) $(3\bar{i} - 2\bar{j} + \bar{k}) \cdot (5\bar{i} + 3\bar{j} + 2\bar{k}) = (3, -2, 1) \cdot (5, 3, 2)$

$= (3)(5) + (-2)(3) + (1)(2)$

$= 15 - 6 + 2$

$= 11$

[9] Outer product / Vector product / Cross product of vectors in R^3 :

The outer product of vectors $\bar{x} = (x_1, x_2, x_3)$ and $\bar{y} = (y_1, y_2, y_3)$ in R^3 is denoted by $\bar{x} \times \bar{y}$, which is defined as follows.

$$\bar{x} \times \bar{y} = (x_1, x_2, x_3) \times (y_1, y_2, y_3)$$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

$$= \left(\begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix}, -\begin{vmatrix} x_1 & x_3 \\ y_1 & y_3 \end{vmatrix}, \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} \right)$$

$$= (x_2 y_3 - x_3 y_2, -(x_1 y_3 - x_3 y_1), x_1 y_2 - x_2 y_1)$$

(Note : Outer product is not defined in R^2)

- **Properties of Outer product :**

For the vectors $\bar{x}$, $\bar{y}$ and $\bar{z}$ of R^3

(i) $\bar{x} \times \bar{y} = -\bar{y} \times \bar{x}$

(ii) $\bar{x} \times \bar{x} = \bar{0}$

(iii) $\bar{x} \times k\bar{y} = k\bar{x} \times \bar{y} = k(\bar{x} \times \bar{y})$

(iv) $\bar{x} \times (\bar{y} + \bar{z}) = \bar{x} \times \bar{y} + \bar{x} \times \bar{z}$

Note that outer product of vectors is a vector and outer product is not commutative.

For the unit vectors $\bar{i} = (1, 0, 0)$, $\bar{j} = (0, 1, 0)$ and $\bar{k} = (0, 0, 1)$ of R^3

(i) $\bar{i} \times \bar{j} = \bar{k}$, $\bar{j} \times \bar{k} = \bar{i}$, $\bar{k} \times \bar{i} = \bar{j}$

(ii) $\bar{i} \times \bar{i} = \bar{j} \times \bar{j} = \bar{k} \times \bar{k} = \bar{0}$

Example 4 : Do as directed

If $\bar{x} = (2, -1, 3)$ and $\bar{y} = (1, 2, -2)$, then find $(\bar{x} + \bar{y}) \times (\bar{x} - \bar{y})$. **(GTU : Jan. 2013)**

Solution :

$\bar{x} + \bar{y} = (2, -1, 3) + (1, 2, -2)$

$= (2 + 1, -1 + 2, 3 - 2) = (3, 1, 1)$

$\bar{x} - \bar{y} = \bar{x} + (-1)\bar{y}$

$= (2, -1, 3) + (-1)(1, 2, -2)$

$= (2, -1, 3) + (-1, -2, 2)$

$= (2 - 1, -1 - 2, 3 + 2) = (1, -3, 5)$

$\therefore (\bar{x} + \bar{y}) \times (\bar{x} - \bar{y}) = (3, 1, 1) \times (1, -3, 5)$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 1 & 1 \\ 1 & -3 & 5 \end{vmatrix}$$

$= \bar{i}(5 + 3) - \bar{j}(15 - 1) + \bar{k}(-9-1)$

$= 8\bar{i} - 14\bar{j} - 10\bar{k}$

$= (8, -14, -10)$

[10] Angle between vectors :

Angle beween two vectors $\bar{x}$ and $\bar{y}$ is denoted by $(\bar{x} \wedge \bar{y})$.

If $(\bar{x} \wedge \bar{y}) = \alpha$, then

(i) $\cos\alpha = \dfrac{\bar{x} \cdot \bar{y}}{|\bar{x}||\bar{y}|}$ $\therefore \alpha = \cos^{-1}\left(\dfrac{\bar{x} \cdot \bar{y}}{|\bar{x}||\bar{y}|}\right)$

where $\alpha \in [0, \pi]$

(ii) $\sin\alpha = \dfrac{|\bar{x}\times\bar{y}|}{|\bar{x}|\,|\bar{y}|}$ $\quad\therefore\ \alpha = \sin^{-1}\left(\dfrac{|\bar{x}\times\bar{y}|}{|\bar{x}|\,|\bar{y}|}\right)$

where $\alpha \in [0, \pi]$

We will accept both the above results without proof.

- **Perpendicular Vectors :**

If $\bar{x} \neq \bar{0}$, $\bar{y} \neq \bar{0}$ and $(\bar{x} \wedge \bar{y}) = \frac{\pi}{2}$, then $\bar{x}$ and $\bar{y}$ are called perpendicular vectors to each other. It is denoted by $\bar{x} \perp \bar{y}$.

Thus, $\bar{x} \perp \bar{y} \Leftrightarrow (\bar{x} \wedge \bar{y}) = \frac{\pi}{2}$

$\Leftrightarrow \cos(\bar{x} \wedge \bar{y}) = \cos\frac{\pi}{2}$

$\Leftrightarrow \dfrac{\bar{x}\cdot\bar{y}}{|\bar{x}|\,|\bar{y}|} = 0$

$\Leftrightarrow \bar{x}\cdot\bar{y} = 0$

Thus $\bar{x}\cdot\bar{y} = 0$ if and only if $\bar{x} \perp \bar{y}$.

Example-5 : Do as directed :

(i) If $\bar{A} = \bar{i} - \bar{j} - 3\bar{k}$ and $\bar{B} = \bar{j} + 2\bar{i} - \bar{k}$, then prove that $\bar{A} \times \bar{B}$ is perpendicular to $\bar{A}$.

Solution :

(i) $\bar{A} = \bar{i} - \bar{j} - 3\bar{k} = (1, -1, -3)$

$\bar{B} = \bar{j} + 2\bar{i} - \bar{k} = (2, 1, -1)$

$\therefore\ \bar{A}\times\bar{B} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -1 & -3 \\ 2 & 1 & -1 \end{vmatrix}$

$= \bar{i}\,(1+3) - \bar{j}\,(-1+6) + \bar{k}\,(1+2)$

$= 4\bar{i} - 5\bar{j} + 3\bar{k}$

$= (4, -5, 3)$

Now

$(\bar{A}\times\bar{B})\cdot\bar{A} = (4, -5, 3) . (1, -1, -3)$

$= (4)(1) + (-5)(-1) + (3)(-3)$

$= 4 + 5 - 9$

$= 0$

Thus as $(\bar{A}\times\bar{B})\cdot\bar{A} = 0$, $(\bar{A}\times\bar{B}) \perp \bar{A}$

Multiple Choice Questions (MCQs) (Solution with Explanation)

1. .... is a unit vector.

(A) $\left(\frac{3}{5}, \frac{4}{5}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(C) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ (D) $\left(\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right)$

Ans. : (A)

Here, $\left|\left(\frac{3}{5}, \frac{4}{5}\right)\right| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$

$= \sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = 1$.

$\therefore\ \left(\frac{3}{5}, \frac{4}{5}\right)$ is a unit vector.

2. If $\bar{a} = 3\bar{i} - \bar{j}$ and $\bar{b} = \bar{i} + 3\bar{j}$, then $(\bar{a} + 2\bar{b}) =$ .....

(A) (5, 5) (B) (5, –7)

(C) (–5, 5) (D) (7, –5)

Ans. : (A)

$\bar{a} + 2\bar{b} = 3\bar{i} - \bar{j} + 2(\bar{i} + 3\bar{j})$

$= 3\bar{i} - \bar{j} + 2\bar{i} + 6\bar{j}$

$= 5\bar{i} + 5\bar{j}$

$= (5, 5)$

3. If $\bar{a} = (2, 1, -1)$ and $\bar{b} = (1, -3, 2)$, then $|2\bar{a} - 3\bar{b}| =$ ......

(A) $\sqrt{186}$ (B) $\sqrt{196}$

(C) $\sqrt{83}$ (D) 21

Ans. : (A)

$2\bar{a} - 3\bar{b} = 2\bar{a} + (-3)\,\bar{b}$

$= 2(2, 1, -1) + (-3)(1, -3, 2)$

$= (4, 2, -2) + (-3, 9, -6)$

$= (4-3, 2+9, -2-6)$
$= (1, 11, -8)$

$\therefore \ |2\bar{a} - 3\bar{b}| = \sqrt{1+121+64} = \sqrt{186}$

4. If $\bar{a} = 2\bar{i} - 2\bar{j} + \bar{k}$ and $\bar{b} = \bar{i} + 3\bar{j} + 4\bar{k}$, then $\bar{a} \cdot \bar{b} =$ ....

(A) -1 (B) 0
(C) 1 (D) -2

Ans. : (B)

$\bar{a} \cdot \bar{b} = (2\bar{i} - 2\bar{j} + \bar{k}) \cdot (\bar{i} + 3\bar{j} + 4\bar{k})$
$= (2)(1) + (-2)(3) + (1)(4)$
$= 2 - 6 + 4$
$= 0$

5. $\bar{x} \times (2\bar{x}) =$ ....

(A) $2|\bar{x}|^2$ (B) $2\bar{x}$
(C) 2 (D) $\bar{0}$

Ans. : (D)

$\bar{x} \times (2\bar{x}) = 2(\bar{x} \times \bar{x}) = 2(\bar{0}) = \bar{0}$

6. $(3\bar{i} - \bar{j} + 2\bar{k}) \cdot (2\bar{i} + \bar{j} - \bar{k}) =$ ......

(A) -3 (B) $(6, -1, -2)$
(C) 0 (D) 3

Ans. : (D)

$(3\bar{i} - \bar{j} + 2\bar{k}) \cdot (2\bar{i} + \bar{j} - \bar{k}) = (3, -1, 2) \cdot (2, 1, -1)$
$= (3)(2) + (-1)(1) + (2)(-1)$
$= 6 - 1 - 2$
$= 3$

7. If $|\bar{x}| = 2$, then $|-3\bar{x}| =$ ......

(A) 6 (B) -6
(C) 12 (D) 18

Ans. : (A)

$|-3\bar{x}| = |-3||\bar{x}| = 3(2) = 6$

8. If $|\bar{x}| = 2$ and $|\bar{y}| = 1$, then $(\bar{x} + \bar{y}) \cdot (\bar{x} - \bar{y}) =$ ....

(A) 5 (B) 4
(C) 3 (D) 2

Ans. : (C)

$(\bar{x} + \bar{y}) \cdot (\bar{x} - \bar{y})$
$= \bar{x} \cdot \bar{x} - \bar{x} \cdot \bar{y} + \bar{y} \cdot \bar{x} - \bar{y} \cdot \bar{y}$
$= |x|^2 - |\bar{y}|^2$ $(\because \bar{x} \cdot \bar{y} = \bar{y} \cdot \bar{x}$ and $|\bar{x}|^2 = \bar{x} \cdot \bar{x})$
$= 2 - (1)^2$
$= 4 - 1 = 3$

9. $\bar{x} \times (\bar{x} - \bar{y}) =$ ......

(A) $|\bar{x}|^2 - (\bar{x} \times \bar{y})$ (B) $\bar{x} \times \bar{y}$
(C) $\bar{y} \times \bar{x}$ (D) $\bar{0}$

Ans. : (C)

$\bar{x} \times (\bar{x} - \bar{y})$
$= \bar{x} \times \bar{x} - \bar{x} \times \bar{y}$
$= \bar{0} + \bar{y} \times \bar{x}$ $(\because \bar{x} \times \bar{x} = \bar{0}$ and $\bar{x} \times \bar{y} = -\bar{y} \times \bar{x})$
$= \bar{y} \times \bar{x}$

10. If $\bar{x} \perp \bar{y}$, then $\bar{x} \cdot (\bar{x} + \bar{y}) =$ ......

(A) $|\bar{x}|^2$ (B) 0
(C) $|\bar{x}|$ (D) $|\bar{x}|\ |\bar{y}|$

Ans. : (A)

$\bar{x} \cdot (\bar{x} + \bar{y})$
$= \bar{x} \cdot \bar{x} + \bar{x} \cdot \bar{y}$
$= |\bar{x}|^2 + 0$
$= |\bar{x}|^2$ $(\because \bar{x} \perp \bar{y} \Rightarrow \bar{x} \cdot \bar{y} = 0$ and $\bar{x} \cdot \bar{x} = |\bar{x}|^2)$

11. If $\bar{x} \times \bar{y} = (1, -2, -5)$, then $\bar{y} \times (\bar{x} + \bar{y}) =$ ....

(A) $(-1, 2, 5)$ (B) $(-1, -2, -5)$
(C) $(-5, -2, 1)$ (D) $(5, 2, -1)$

Ans. : (A)

$\bar{y} \times (\bar{x} + \bar{y}) = (\bar{y} \times \bar{x}) + (\bar{y} \times \bar{y})$
$= -(\bar{x} \times \bar{y}) + \bar{0}$
$= -(\bar{x} \times \bar{y})$
$= -(1, -2, -5)$
$= (-1, 2, 5)$

12. Angle between the vector $\bar{x} = \bar{j} + \bar{k}$ and Y-axis is ......

(A) 0 (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Ans. : (C)

$\bar{x} = \bar{j} + \bar{k} = (0, 1, 1)$ and

Unit vector in the direction of Y-axis $= \bar{j} = (0, 1, 0)$

$\therefore$ Angle between $\bar{x}$ and Y-axis $= \cos^{-1}\left(\frac{\bar{x} \cdot \bar{j}}{|\bar{x}||\bar{j}|}\right)$

$$= \cos^{-1}\left(\frac{(0,1,1)\cdot(0,1,0)}{\sqrt{0+1+1}\cdot\sqrt{0+1+0}}\right)$$

$$= \cos^{-1}\left(\frac{0+1+0}{\sqrt{2}\,(1)}\right)$$

$$= \cos^{-1}\frac{1}{\sqrt{2}}$$

$$= \frac{\pi}{4}$$

Multiple Choice Questions (MCQ's) with (Final Answers)

1. .... is a unit vector.

(A) $(1, -1)$ (B) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(C) $(\sin\theta, \cos\theta)$ (D) $(-1, 1)$

2. $|3\bar{i} + 4\bar{j} + 12\bar{k}| =$ ....

(A) $\sqrt{50}$ (B) 12

(C) 13 (D) 15

3. $(1, 0, 0) \times (0, 0, 1) =$ ....

(A) $(0, 1, 0)$ (B) $(0, -1, 0)$

(C) $\bar{0}$ (D) $(-1, 0, 0)$

4. For every vector $\bar{x} \neq \bar{0}$ of R^2, $\frac{\bar{x}}{|\bar{x}|}$ is ....

(A) zero vector (B) scalar

(C) unit vector (D) constant

5. $\bar{j} \times \bar{k} =$ ......

(A) $-\bar{i}$ (B) $\bar{i}$

(C) $\bar{0}$ (D) 1

6. If $\bar{a} = (1, 0)$, $\bar{b} = (2, 0)$, then $|\bar{a} + \bar{b}|$ .... $|\bar{a}| + |\bar{b}|$

(A) > (B) <

(C) = (D) none of these

7. If $(1, -2, 3) \cdot (4, 5, m) = 0$, then $m =$ ....

(A) 2 (B) -2

(C) 3 (D) 0

8. $(\bar{i} \wedge \bar{j}) =$ .... $(\bar{i}, \bar{j} \in R^2)$

(A) 0 (B) $\frac{\pi}{2}$

(C) π (D) $\frac{\pi}{3}$

9. If $\bar{x} \cdot \bar{y} = |\bar{x}| = |\bar{y}| = \sqrt{2}$, then $(\bar{x} \wedge \bar{y}) =$ ....

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) 0

10. $|\bar{x}|^2 =$ ....

(A) $\bar{x} \times \bar{x}$ (B) $\bar{x} \cdot \bar{x}$

(C) $\bar{x} + \bar{x}$ (D) $|\bar{x}|\,\bar{x}$

11. $(3\bar{x}) \times (3\bar{y}) =$ ....

(A) $\bar{0}$ (B) $3(\bar{x} \times \bar{y})$

(C) $3(\bar{x} \cdot \bar{y})$ (D) $9(\bar{x} \times \bar{y})$

12. $(\sec\theta, \tan\theta, -1) \cdot (\sec\theta, -\tan\theta, 1) =$ ......

(A) 1 (B) 0

(C) -1 (D) 2

13. If $|\bar{x}| = 4$, then $\bar{x} \cdot (5\bar{x}) =$ ....

(A) 16 (B) 25

(C) 20 (D) 80

14. $\bar{i} \cdot \bar{j} + \bar{j} \cdot \bar{k} + \bar{k} \cdot \bar{i} =$ ......

(A) 1 (B) 3

(C) 0 (D) $\bar{i} + \bar{j} + \bar{k}$

15. $\bar{i} \times \bar{j} + j \times \bar{k} + \bar{k} \times \bar{i} =$ ......

(A) $\bar{i} + \bar{j} + \bar{k}$ (B) 3

(C) $-\bar{i} - \bar{j} - \bar{k}$ (D) $\bar{0}$

Answers

(1) C	(2) C	(3) B	(4) C	(5) A
(6) C	(7) A	(8) B	(9) A	(10) B
(11) D	(12) B	(13) D	(14) C	(15) A

★★★