SECTION-02 - BE-02

APTITUDE TEST (MATHEMATICS & SOFT SKILL)

MATHEMATICS

1. Determinant and Matrices
2. Trigonometry
3. Vectors
4. Co-ordinate Geometry
5. Function & Limit
6. Differentiation and its Applications
7. Integration
8. Logarithm
9. Statistics

ENGLISH

10. Comprehension of Unseen Passage
11. Theory of Communication
12. Grammar
13. Correction of Incorrect Words and Sentences

5. Function & Limit

FUNCTION

[1] Introduction :

Mathematics began with number system and arithmatic. The development of abstract form of mathematics led to the development of algebra.

Set theory is developed to provide a basis for the various branches of mathematics and as the development increased an important concept of function developed.

French mathematician 'Descartes' used the word 'function' first time in the year 1637. At that time he had only referred to x^n, $n \in N$. James Gregory gave the definition of a function in 1667. In 1673 Liebnitz gave the defintion of a function in the context of the co-ordinates, the slope of a tangent and the slope of a normal at a point on a curve as a quantity varying at every point. In 1734, Euler introduced the notation for function. The definition of a function used now a days was given by Dirichlet. George Cantor gave definition of a function with the help of sets. Thus many mathematicians have contributed to the development of function.

[2] Relation :

If A and B are any two non empty sets, then any subset of $A \times B$ is called a relation from set A to set B.

For example, $A = \{1, 3, 5\}$ and $B = \{2, 4\}$

$\therefore \quad A \times B = \{(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), (5, 4)\}$

Now consider a subset $R = \{(1, 2), (5, 2), (5, 4)\}$ of $A \times B$, then R is called a relation from set A to set B.

[3] Function :

We have discussed about 'relation' in the above topic. Function is a special type of relation.

Suppose A and B are any two non empty sets and $R \subset A \times B$ is a relation from A to B. If this relation R satisfies the following conditions, then it is called a function.

(1) For each element of A, there should be an order pair containing that element in R.

(2) For each element of A, there should be an unique order pair containing that element in R.

In the example shown in the section 1.7, relation $R = \{(1, 2), (5, 2), (5, 4)\}$. Here, there is no any order pair containing an element 3 of set A in R. Thus it is not a function.

But if we take a subset $R_1 = \{(1, 4), (3, 2), (5, 2)\}$ of $A \times B$, then there is an order pair containing all the three element 1, 3 and 5 of set A in R_1 and there is one and only one such pair for each element. Thus this relation R_1 is a function.

Thus the definition of a function can be given as follows :

Suppose $A \neq \phi$, $B \neq \phi$, $f \subset (A \times B)$ and $f \neq \phi$. If for all $x \in A$, there corresponds a unique order pair $(x, y) \in f$, then $f : A \to B$ is called a function.

In other words, any correspondence conncecting each element of set A with the unique element of set B is called a function from set A to set B.

→ Function from set A to set B is written as $f : A \to B$ and read as 'Function f from set A to set B.'

→ Set A is called the domain of the function $f : A \to B$ and it is denoted by D_f.

→ Set B is called the codomain of the function $f : A \to B$ and is denoted by CoD_f.

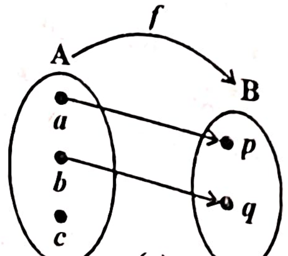
→ Set $\{y \mid (x, y) \in f\}$ for the function $f : A \to B$ is called the range of the function and is denoted by R_f.

Thus the range is a set formed by the second element of each order pair of relation f. The range of a function f is always a subset of the codomain of the function f. i.e. $R_f \subset CoD_f$.

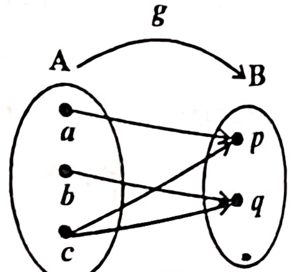
[4] Understanding of function by Venn diagram :

We know that a set can be represented as Venn diagram. By representing set A and set B as Venn diagram and correspondance between their elements by arrow (→), we can understand the concept of function.

Four different relations between the set $A = \{a, b, c\}$ and set $B = \{p, q\}$ are shown in the following figure.



(a)



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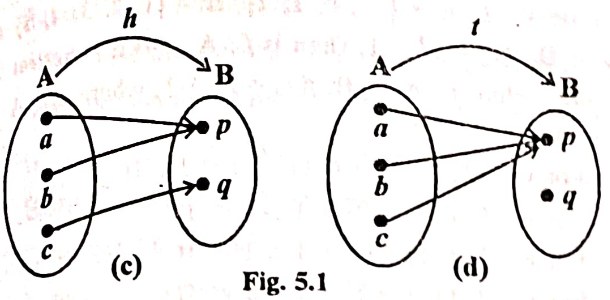


Fig. 5.1

Relation shown in the figure 5.1 (a) can be written in the form of set as follows :

$f = \{(a, p), (b, q)\}$

Here, from the figure 5.1 (a), it can be seen that there is no arrow from an element c of the set A. i.e. an element c of set A is not associated with any element of set B. Thus f is not a function.

Relation shown in the figure 5.1 (b) can be written in the form of set as follows.

$g = \{(a, p), (b, q), (c, p), (c, q)\}$.

Here, it can be seen that there are two arrows from an element c of set A. i.e. element c is associated with two elements of set B. Thus relation g is also not a function.

Relation shown in the figure 5.1 (c) can be written in the form of set as follows.

$h = \{(a, p), (b, p), (c, q)\}$.

Here, it can be seen that there is unique arrow from each element of set A. Thus h is a function.

Relation shown in the figure-5.1 (d) can be written in the form of set as follows.

$t = \{(a, p), (b, p), (c, p)\}$.

Here, also it can be seen that there is unique arrow from each element of set A. Thus t is also a function.

(**Note :** To determine whether the given relation is a function or not using Venn diagram, we have to concentrate only on the set A. There must be an unique arrow from each element of set A.)

Example-1 : If A = {1, 2, 3, 4}, B = {15, 16, 17, 18, 19} and relation f = {(1, 16), (2, 19), (3, 15), (4, 18), (2, 15)}, then is $f : A \rightarrow B$ a function ?

Solution : Here, relation

$f = \{(1, 16), (2, 19), (3, 15), (4, 18), (2, 15)\}$ is given

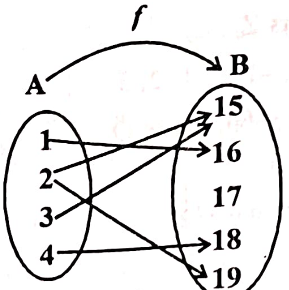


Fig. 5.2

If we represent this relation using Venn diagram as shown in the figure 5.2, then it can be seen that there are two arrows from an element 2 of the set A. i.e. There are two order pairs (2,19) and (2, 15) containing the element 2 of set A. Thus the given relation f is not a function.

Example-2 : If M = {–2, 1, 4}, N = {2, 3} and relation g = {(–2, 2), (1, 3)}, then is $g : M \rightarrow N$ a function ?

Solution : Here, relation $g = \{(-2, 2), (1, 3)\}$ is given.

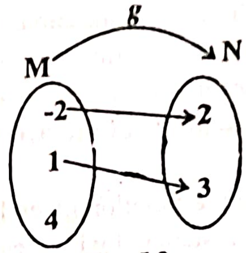


Fig. 5.3

If we represent this relation using Venn diagram as shown in the above figure, then it can be seen that there is no arrow from an element 4 of the set M. i.e. An element 4 of set M is not associated with any element of set N. Thus there is no order pair containing 4 in g. Thus relation g is not a function.

Example-3 : If A = {2, 3, 4}, B = {5, 7, 9, 11} and relation h = {(2, 5), (3, 7), (4, 9)}, then is $h : A \rightarrow B$ a fuction ? If h is a function then find it's domain, codomain and range.

Solution : Here, relation $h = \{(2, 5), (3, 7), (4, 9)\}$ is given.

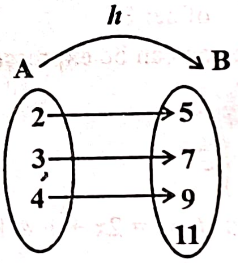


Fig. 5.4

It can be clearly seen from the set form and the above figure of relation h that every element of set A is associated with the unique element of set B. Thus $h : A \rightarrow B$ is a function.

D_h = Domain of function h = A = {2, 3, 4}

CoD_h = Codomain of function h = B = {5, 7, 9, 11}

The range of function h is the set of all those elements of set B which are associated with the elements of set A. In other words the set of the second element of all the order pairs in h.

Thus R_h = Range of function h = {5, 7, 9}

- Many times a function is expressed using formula instead of set form.

For example, Let $f: A \to B$, $f = \{(2, 5), (3, 7), (4, 9)\}$ be a function, where $A = \{2, 3, 4\}$ and $B = \{5, 7, 9, 11\}$

Here, we have used order pairs to show the associations of element of set A with the elements of set B.

This method is easier if the number of elements in set A is less. But when the number of elements in set A is very large, it becomes tedious or impossible to write the order pairs that associates every element of set A with the elements of set B. At such times it is easier and more effective if the realtion between the elements of the two sets is represented by a general rule.

For example, in the function given here, elements 2, 3 and 4 of the set A are associated with elements 5, 7 and 9 of set B respectively. Now this relation can also be represented by the following general rule.

Relation : The element x of the set A is associated with the element $2x + 1$ of set B.

Here x is an element of set A. So the values of x are 2, 3 and 4.

If we take $x = 2$, then according to the above relation the element 2 of set A is associated with an element $2(2) + 1 = 5$ of set B.

If we take $x = 3$, then the element 3 of set A is associated with an element $2(3) + 1 = 7$ of set B and if we take $x = 5$, then the element 5 of set A is associated with an element $2(5) + 1 = 11$ of set B.

The above relation can be expressed using notation as follows :

$f: A \to B$

$f(x) = 2x + 1$

Here $x \in A$ and $f(x) = 2x + 1 \in B$

i.e. An element x of set A is associated with an element $f(x) = 2x + 1$ of B by the function f. Thus the function can be expressed using formula insted of order pairs.

Thus if function $f: A \to B$, $f(x) = y$, then an element x of A is associated with the element $f(x) = y$ of set B by the function f.

Here $y = f(x)$ is called the image of x and x is called the pre-image of $y = f(x)$.

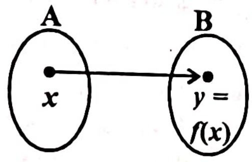


Fig. 5.5

Example-4 : If $A = \{-1, 0, 1, 2\}$, $B = \{1, 2, 3, 4, 5\}$ and $f: A \to B$, $f(x) = x^2 + 1$, then is $f: A \to B$ a function ?

Solution : Here $f: A \to B$, $f(x) = x^2 + 1$, where $x \in A$ and $f(x) \in B$.

For $x = -1$, $f(-1) = (-1)^2 + 1 = 2 \in B$

For $x = 0$, $f(0) = 0^2 + 1 = 1 \in B$

For $x = 1$, $f(1) = 1^2 + 1 = 2 \in B$

For $x = 2$, $f(2) = 2^2 + 1 = 5 \in B$

Thus from the above calculation, it is possible to know that which element of set A is associated with which element of set B, which is shown below in the form of diagram.

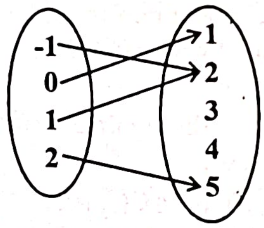


Fig. 5.6

It can be seen here that each element of set A is associated with unique element of set B. Hence $f: A \to B$ is a function.

Example-5 : If $A = \{1, 2, 3\}$, $B = \{2, 3, 7, 8, 9\}$ and $f: A \to B$, $f(x) = x^2 + x + 1$, then is $f: A \to B$ a function ?

Solution : Here $f: A \to B$, $f(x) = x^2 + x + 1$, where $x \in A$ and $f(x) = x^2 + x + 1 \in B$.

For $x = 1$, $f(1) = 1^2 + 1 + 1 = 3 \in B$

For $x = 2$, $f(2) = 2^2 + 2 + 1 = 7 \in B$

For $x = 3$, $f(3) = 3^2 + 3 + 1 = 13 \notin B$

Here an element 3 of set A is assocaited with the element 13. But 13 is not an element of B. Hence an element 3 of set A is not associated with any element of set B. Thus $f: A \to B$ is not a function.

[5] Some useful sets :

(1) Set of Natural Numbers N

$= \{1, 2, 3, 4, 5, ...\}$

(2) Set of Integers Z

$= \{.... -3, -2, -1, 0, 1, 2, 3, ...\}$

(3) Set of Rational Numbers Q

$= \left\{\frac{p}{q} \mid p \in z, q \in N\right\}$

(4) Set of Real Numbers R

= All real numbers in which rational numbers and irrational numbers (like $\sqrt{2}$, π, e ... etc.) are included.

Here $N \subset Z \subset Q \subset R$.

[6] Some useful Functions :

(1) **Constant Function :**

A function whose range is singleton set is called a constant function.

For example, $f : N \to N$ $f(x) = 10$ is a constant function.

(2) **Modulus Function :**

$f : R \to R^+ \cup \{0\}$, $f(x) = |x|$ is called a modulus function.

Where $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

(3) **Identity Function :**

$I_A : A \to A$, $I_A(x) = x$ is called identity function.

(4) **Exponential Function :**

$f : R \to R^+$, $f(x) = a^x$, $a \in R^+$ is called exponential function.

e.g. $f(x) = 2^x$, $f(x) = 5^x$ are exponential functions.

(5) **Logarithmic Function :**

$f : R^+ \to R$, $f(x) = \log_a x$, $a \in R^+ - \{1\}$ is called logarithmic function.

(6) **Trigonometric Functions :**

(7) **Polynomial Function :**

$f : A \to R$,

$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$, $a_i \in R$ $(i = 0, 1, 2, ..., n)$, $(a_n \neq 0)$ is called a polynomial function on A.

where $A \subset R$ and $n \in N \cup \{0\}$.

e.g. $f(x) = 5x^3 - 3x^2 + 2x - 7$ is a polynomial function.

(8) **Rational Function :**

$f : A \to R$, $f(x) = \dfrac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions on A and $q(x) \neq 0$, $\forall x \in A$ is called rational function.

e.g. $f(x) = \dfrac{2x^2 + 3x - 4}{4x^3 - 5x^2 + 2x + 3}$ is a rational function.

[7] Composition of functions :

Let $f : A \to B$ and $g : C \to D$ are functions and $R_f \subset D_g$. So for each $x \in A$, $f(x) \in C$ and $g(f(x)) \in D$. Thus associated with each $x \in A$, there exists unique $g(f(x)) \in D$. Thus the function $gof : A \to D$, $(gof)(x) = g(f(x))$ can be defined where $R_f \subset D_g$. This function gof is called the composition of f and g.

For example,

$f : R \to R$, $f(x) = x + 1$ and $g : R \to R$, $g(x) = x^2$

Here, $R_f \subset R = D_g$

∴ gof exists and $gof : R \to R$

Also, $(gof)(x) = g(f(x))$

$= g(x + 1)$

$= (x + 1)^2$

$= x^2 + 2x + 1$

Example-6 : Do as directed :

(1) **If $f(x) = x^3 - 3$, then find the image of 2 wr.t. f.**

OR

If $f(x) = x^3 - 3$, then find $f(2)$.

(2) **If $f(x) = x^3 + 5$, then find $f(2)$.**

(3) **If $f(x) = x^2 - 1$, then find $f(-1)$.**

(4) **If $f(x) = x^3 - 1$ then find $f(3) + f(-2)$.**

Solution :

(1) Here, $f(x) = x^3 - 3$

∴ $f(2) = (2)^3 - 3$ (replacing x with 2)

$= 8 - 3$

$= 5$

(2) Here, $f(x) = x^3 + 5$

∴ $f(2) = 2^3 + 5$

$= 8 + 5$

$= 13$

(3) Here, $f(x) = x^2 - 1$

∴ $f(-1) = (-1)^2 - 1$

$= 1 - 1$

$= 0$

(4) Here, $f(x) = x^3 - 1$

∴ $f(3) = 3^3 - 1 = 27 - 1 = 26$ and

$f(-2) = (-2)^3 - 1 = -8 - 1 = -9$

∴ $f(3) + f(-2) = 26 + (-9) = 26 - 9 = 17$

Multiple Choice Questions (MCQs) (Solution with Explanation)

1. If $f(x) = \log x$ then $f(x) + f(y) =$ ....

(A) $f(x + y)$ (B) $f(x - y)$

(C) $f(xy)$ (D) $f\left(\frac{x}{y}\right)$

Ans. : (C)

Explanation : Here, $f(x) = \log x$ $\therefore f(y) = \log y$

$f(x) + f(y) = \log x + \log y$

$= \log (xy)$ (Multiplication law)

$= f(xy)$

2. If $f(x) = \log\left(\frac{x-1}{x}\right)$, then $f(-x) =$ ....

(A) $\log\left(\frac{x-1}{x}\right)$ (B) $\log\left(\frac{x+1}{x}\right)$

(C) $\log\left(\frac{x}{x+1}\right)$ (D) $\log\left(\frac{x}{x-1}\right)$

Ans. : (B)

Explanation :

Here $f(x) = \log\left(\frac{x-1}{x}\right)$

$\therefore \quad f(-x) = \log\left(\frac{-x-1}{-x}\right) = \log\left(\frac{x+1}{x}\right)$

3. If $\sqrt{\log_3 x} = 2$, then $x =$ ....

(A) 9 (B) 81

(C) $\sqrt{3}$ (D) 6

Ans. : (B)

Explanation : $\sqrt{\log_3 x} = 2$

$\therefore \log_3 x = 4$

$\therefore x = 3^4 = 81$

LIMIT

[8] Introduction :

The study we have done so far has been pre-calculus study. We will now try to get an introduction to calculus. One quantity 'tends to' the another quantity is discussed in calculus. So before we start studying this we will look at the main ideas from which the idea of 'Limit' arises.

About 2500 years ago the Greeks divided the inner and outer polygonal closed region of an irregularly shaped closed region in to a triangular regions to find its area and by the sum of their areas obtained the area of this polygonal region and by their limit they found the area of irregular closed region.

Limit is also used in questions of growth rate, velocity, acceleration... etc. Newton said that limit is the bases of calculus. Then Cauchy clarified the concepts of limit. Leibnitz and Netwon discovered calculus independently of each other, but Newton did not publish his work immediately for fear of controversy. In 1684 first Leibnitz published his work on calculus.

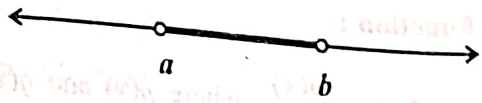
[9] Interval :

Let us know about the following intervals.

For $a, b, \in R$

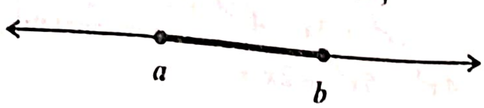
(1) **Open interval :**

$(a, b) = \{x \mid a < x < b, x \in R\}$



(2) **Closed interval :**

$[a, b] = \{x \mid a \le x \le b, x \in R\}$

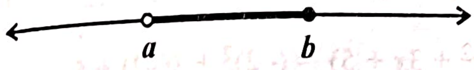


(3) **Closed – open interval :**

$[a, b) = \{x \mid a \le x < b, x \in R\}$

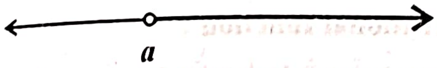
(4) **Open–closed interval :**

$(a, b] = \{x \mid a < x \le b, x \in R\}$



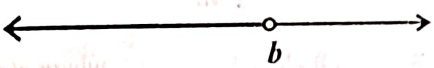
Let us know about the following 'infinite' intervals.

(5) $(a, \infty) = \{x \mid x > a, x \in R\}$

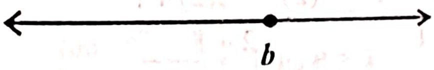


(6) $[a, \infty) = \{x \mid x \ge a, x \in R\}$

(7) $(-\infty, -b) = \{x \mid x < b, x \in R\}$



(8) $(-\infty, b] = \{x / x \le b, x \in R\}$



[10] Limit of a function :

Now let's focus on 'Limit of a function' - a central thought of this chapter.

The graph of the function $f: R \to R$, $f(x) = 4x - 5$ is shown in the figure 5.7. Suppose the values of x get closer and closer to 2 by getting the values smaller or larger then 2.

$x < 2$

x	$f(x)$
1.5	1
1.8	2.2
1.9	2.6
1.99	2.96
1.999	2.996

(1)

$x > 2$

x	$f(x)$
2.5	5
2.2	3.8
2.1	3.4
2.01	3.04
2.001	3.004

(2)

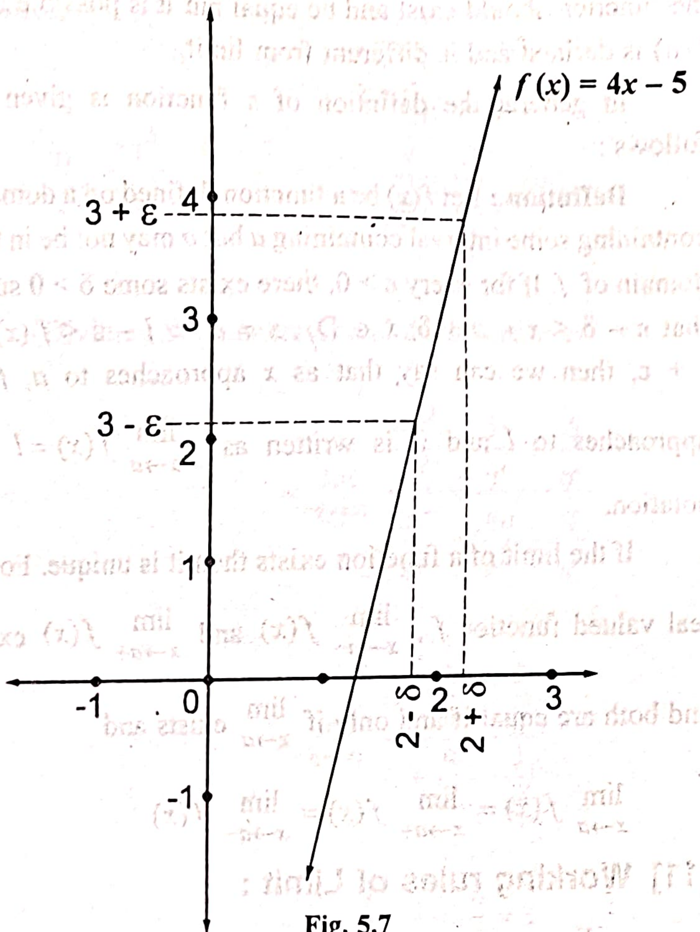


Fig. 5.7

It can be seen from the above table that as the values of x get closer to 2, the values of $4x - 5$ get closer to 3.

The following statement can be given to explain the situation of table (1).

'The limit of $f(x) = 4x - 5$ is 3, as x approaches 2 from left'. For this we write $\lim_{x \to 2-} f(x) = 3$ in notation and can say that the left limit of the function is 3.

Similarly the following statement can be given to explain the situation of table (2).

'The limit of $f(x) = 4x - 5$ is 3 as x approaches 2 from right'. For this we write $\lim_{x \to 2+} f(x) = 3$ in notation and can say that the right limit of the function is 3.

If left limit and right limit exist and are equal as in the above example then we can say that the limit of the function exists. For the above example, it is denoted by $\lim_{x \to 2} f(x) = 3$.

Notice that when we talk about the limit of a function if we say $x \to a$ (read : x tends to a) then x approaches to a through the values less than a or greater than a, $f(x)$ must be defined for such values of x. But the function is not necessarily defined for $x = a$. Also left limit and right limit of the function should exist and be equal but it is possible that $f(a)$ is defined and is different from limit.

In general the defintion of a function is given as follows :

Defintion : Let $f(x)$ be a function defined on a domain containing some interval containing a but a may not be in the domain of f. If for every $\varepsilon > 0$, there exists some $\delta > 0$ such that $a - \delta < x < a + \delta$, $x \in D_f$, $x \neq a \Rightarrow l - \varepsilon < f(x) < l + \varepsilon$, then we can say that as x approaches to a, $f(x)$ approaches to l and it is written as $\lim_{x \to a} f(x) = l$ in notation.

If the limit of a function exists then it is unique. For a real valued function f, $\lim_{x \to a-} f(x)$ and $\lim_{x \to a+} f(x)$ exist and both are equal if and only if $\lim_{x \to a}$ exists and

$$\lim_{x \to a} f(x) = \lim_{x \to a+} f(x) = \lim_{x \to a-} f(x)$$

[11] Working rules of Limit :

If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then

(1) $\lim_{x \to a} (f(x) + g(x)) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

(2) $\lim_{x \to a} f(x) \cdot g(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

(3) $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, where $\lim_{x \to a} g(x) \neq 0$

(4) $\lim_{x \to a} k = k$, where k is any constant number.

(5) $\lim_{x \to a} k\, f(x) = k \lim_{x \to a} f(x)$

(6) $\lim_{x \to a} (f(x) - g(x)) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

[12] Some Standard forms :

(1) $\lim_{x \to a} x^n = a^n$, $n \in R$

For example $\lim_{x \to 3} x^2 = 3^2 = 9$

(2) Limit of a polynomial function :

If $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1 x + a_0$, $a_i \in R$ where $i = 1, 2, 3, \dots, n$ and $a_n \neq 0$ is a polynomial on R, then

$$\lim_{x \to b} p(x) = p(b)$$

For example

$$\lim_{x \to -2} (x^2 + 3x + 5) = (-2)^2 + 3(-2) + 5$$
$$= 4 - 6 + 5$$
$$= 3$$

(3) Limit of a rational function :

If $p(x)$ and $q(x)$ are polynomial function, then for the rational function $f(x) = \dfrac{p(x)}{q(x)}$, $q(x) \neq 0$.

$$\lim_{x \to a} f(x) = \lim_{x \to a} \frac{p(x)}{q(x)} = \frac{\lim_{x \to a} p(x)}{\lim_{x \to a} q(x)} = \frac{p(a)}{q(a)} = f(a)$$

where $q(a) \neq 0$

For example

$$\lim_{x \to 2} \frac{x^2 - 3x + 1}{x + 1} = \frac{(2)^2 - 3(2) + 1}{2 + 1}$$

$$= \frac{4 - 6 + 1}{3} = -\frac{1}{3}$$

(4) $\lim_{x \to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$, $n \in R$, $x \in R - \{a\}$

Example-7 : Find the following limits :

(i) $\lim_{x \to 1} (x^3 - 3x^2 + 5x - 6)$

(ii) $\lim_{x \to 1} \dfrac{x^2 + x + 1}{x + 1}$

(iii) $\lim_{x \to 2} \dfrac{x^3 + 5}{5x + 3}$

(iv) $\lim_{x \to 2} \dfrac{x^2 - 1}{x - 1}$

Solution :

(i) $\lim_{x\to 1}(x^3-3x^2-5x-6) = (1)^3\, 3(1)^2+5(1)-6$

$= 1-3+5=6$

$= -3$

(ii) $\lim_{x\to 1}\frac{x^2+x+1}{x+1} = \frac{(1)^2+(1)+1}{(1)+1} = \frac{3}{2}$

(iii) $\lim_{x\to 2}\frac{x^3+5}{5x+3} = \frac{(2)^3+5}{5(2)+3} = \frac{8+5}{10+3} = \frac{13}{13} = 1$

(iv) $\lim_{x\to 2}\frac{x^2-1}{x-1} = \frac{(2)^2-1}{2-1} = \frac{4-1}{1} = 3$

[13] Some more standard forms :

(1) $\lim_{n\to\infty}\frac{1}{n} = 0$

(2) $\lim_{h\to 0}\frac{a^h-1}{h} = \log_e a, \quad a\in R^+$

By substituting $a = e$ in the above result

$$\lim_{h\to 0}\frac{e^h-1}{h} = \log_e e = 1$$

(3) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

Let $\frac{1}{n} = x$ in the above result, as $n\to\infty, x\to 0$

$$\therefore \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$$

Example-8 : Find the following limits :

$\left(\text{Realted to } \lim_{n\to\infty}\frac{1}{n} = 0\right)$

$$\lim_{n\to\infty}\frac{6n^2-3n+5}{2n^2+4n-3}$$

Solution :

$$\lim_{n\to\infty}\frac{6n^2-3n+5}{2n^2+4n-3} = \lim_{n\to\infty}\frac{\frac{6n^2-3n+5}{n^2}}{\frac{2n^2+4n-3}{n^2}}$$

(Dividing the numerator and denominator by n^2)

$$= \lim_{n\to\infty}\frac{\frac{6n^2}{n^2}-\frac{3n}{n^2}+\frac{5}{n^2}}{\frac{2n^2}{n^2}+\frac{4n}{n^2}-\frac{3}{n^2}}$$

$$= \lim_{n\to\infty}\frac{6-\frac{3}{n}+\frac{5}{n^2}}{2+\frac{4}{n}-\frac{3}{n^2}}$$

$$= \frac{6-0+0}{2+0-0} = \frac{6}{2} = 3$$

Example-9 : Find the following limits :

$\left(\text{Related to } \lim_{h\to 0}\frac{a^h-1}{h} = \log_e a\right)$

(i) $\lim_{x\to 0}\frac{5^x-1}{x}$

(ii) $\lim_{x\to 0}\frac{e^{3x}-1}{x}$

Solution :

(i) $\lim_{x\to 0}\frac{5^x-1}{x} = \log_e 5$

$\left(\because \text{ By comparing with } \lim_{h\to 0}\frac{a^h-1}{h}, a=5\right)$

(ii) $\lim_{x\to 0}\frac{e^{3x}-1}{x} = \lim_{x\to 0}3\left(\frac{e^{3x}-1}{3x}\right)$

$= 3\cdot\log_e e = 3(1) = 3$

Multiple Choice Questions (MCQs) (Solution with Explanation)

1. $\lim_{x \to 1} (x^3 - 3x^2 + 5x - 6) = \ldots\ldots .$

(A) 1 (B) 6 (C) 3 (D) −3

Ans. : (D)

Explanation :

$$\lim_{x \to 1} (x^3 - 3x^2 - 5x - 6) = (1)^3 - 3(1)^2 + 5(1) - 6$$
$$= 1 - 3 + 5 - 6$$
$$= -3$$

2. $\lim_{x \to 1} \frac{x^2 + x + 1}{x + 1} = \ldots\ldots .$

(A) $\frac{3}{2}$ (B) 1 (C) 3 (D) $\frac{2}{3}$

Ans. : (A)

Explanation : $\lim_{x \to 1} \frac{x^2 + x + 1}{x + 1} = \frac{(1)^2 + (1) + 1}{(1) + 1} = \frac{3}{2}$

3. $\lim_{x \to 2} \frac{x^3 + 5}{5x + 3} = \ldots\ldots .$

(A) $\frac{13}{7}$ (B) 1 (C) $\frac{3}{4}$ (D) $\frac{7}{13}$

Ans. : (B)

Explanation :

$$\lim_{x \to 2} \frac{x^3 + 5}{5x + 3} = \frac{(2)^3 + 5}{5(2) + 3} = \frac{8 + 5}{10 + 3} = \frac{13}{13} = 1$$

4. $\lim_{x \to 2} \frac{x^2 - 1}{x - 1} = \ldots\ldots .$

(A) 3 (B) $\frac{1}{3}$

(C) 2 (D) 1

Ans. : (A)

Explanation : $\lim_{x \to 2} \frac{x^2 - 1}{x - 1} = \frac{(2)^2 - 1}{2 - 1} = \frac{4 - 1}{1} = 3$

5. $\lim_{x \to 2} \frac{x^3 - 8}{x - 2} = \ldots\ldots .$

(A) −12 (B) 0 (C) 8 (D) 12

Ans. : (D)

Explanation :

$$\lim_{x \to 2} \frac{x^3 - 8}{x - 2} = \lim_{x \to 2} \frac{x^3 - 2^3}{x - 2}$$

(By comparing with standard form, $a = 2, n = 3$)

$$= 3 \cdot 2^{3-1}$$
$$= 3 \cdot 2^2 = 12$$

6. $\lim_{x \to 2} \frac{x^4 - 16}{x - 2} = \ldots\ldots .$

(A) 14 (B) 12 (C) 32 (D) None of these

Ans. : (C)

Explanation : $\lim_{x \to 2} \frac{x^4 - 16}{x - 2} = \lim_{x \to 2} \frac{x^4 - 2^4}{x - 2}$

$$= 4 \cdot 2^{4-1} = 4 \cdot 2^3 = 32$$

7. $\lim_{x \to 0} \frac{e^{3x} - 1}{x} = \ldots\ldots .$

(A) 1 (B) 3 (C) $\frac{1}{3}$ (D) None of these

Ans. : (B)

Explanation : $\lim_{x \to 0} \frac{e^{3x} - 1}{x} = \lim_{x \to 0} 3\left(\frac{e^{3x} - 1}{3x}\right)$

$$= 3 \cdot \log_e e = 3(1) = 3$$

8. $\lim_{x \to 0} \frac{5^x - 1}{x} = \ldots\ldots .$

(A) $\log_5 e$ (B) $\log_e 5$ (C) 1 (D) 0

Ans. : (B)

Explanation : $\lim_{x \to 0} \frac{5^x - 1}{x} = \log_e 5$

($\because$ By comparing with $\lim_{h \to 0} \frac{a^h - 1}{h}$, $a = 5$)

9. $\lim_{x \to 1} (x)^{\frac{1}{x-1}} = \ldots\ldots$.

(A) 1 (B) e (C) 0 (D) $\frac{1}{e}$

Ans. : (B)

Explanation : $\lim_{x \to 1} (x)^{\frac{1}{x-1}}$

By substituting $x - 1 = y$, as $x \to 1$, $y \to 0$ and $x = 1 + y$

$$\therefore \lim_{x \to 1} (x)^{\frac{1}{x-1}} = \lim_{y \to 0} (1 + y)^{\frac{1}{y}} = e$$

10. $\lim_{n \to \infty} \frac{6n^2 - 3n + 5}{2n^2 + 4n - 3} = \ldots\ldots$.

(A) 0 (B) 6 (C) ∞ (D) 3

Ans. : (D)

Explanation : $\lim_{n \to \infty} \frac{6n^2 - 3n + 5}{2n^2 + 4n - 3} = \lim_{n \to \infty} \frac{\frac{6n^2 - 3n + 5}{n^2}}{\frac{2n^2 + 4n - 3}{n^2}}$

(Dividing numerator and denominator by n^2)

$$= \lim_{n \to \infty} \frac{\frac{6n^2}{n^2} - \frac{3n}{n^2} + \frac{5}{n^2}}{\frac{2n^2}{n^2} + \frac{4n}{n^2} - \frac{3}{n^2}}$$

$$= \lim_{n \to \infty} \frac{6 - \frac{3}{n} + \frac{5}{n^2}}{2 + \frac{4}{n} - \frac{3}{n^2}}$$

$$= \frac{6 - 0 + 0}{2 + 0 - 0} = \frac{6}{2} = 3$$

11. $\lim_{\theta \to 0} \frac{\sin m\theta}{\theta} = \ldots\ldots$.

(A) 0 (B) m (C) 1 (D) θ

Ans. : (B)

Explanation : $\lim_{\theta \to 0} \frac{\sin m\theta}{\theta} = \lim_{\theta \to 0} \frac{m \cdot \sin m\theta}{m\theta}$

$$= m \cdot \lim_{\theta \to 0} \frac{\sin m\theta}{\theta} = m(1) = m$$

12. $\lim_{x \to 0} \frac{\tan 8x}{\sin 3x} = \ldots\ldots$.

(A) 1 (B) $\frac{3}{8}$

(C) $\frac{8}{3}$ (D) None of these

Ans. : (C)

Explanation :

$$\lim_{x \to 0} \frac{\tan 8x}{\sin 3x} = \lim_{x \to 0} \frac{\left(\frac{\tan 8x}{x}\right)}{\left(\frac{\sin 3x}{x}\right)} = \lim_{x \to 0} \frac{8 \cdot \left(\frac{\tan 8x}{8x}\right)}{3 \cdot \left(\frac{\sin 3x}{3x}\right)}$$

$$= \frac{8\,(1)}{3(1)} = \frac{8}{3}$$

13. $\lim_{x \to 0} (\sec^2 x - \tan^2 x) = \ldots\ldots$.

(A) -1 (B) 0 (C) 1 (D) 5

Ans. : (C)

Explanation : $\lim_{x \to 0} (\sec^2 x - \tan^2 x) = \lim_{x \to 0} 1 = 1$

14. $\lim_{\theta \to 0} \frac{\tan^2\left(\frac{\theta}{2}\right)}{\theta^2} = \ldots\ldots$.

(A) 4 (B) $\frac{1}{4}$ (C) 2 (D) $\frac{1}{2}$

Ans. : (B)

Explanation : $\lim_{\theta \to 0} \frac{\tan^2\left(\frac{\theta}{2}\right)}{\theta^2} = \lim_{\theta \to 0} \left(\frac{\tan\frac{\theta}{2}}{\theta}\right)^2$

$$= \lim_{\theta \to 0} \left(\frac{\tan\frac{\theta}{2}}{2 \cdot \frac{\theta}{2}}\right)^2$$

$$= \lim_{\theta \to 0} \frac{1}{4} \cdot \left(\frac{\tan\frac{\theta}{2}}{\frac{\theta}{2}}\right)^2$$

$$= \frac{1}{4} \cdot (1)^2 = \frac{1}{4}$$

15. $\lim_{x \to 0} \frac{\sin x^o}{x} = \ldots\ldots$.

(A) 1 (B) 0

(C) $\frac{\pi}{180}$ (D) $\frac{180}{\pi}$

Ans. : (C)

Explanation :

$$\lim_{x \to 0} \frac{\sin x^o}{x} = \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x} \quad \left(\because x^o = \left(\frac{\pi x}{180}\right)^R\right)$$

$$= \lim_{x \to 0} \frac{\pi}{180} \frac{\sin\left(\frac{\pi x}{180}\right)}{\left(\frac{\pi x}{180}\right)}$$

$$= \frac{\pi}{180} \cdot (1) = \frac{\pi}{180}$$

16. $\lim_{x \to 0} \frac{x^2}{\sin^2 x + x^2 \cos^2 x} = \ldots\ldots$.

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) 2

Ans. : (B)

Explanation :

$$\lim_{x \to 0} \frac{x^2}{\sin^2 x + x^2 \cos^2 x} = \lim_{x \to 0} \frac{1}{\left(\frac{\sin^2 x + x^2 \cos^2 x}{x^2}\right)}$$

$$= \lim_{x \to 0} \frac{1}{\frac{\sin^2 x}{x^2} + \cos^2 x}$$

$$= \lim_{x \to 0} \frac{1}{\left(\frac{\sin x}{x}\right)^2 + \cos^2 x}$$

$$= \frac{1}{(1)^2 + \cos^2 0} = \frac{1}{1+1} = \frac{1}{2}$$

Multiple Choice Questions (MCQ's) with (Final Answers)

1. $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \ldots\ldots$.

(A) –4 (B) 4 (C) 1 (D) 0

2. $\lim_{n \to \infty} \left(\frac{3n + 2}{2n + 3}\right) = \ldots\ldots$.

(A) 1 (B) 0 (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

3. $\lim_{x \to 2} \frac{x^5 - 32}{x - 2} = \ldots\ldots$.

(A) 80 (B) 160 (C) 40 (D) 0

4. $\lim_{h \to 0} \frac{a^h - 1}{h} = \ldots\ldots$.

(A) $\log_e a$ (B) e (C) a (D) 0

5. $\lim_{x \to 0} \frac{e^x - 1}{x} = \ldots\ldots$.

(A) e (B) 1 (C) –1 (D) 0

6. $\lim_{x \to 0} \frac{2^x - 1}{x} = \ldots\ldots$.

(A) $\log_e 2$ (B) $\log_2 e$

(C) 1 (D) 0

7. $\lim_{x \to 0} \frac{3^x - 1}{x} = \ldots\ldots$.

(A) 3 (B) $\log_e 3$ (C) $\log_3 e$ (D) –3

8. $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = \ldots\ldots$.

(A) e (B) 0 (C) 1 (D) –1

9. $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = \ldots\ldots$.

(A) –1 (B) 0 (C) 1 (D) ∞

10. $\lim_{x \to 0} \frac{x}{\sin x} = \ldots\ldots$.

(A) 0 (B) $\sin x$ (C) x (D) 1

11. $\lim_{\theta\to0}\frac{\tan\theta}{\theta}=$ ....

(A) -1 (B) 0 (C) 1 (D) ∞

12. $\lim_{x\to0}\frac{\sin 2x}{x}=$ ....

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) 0

13. $\lim_{x\to0}\frac{\sin 3x}{x}=$ ....

(A) 1 (B) 3

(C) $\frac{1}{3}$ (D) Does not exist

14. $\lim_{x\to0}\frac{\sin 4x}{x}=$ ....

(A) 0 (B) 1 (C) 4 (D) $\frac{1}{4}$

15. $\lim_{\theta\to0}\frac{\sin 5\theta}{\theta}=$ ....

(A) 2 (B) 3 (C) 4 (D) 5

16. $\lim_{\theta\to0}\frac{\sin 9\theta}{2\theta}=$ ....

(A) $\frac{9}{2}$ (B) $\frac{2}{9}$ (C) $\frac{1}{9}$ (D) $\frac{1}{2}$

17. $\lim_{\theta\to0}\frac{\theta}{\tan 3\theta}=$ ....

(A) 3 (B) $\frac{1}{3}$ (C) 1 (D) 0

18. $\lim_{\theta\to0}\frac{\tan 3\theta}{4\theta}=$ ....

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) 12 (D) $\frac{1}{12}$

Answers

(1) B	(2) D	(3) A	(4) A	(5) B
(6) A	(7) B	(8) A	(9) C	(10) D
(11) C	(12) B	(13) B	(14) C	(15) D
(16) A	(17) B	(18) A		

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