SECTION-02 - BE-02

APTITUDE TEST (MATHEMATICS & SOFT SKILL)

MATHEMATICS

1. Determinant and Matrices
2. Trigonometry
3. Vectors
4. Co-ordinate Geometry
5. Function & Limit
6. Differentiation and its Applications
7. Integration
8. Logarithm
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ENGLISH

10. Comprehension of Unseen Passage
11. Theory of Communication
12. Grammar
13. Correction of Incorrect Words and Sentences

6. Differentiation and its Applications

[1] Introduction :

To find tangents in geometry, velocity and acceleration in physics, rates of change of quantities etc. a special type of limit is used. In this chapter, we will study this special type of limit. In 1660 B.C., Isaac Newton had started to use such method of derivative. Newton, Pierre de Fermat and Isaac Barrow were familiar with the method of obtaining tangents and that is how the calculus was invented. In 1684 B.C., Leibnitz had started to use the notations of derivative and rules now in use. Cauchy gave the definition of the derivative of a function using D' Alembert's concept of limit. Differentiation is widely used in physics, chemistry, economics, biology, engineering etc.

[2] Differentiation :

Let $f: (a, b) \to R$ is a function. $x \in (a, b)$ is fixed and $t \in (a, b)$ is a variable. For fixed x, t takes the values from (a, b) other than x. That is the ratio $\frac{f(t) - f(x)}{t - x}$ is defined for $t \in (a, b) - \{x\}$. $F(t) = \frac{f(t) - f(x)}{t - x}$ is a function on the region $(a, b) - \{x\}$. If $\lim_{t \to x} F(t)$ exists, then f is said to be differentiable at x and $\lim_{t \to x} F(t) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x}$ is called the derivative of f at (or w.r.t.) x and it is denoted by $f'(x)$ or $\frac{d}{dx} f(x)$. If $y = f(x)$, then $\frac{d}{dx} f(x)$ can also be written as $\frac{dy}{dx}$, y', y_1.... etc.

$$\text{Thus, } f'(x) = \frac{d}{dx} f(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x} \quad ...(1)$$

If $t = x + h$, then as $t \to x$, $h \to 0$

$$\therefore \quad f'(x) = \frac{d}{dx} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad ...(2)$$

If we want to find the derviative at some fixed number c, then replacing t by variable x and x by c in the result (1) we have

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c} \quad ...(3)$$

Example-1 : Find the derivatives of the following functions using definition :

(1) x^3

(2) e^x

(3) $\sin x$

Solution :

(1) Here, $f(x) = x^3$ $\quad \therefore f(t) = t^3$

$$\therefore \quad f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x}$$

$$= \lim_{t \to x} \frac{t^3 - x^3}{t - x}$$

$$= 3x^{3-1} \left(\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1} \right)$$

$$= 3x^2$$

(2) Here, $f(x) = e^x$ $\quad \therefore f(x+h) = e^{x+h}$

$$\therefore \quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$= \lim_{h \to 0} e^x \left(\frac{e^h - 1}{h} \right)$$

$$= e^x \cdot \log_e e$$

$$= e^x \qquad (\because \log_e e = 1)$$

(3) Here, $f(x) = \sin x$ $\quad \therefore f(x+h) = \sin(x+h)$

$$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{2x+h}{2}\right)\cdot \sin\left(\frac{h}{2}\right)}{h}$$

$$= \lim_{h\to 0} \cos\left(\frac{2x+h}{2}\right)\cdot \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$$

$$= \cos x\left(\frac{2x+0}{2}\right)\cdot (1)$$

$$= \cos x$$

[3] Working Rules of Differentiation :

If f and g be the real functions defined on (a, b), are differentiable at $x \in (a, b)$, then

(1) $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$

(2) $\frac{d}{dx} k f(x) = k \cdot \frac{d}{dx} f(x), \quad k \in R$

(3) $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx} f(x) - \frac{d}{dx} g(x)$

(4) $\frac{d}{dx} f(x)\, g(x) = f(x) \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x)$

(5) $\frac{d}{dx} f(x)\, g(x)\, h(x) = f(x)\, g(x) \frac{d}{dx} h(x) +$

$$f(x)\, h(x) \frac{d}{dx} g(x) + g(x)\, h(x) \frac{d}{dx} f(x)$$

(6) $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{[g(x)]^2}, \quad g(x) \neq 0$

[4] Some Standard forms :

(1) Derivative of the constant function is 0. That is $\frac{d}{dx} c = 0$, where c is a constant number.

(2) $\frac{d}{dx} x^n = n\, x^{n-1}, x \in R^+; n \in R$

Remember : $\frac{d}{dx} x = 1, \frac{d}{dx} x^2 = 2x, \frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}},$

$\frac{d}{dx} \frac{1}{x} = -\frac{1}{x^2})$

(3) $\frac{d}{dx} a^x = a^x \cdot \log_e a, x \in R, a \in R^+ - \{1\}$

(4) $\frac{d}{dx} e^x = e^x, x \in R$

(5) $\frac{d}{dx} \sin x = \cos x, x \in R$

(6) $\frac{d}{dx} \cos x = -\sin x, x \in R$

(7) $\frac{d}{dx} \tan x = \sec^2 x, x \in (a, b), (2n-1)\frac{\pi}{2} \notin (a, b), n \in Z$

(8) $\frac{d}{dx} \cot x = -\text{cosec}^2 x, x \in (a, b), n\pi \notin (a, b), n \in Z$

(9) $\frac{d}{dx} \sec x = \sec x \cdot \tan x, x \in (a, b), (2n-1)\frac{\pi}{2} \notin (a, b), n \in Z$

(10) $\frac{d}{dx} \text{cosec } x = -\text{cosec } x \cdot \cot x \cdot x \in (a, b), n\pi \notin (a, b), n \in Z$

(11) $\frac{d}{dx} \log_e x = \frac{1}{x}, x \in R^+$

Example-2 : Find the derivatives of the following functions with respect to x :

(1) $x^3 + 3x + 5$

(2) $x^2 + 2^x + 2^2$

(3) $\log (e^{\sin x})$

Solution :

(1) Here, $y = x^3 + 3x + 5$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^3 + 3x + 5)$$

$$= \frac{d}{dx}x^3 + \frac{d}{dx}3x + \frac{d}{dx}5$$

$$= \frac{d}{dx}x^3 + 3\frac{d}{dx}x + \frac{d}{dx}5$$

$$= 3x^{3-1} + 3(1) + 0$$

$$= 3x^2 + 3$$

(2) Here, $y = x^2 + 2^x + 2^2$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^2 + 2^x + 2^2)$$

$$= \frac{d}{dx}x^2 + \frac{d}{dx}2^x + \frac{d}{dx}2^2$$

$$= 2x + 2^x \log 2 + 0$$

$$= 2x + 2^x \log 2$$

(3) Here, $y = \log(e^{\sin x})$

$\therefore$ $y = \sin x \log e$

$\therefore$ $y = \sin x$ $\quad (\because \log e = 1)$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\sin x = \cos x$$

Example-3 : Find the derivative of the following function with respect to x :

$$\frac{\log x}{x}$$

Solution :

Here, $y = \dfrac{\log x}{x}$

$$\therefore \frac{dy}{dx} = \frac{x\frac{d}{dx}\log x - \log x\frac{d}{dx}x}{x^2}$$

$(\because$ Division rule)

$$= \frac{x \cdot \frac{1}{x} - \log x\,(1)}{x^2}$$

$$= \frac{1 - \log x}{x^2}$$

Example-4 : Do as directed

(1) **If $f(x) = \sin x$, then find $f'\left(\frac{\pi}{2}\right)$.**

(2) **If $f(x) = \dfrac{\log x}{x}$, then find $f'(1)$.**

(3) **Without using the derivatives of trigonometric functions, prove that the derivatives of**

$$f(x) = \frac{\sin x}{\sin x - \cos x} \text{ and } g(x) = \frac{\cos x}{\sin x - \cos x} \text{ are}$$

equal.

Solution :

(1) Here, $f(x) = \sin x$

$\therefore$ $f'(x) = \cos x$

$$\therefore f'\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$$

(2) Here, $f(x) = \dfrac{\log x}{x}$

$$\therefore f'(x) = \frac{x\frac{d}{dx}\log x - \log x\frac{d}{dx}x}{x^2}$$

$$= \frac{x\left(\frac{1}{x}\right) - \log x\,(1)}{x^2}$$

$$= \frac{1 - \log x}{x^2}$$

$$\therefore f'(1) = \frac{1 - \log 1}{(1)^2} = \frac{1 - 0}{1} = 1$$

(3) Here, $f(x) = \dfrac{\sin x}{\sin x - \cos x}$ and $g(x) = \dfrac{\cos x}{\sin x - \cos x}$

$$\therefore f(x) - g(x) = \frac{\sin x}{\sin x - \cos x} - \frac{\cos x}{\sin x - \cos x}$$

$$= \frac{\sin x - \cos x}{\sin x - \cos x}$$

$= 1$

$\therefore \; f(x) - g(x) = 1$

$\therefore \; f(x) = g(x) + 1$

$$\therefore \; \frac{d}{dx} f(x) = \frac{d}{dx}(g(x) + 1)$$

$$= \frac{d}{dx} g(x) + \frac{d}{dx}(1)$$

$$= \frac{d}{dx} g(x) + 0$$

$$= \frac{d}{dx} g(x)$$

Thus, the derivatives of $f(x)$ and $g(x)$ are equal.

[5] Differentiation of Composite Function (Chain Rule) :

If $f : (a, b) \to R$ is differentiable at x, $f(a, b) \subset (c, d)$, $g : (c, d) \to R$ is a function and g is differentiable at $f(x)$, then gof is also differentiable at x and

$(gof)'(x) = g'(f(x)) \cdot f'(x)$"

We will represent this rule in a different form.

Let $u = f(x)$

and $y = (gof)(x) = g(f(x)) = g(u)$

$$\therefore \; \frac{dy}{du} = g'(u) = g'(f(x)) \text{ and } \frac{du}{dx} = f'(x) \text{ and}$$

$$\frac{dy}{dx} = (gof)'(x)$$

$$= g'(f(x)) \cdot f'(x)$$

$$= g'(u) \cdot f'(x)$$

$$= \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\therefore \; \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Similarly, if y is a function of u, u is a function of v, v is a function of w and w is a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}.$$

Since the rule is written in the form of a chain, it is called a chain rule.

This rule can also be remenbered in the following way.

$$\underbrace{\frac{d}{dx}}_{\text{External function}} \; \underbrace{f}_{\text{Internal function}} \; (g(x)) = \underbrace{f'}_{\text{Derivative of external function}} \; \underbrace{(g(x))}_{\text{At internal function}} \; \underbrace{g'(x)}_{\text{Derivative of internal function}}$$

Example-5 : Find the derivatives of the following functions with respect to x :

(1) $\sin(x^2)$

(2) $\sin^2 x$

Solution :

(1) Here, in $y = \sin(x^2)$

Let $u = x^2$ and $y = \sin(x^2) = \sin u$

$$\therefore \; \frac{du}{dx} = 2x \text{ and } \frac{dy}{du} = \cos u$$

$\therefore$ According to the chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \cos u \,.\, 2x$$

$$= 2x \cos(x^2)$$

Direct Method : Here, $y = \sin(x^2)$ has the external function sin ( ) and its variable is x^2 and the internal function is x^2.

$\therefore$ Derivative of the external function sin ( ) = cos ( ) = $\cos(x^2)$ and the derivative of the internal function $x^2 = 2x$

$$\therefore \; \frac{dy}{dx} = \cos(x^2) \cdot 2x = 2x \cos x^2$$

(2) Here, in $y = \sin^2 x$

Let $u = \sin x$ and $y = \sin^2 x = u^2$

$$\therefore \; \frac{du}{dx} = \cos x \text{ and } \frac{dy}{du} = 2u$$

$\therefore$ According to the chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2u \cdot \cos x$$

$= 2 \sin x \cos x$

$= \sin 2x$

Direct Method : Here, $y = \sin^2 x$ has the external function is $(\ )^2$ and its variable is $\sin x$ and the internal function is $\sin x$.

$\therefore$ Derivative of the external fuction $(\ )^2 = 2(\ ) = 2 \sin x$ and derivative of the internal function $\sin x = \cos x$

$$\therefore \frac{dy}{dx} = 2 \sin x \cdot \cos x = \sin 2x$$

[6] Logarithmic Differentiation :

We have learned the method of differentiation of the functions like $a^{f(x)}$, and $(f(x))^a$. Now we will learn to find the derivative of the function like $f(x)^{g(x)}$ in which base and exponent both are functions of x.

We know that, $a^{\log_a x} = x$.

From this result

$$(f(x))^{g(x)} = e^{\log_e (f(x))^{g(x)}} = e^{g(x) \log_e f(x)}.$$

The above form can now be differentiable. Let us understand this, through the following examples.

Example-6 : Find the derivatives of the following functions with respect to x :

(1) x^x

(2) $(\sin x)^x$

Solution :

(1) Here, $y = x^x$

$$\therefore y = e^{x \log_e x}$$

$$\therefore \frac{dy}{dx} = e^{x \log_e x} \cdot \frac{d}{dx}(x \log x)$$

$$= x^x \cdot \left(x \cdot \frac{1}{x} + \log x\right)$$

$$= x^x (1 + \log x)$$

(2) Here, $y = (\sin x)^x$

$$\therefore y = e^{x \log_e \sin x}$$

$$\therefore \frac{dy}{dx} = e^{x \log_e \sin x} \cdot \frac{d}{dx}(x \log \sin x)$$

$$= (\sin x)^x \left[x \cdot \frac{1}{\sin x} \cdot \cos x + \log \sin x\right]$$

$$= (\sin x)^x \cdot (x \cot x + \log \sin x)$$

[7] Differentiation of Implicit function :

So far we have seen the functions in the form $y = f(x)$. i.e. studied the functions in which y is represented in the form of x.

e.g. $y = x + 1$, $y = e^x$, ... etc.

But in some relations, y and x are mixed. For example, $x^2 + y^2 = 1$, $x^3 + y^3 = 3xy$, ... etc. In some of these relations, y can not be represented in the form of only x. For example, $x^3 + y^3 = 3xy$. And in some relations, y is associated with x by more than one relation. For example, from $x^2 + y^2 = 1$, $y = \pm \sqrt{1 - x^2}$. Such equation of y are called Implicit functions.

(**Remember :** $\frac{d}{dx} y = \frac{dy}{dx}$, $\frac{d}{dx} y^2 = 2y \frac{dy}{dx}$, $\frac{d}{dx} y^3 = 3y^2 \frac{dy}{dx}$)

Example-7 : Find $\frac{dy}{dx}$ for the following function :

$x^2 + y^2 = 1$

Solution :

Here, $x^2 + y^2 = 1$

$$\therefore \frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(1)$$

$$\therefore 2x + 2y \frac{dy}{dx} = 0$$

$$\therefore 2y \frac{dy}{dx} = -2x$$

$$\therefore \frac{dy}{dx} = \frac{-2x}{2y} \qquad \therefore \frac{dy}{dx} = -\frac{x}{y}$$

[8] Differentiation of Parametric function :

Sometimes a third variable is used to express the relation between x and y.

For example,

(1) $x = at^2, y = 2at$

(2) $x = a\cos\theta, y = b\sin\theta$

The above type of equation are known as parametric equations and this third variable t or θ is called a parameter.

Let a parametric equation be as follows.

$x = f(t)$ and $y = g(t), t \in [a, b]$

$$\therefore \frac{dx}{dt} = f'(t) \text{ and } \frac{dy}{dt} = g'(t)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}, \text{ where } f'(t) \neq 0$$

This rule is called the rule for differentiation of parametric functions.

Example-8 : Find $\frac{dy}{dx}$ for the following parametric equations :

(1) $x = \cos\theta, y = \sin\theta$

(2) $x = \sin^{-1}t, y = \cos^{-1}t$

Solution :

(1) Here, $x = \cos\theta, y = \sin\theta$

$$\therefore \frac{dx}{d\theta} = -\sin\theta, \quad \frac{dy}{d\theta} = \cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cos\theta}{-\sin\theta} = -\cot\theta$$

(2) Here, $x = \sin^{-1}t, y = \cos^{-1}t$

$$\therefore \frac{dx}{dt} = \frac{1}{\sqrt{1-t^2}}, \quad \frac{dy}{dt} = \frac{-1}{\sqrt{1-t^2}}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\frac{1}{\sqrt{1-t^2}}}{\frac{1}{\sqrt{1-t^2}}} = -1$$

[9] Higher order derivatives :

Let $f : (a, b) \to R$ be any differentiable function and if $f' : (a, b) \to R$ is also differentiable, then its derivative is called the second derivative of f and it is denoted by $f''(x)$ $\frac{d^2y}{dx^2}$.

$$\text{Thus, } f''(x) = \frac{d}{dx} f(x) = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = y_2$$

$$\left(\textbf{Note : } \frac{d}{dx} y_1 = y_2, \ \frac{d}{dx} y_1^2 = 2y_1 y_2\right)$$

Example-9 : Find $\frac{d^2y}{dx^2}$ for the following functions :

(1) $y = e^x + 4$

(2) $y = x^2 + 2$

(3) $y = \log x$

(4) $y = x \log x$

Solution :

(1) Here, $y = e^x + 4$

$$\therefore \frac{dy}{dx} = e^x + 0 = e^x$$

$$\therefore \frac{d^2y}{dx^2} = e^x$$

(2) Here, $y = x^2 + 2$

$$\therefore \frac{dy}{dx} = 2x + 0 = 2x$$

$$\therefore \frac{d^2y}{dx^2} = 2(1) = 2$$

(3) Here, $y = \log x$

$$\therefore \frac{dy}{dx} = \frac{1}{x}$$

$$\therefore \frac{d^2y}{dx^2} = -\frac{1}{x^2}$$

(4) Here, $y = x \log x$

$$\therefore \frac{dy}{dx} = x \frac{d}{dx} \log x + \log x \frac{d}{dx} x$$

$$= x \frac{1}{x} + \log x \,(1)$$

$$= 1 + \log x$$

$$\therefore \frac{d^2y}{dx^2} = 0 + \frac{1}{x} = \frac{1}{x}$$

Applications of Derivative

[10] Velocity and Acceleration :

The linear motion of a particle depends on time. If the distance of a particle from a certain point at time t is s, then s is a function of t. So we can write $s = f(t)$. This is called the equation of motion of the particle. Thus, if $f(t)$ and $f(t + h)$ are the distances covered by the particle at time t and $t + h$ respectively, then during the period h, the average velocity of the particle

$$= \frac{f(t+h) - f(t)}{(t+h) - t} = \frac{f(t+h) - f(t)}{h}.$$

If we keep the period h smallar and smaller, then $\lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$ gives the velocity of the particle at time t. Thus, if $s = f(t)$ is the equation of motion, then $f'(t) = \frac{ds}{dt}$ is the instantaneous velocity of the particle at time t.

Thus, velocity of the particle is $v = \frac{ds}{dt}$.

Similarly, the rate of change of velocity of a particle w.r.t. time is known as acceleration and acceleration of the particle at time t is $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$.

Example-10 : Do as directed :

(1) If equation of motion of a particle is $s = t^3 - 6t^2 + 8t - 4$, then find its velocity and acceleration at $t = 4$ second.

(2) If equation of motion of a particle is $s = t^3 - 3t^2 + 4t + 3$, then find its velocity and acceleration at $t = 2$ second.

Solution :

(1) Here, $s = t^3 - 6t^2 + 8t - 4$

$\therefore$ velocity $v = \frac{ds}{dt} = 3t^2 - 12t + 8$

and acceleration $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 6t - 12$

$\therefore$ $v_{t=4} = 3(4)^2 - 12(4) + 8 = 48 - 48 + 8$

$= 8$ unit/sec.

and $a_{t=4} = 6(4) - 12 = 24 - 12 = 12$ unit/sec.^{2}

(2) Here, $s = t^3 - 3t^2 + 4t + 3$

$\therefore$ velocity, $v = \frac{ds}{dt} = 3t^2 - 6t + 4$

and acceleration $a = \frac{dv}{dt} = 6t - 6$

$\therefore$ $v_{t=2} = 3(2)^2 - 6(2) + 4 = 12 - 12 + 4 = 4$ unit/sec.

and $a_{t=2} = 6(2) - 6 = 12 - 6 = 6$ unit/sec.^{2}

Multiple Choice Questions (MCQs) (Solution with Explanation)

(1) $\frac{d}{dx}(x^2 + 2x + 3) =$ ....

(A) $x^2 + 2$ (B) $2x + 2$

(C) $2x + 3$ (D) $2x$

Ans. : (B) $2x + 2$

Explanation :

$$\frac{d}{dx}(x^2 + 2x + 3) = 2x + 2(1) + 0 = 2x + 2$$

(2) $\frac{d}{dx}(x^3 + 3x + 5) =$ ....

(A) $3x^2 + 3$ (B) $\frac{x^4}{4} + \frac{3x^2}{2} + 5x$

(C) $3x + 3$ (D) $3x^2 + 5$

Ans. : (A) $3x^2 + 3$

Explanation : Here, $y = x^3 + 3x + 5$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^3 + 3x + 5)$$

$$= \frac{d}{dx}x^3 + \frac{d}{dx}3x + \frac{d}{dx}5$$

$$= \frac{d}{dx}x^3 + 3\frac{d}{dx}x + \frac{d}{dx}5$$

$$= 3x^{3-1} + 3(1) + 0$$

$$= 3x^2 + 3$$

(3) $\frac{d}{dx}(x^2 + 2^x + 2^2) =$ ....

(A) 1 (B) $2x + 2^x + 2^2$

(C) $2x + 2^x \log 2$ (D) 0

Ans. : (C) $2x + 2^x \log 2$

Explanation : Here, $y = x^2 + 2^x + 2^2$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^2 + 2^x + 2^2)$$

$$= \frac{d}{dx}x^2 + \frac{d}{dx}2^x + \frac{d}{dx}2^2$$

$$= 2x + 2^x \log 2 + 0$$

$$= 2x + 2^x \log 2$$

(4) $\frac{d}{dx}(\sin^{-1}x + \cos^{-1}x) =$ ....

(A) $\frac{\pi}{2}$ (B) 1

(C) 0 (D) -1

Ans. : (C) 0

Explanation : Here, $y = \sin^{-1}x + \cos^{-1}x$

$$\therefore y = \frac{\pi}{2} \quad \left(\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right)$$

$$\therefore \frac{dy}{dx} = 0$$

(5) $\frac{d}{dx}(\tan^{-1}x + \cot^{-1}x) =$ ....

(A) 1 (B) -1

(C) 0 (D) $\frac{\pi}{2}$

Ans. : (C) 0

Explanation : Here, $y = \tan^{-1}x + \cos^{-1}x$

$$\therefore y = \frac{\pi}{2} \quad \left(\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right)$$

$$\therefore \frac{dy}{dx} = 0$$

(6) $\frac{d}{dx}(\sin^2x + \cos^2x) =$ ....

(A) -1 (B) 1

(C) $2\sin x \cos x$ (D) 0

Ans. : (D) 0

Explanation : $\frac{d}{dx}(\sin^2x + \cos^2x) = \frac{d}{dx}(1) = 0$

($\because \sin^2x + \cos^2x = 1$, Derivative of the constant function = 0)

(7) $\frac{d}{dx}e^{-\log x} =$ ....

(A) $\frac{1}{x^2}$ (B) $-\frac{1}{x}$

(C) $\frac{1}{x}$ (D) $-\frac{1}{x^2}$

Ans. : (D) $-\frac{1}{x^2}$

Explanation : $\frac{d}{dx}e^{-\log x} = \frac{d}{dx}e^{\log x^{-1}}$

$$= \frac{d}{dx}x^{-1} \quad \left(\because a^{\log_a x} = x\right)$$

$$= (-1)x^{-2} = -\frac{1}{x^2}$$

(8) If $f(x) = e^{2x}$, then $f'(0) =$ ....

(A) $2e$ (B) 2

(C) 1 (D) 0

Ans. : (B) 2

Explanation :

$f(x) = e^{2x}$

$\therefore \quad f'(x) = e^{2x} \cdot \frac{d}{dx}(2x) = 2e^{2x}$

$\therefore \quad f'(0) = 2e^{2(0)} = 2e^0 = 2(1) = 2$

(9) If $f(x) = \sin x$, then $f'\left(\frac{\pi}{2}\right) =$ ....

(A) 0 (B) 1

(C) $\cos x$ (D) $\frac{1}{2}$

Ans. : (A) 0

Explanation : Here, $f(x) = \sin x$

$\therefore \quad f'(x) = \cos x$

$\therefore \quad f'\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$

(10) If $f(x) = \log x$, then $f'(1) =$ ....

(A) log 1 (B) 0

(C) 1 (D) –1

Ans. : (C) 1

Explanation : Here, $f(x) = \log x$

$\therefore f'(x) = \frac{1}{x}$

$\therefore \quad f'(1) = \frac{1}{1} = 1$

(11) $\frac{d}{dx}\log\cos x =$ ....

(A) $\tan x$ (B) $\cot x$

(C) $-\tan x$ (D) $-\cot x$

Ans. : (C) $-\tan x$

Explanation :

Here, $y = \log\cos x$ has the external function log ( ) and its variable is $\cos x$ and the internal function is $\cos x$.

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\log\cos x$

$$= \underbrace{\frac{1}{\cos x}}_{\substack{\text{Derivative of the}\\ \text{external function}\\ \log(\,)\text{ w.r.t. its}\\ \text{variable }\cos x}} \cdot \underbrace{(-\sin x)}_{\substack{\text{Derivative of the}\\ \text{internal function}\\ \cos x\,(\,)\text{ w.r.t. its}\\ \text{variable }x}}$$

$= -\tan x$

(12) $\frac{d}{dx}(3\sin x - 4\sin^3 x) =$ ....

(A) $-\cos 3x$ (B) $3\cos 3x$

(C) $3\cos x - 4\cos^3 x$ (D) $\sin 3x$

Ans. : (B) 3 cos 3x

Explanation :

$\frac{d}{dx}(3\sin x - 4\sin^3 x) = \frac{d}{dx}(\sin 3x)$

$= \cos 3x \cdot \frac{d}{dx}(3x)$

$= 3\cos 3x$

(13) $\frac{d}{dx}\sin(\log x) =$ ....

(A) $-\cos(\log x)$ (B) $x\cos(\log x)$

(C) $\frac{1}{x}\cos(\log x)$ (D) $-\frac{1}{x}\cos(\log x)$

Ans. : (C) $\frac{1}{x}$ cos (log x)

Explanation : Here, $y = \sin(\log x)$ has the external function sin ( ) and its variable is $\log x$ and the internal function is $\log x$.

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\sin(\log x)$

$$= \underbrace{\cos(\log x)}_{\substack{\text{Derivative of the}\\ \text{external function}\\ \sin(\,)\text{ w.r.t. its}\\ \text{variable }\log x}} \cdot \underbrace{\frac{1}{x}}_{\substack{\text{Derivative of the}\\ \text{internal function}\\ \log x\text{ w.r.t. its}\\ \text{variable }x}}$$

$= \frac{1}{x}\cos(\log x)$

(14) $\frac{d}{dx} \log \sqrt{x^2 + a^2} =$ ....

(A) $\frac{x}{\sqrt{x^2 + a^2}}$ (B) $\frac{2x}{\sqrt{x^2 + a^2}}$

(C) $\frac{2x}{x^2 + a^2}$ (D) $\frac{x}{x^2 + a^2}$

Ans. : (D) $\frac{x}{x^2 + a^2}$

Explanation : Here, $y = \log \sqrt{x^2 + a^2}$

$$= \frac{1}{2} \log (x^2 + a^2)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{(x^2 + a^2)} \cdot \frac{d}{dx} (x^2 + a^2)$$

$$= \frac{2x}{2(x^2 + a^2)}$$

$$= \frac{x}{x^2 + a^2}$$

(15) If $f(x) = \log \sqrt{x^2 + 1}$, then $f'(0) =$ ....

(A) 0 (B) 1

(C) 2 (D) −1

Ans. : (A) 0

Explanation :

$$f(x) = \log \sqrt{x^2 + 1} = \log (x^2 + 1)^{1/2}$$

$$= \frac{1}{2} \log (x^2 + 1)$$

$$\therefore f'(x) = \frac{1}{2} \cdot \frac{1}{x^2 + 1} \cdot \frac{d}{dx} (x^2 + 1) = \frac{1}{2} \cdot \frac{1}{x^2 + 1} \cdot 2x$$

$$\therefore f'(x) = \frac{x}{x^2 + 1} \qquad \therefore f'(0) = \frac{0}{0 + 1} = 0$$

(16) If $x^2 + y^2 = 29$, then at point (2, 5), $\frac{dy}{dx} =$ ....

(A) $\frac{2}{5}$ (B) $-\frac{2}{5}$

(C) $-\frac{5}{2}$ (D) none of these

Ans. : (B) $-\frac{2}{5}$

Explanation : Here, $x^2 + y^2 = 29$

$$\therefore 2x + 2y \frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -\frac{x}{y}$$

$$\therefore \left(\frac{dy}{dx}\right)_{(2, 5)} = -\frac{2}{5}$$

(17) If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, then $\frac{dy}{dx} =$ ....

(A) $-\sqrt{\frac{y}{x}}$ (B) $-\sqrt{\frac{x}{y}}$

(C) $\sqrt{\frac{x}{y}}$ (D) none of these

Ans. : (A) $-\sqrt{\frac{y}{x}}$

Explanation : Here, $\sqrt{x} + \sqrt{y} = \sqrt{a}$

$$\therefore \frac{d}{dx} (\sqrt{x} + \sqrt{y}) = \frac{d}{dx} (\sqrt{a})$$

$$\therefore \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$$

$$\therefore \frac{1}{2\sqrt{y}} \frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = -\frac{2\sqrt{y}}{2\sqrt{x}} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = -\sqrt{\frac{y}{x}}$$

(18) If $x^2 + y^2 = 1$, then $\frac{dy}{dx} =$ ....

(A) $\frac{x}{y}$ (B) $-\frac{x}{y}$

(C) $\frac{y}{x}$ (D) $-\frac{y}{x}$

Ans. : (B) $-\frac{x}{y}$

Explanation : Here, $x^2 + y^2 = 1$

$\therefore \frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(1)$

$\therefore 2x + 2y\frac{dy}{dx} = 0$

$\therefore 2y\frac{dy}{dx} = -2x$

$\therefore \frac{dy}{dx} = \frac{-2x}{2y}$ $\quad \therefore \frac{dy}{dx} = -\frac{x}{y}$

(19) If $x = \sec\theta + \tan\theta$ and $y = \sec\theta - \tan\theta$, then $\frac{dy}{dx}$ = ...

(A) $\frac{y}{x}$ (B) $-\frac{1}{x^2}$

(C) $\frac{x}{y}$ (D) 0

Ans. : (B) $-\frac{1}{x^2}$

Explanation : Here, $x = \sec\theta + \tan\theta$,

$y = \sec\theta - \tan\theta$

$\therefore xy = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$= \sec^2\theta - \tan^2\theta$

$\therefore xy = 1$

$\therefore y = \frac{1}{x}$

$\therefore \frac{dy}{dx} = -\frac{1}{x^2}$

(20) If $x = at$ and $y = \frac{a}{t}$, then $\frac{dy}{dx}$ = ...

(A) $\frac{x}{y}$ (B) $-\frac{y}{x}$

(C) $\frac{y}{x}$ (D) $-\frac{x}{y}$

Ans. : (B) $-\frac{y}{x}$

Explanation : Here, $x = at,\ y = \frac{a}{t}$

$\therefore xy = (at)\left(\frac{a}{t}\right)$

$\therefore xy = a^2$

$\therefore \frac{d}{dx}(xy) = \frac{d}{dx}a^2$

$\therefore x\frac{dy}{dx} + y = 0$

$\therefore x\frac{dy}{dx} = -y$

$\therefore \frac{dy}{dx} = -\frac{y}{x}$

(21) If $x = \cos\theta$ and $y = \sin\theta$, then $\frac{dy}{dx}$ = ...

(A) $\cot\theta$ (B) $\tan\theta$

(C) $-\cot\theta$ (D) $-\tan\theta$

Ans. : (C) $-\cot\theta$

Explanation : Here, $x = \cos\theta,\ y = \sin\theta$

$\therefore \frac{dx}{d\theta} = -\sin\theta,\ \frac{dy}{d\theta} = \cos\theta$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cos\theta}{-\sin\theta} = -\cot\theta$

(22) If $x = \sin^{-1}t$ and $y = \cos^{-1}t$, then $\frac{dy}{dx}$ = ...

(A) 0 (B) 1

(C) −1 (D) none of these

Ans. : (C) −1

Explanation : Here, $x = \sin^{-1}t,\ y = \cos^{-1}t$

$\therefore \frac{dx}{dt} = \frac{1}{\sqrt{1-t^2}},\ \frac{dy}{dt} = \frac{-1}{\sqrt{1-t^2}}$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\frac{1}{\sqrt{1-t^2}}}{\frac{1}{\sqrt{1-t^2}}} = -1$$

(23) $\frac{d}{dx} x^x = \ldots\ldots\ldots\ .$

(A) $x - \log x$ (B) $x + \log x$

(C) $x^x (1 + \log x)$ (D) $x \cdot x^{x-1}$

Ans. : (C) $x^x (1 + \log x)$

Explanation : Here, $y = x^x$

$$\therefore \quad y = e^{x \log_e x}$$

$$\therefore \frac{dy}{dx} = e^{x \log_e x} \cdot \frac{d}{dx}(x \log x)$$

$$= x^x \cdot \left(x \cdot \frac{1}{x} + \log x\right)$$

$$= x^x (1 + \log x)$$

(24) If $y = x \log x$, then $\frac{d^2y}{dx^2} = \ldots\ldots\ldots$

(A) $1 + \log x$ (B) $\frac{1}{x}$

(C) $\log (e + x)$ (D) $-\frac{1}{x}$

Ans. : (B) $\frac{1}{x}$

Explanation : Here, $y = x \log x$

$$\therefore \frac{dy}{dx} = x \frac{d}{dx} \log x + \log x \frac{d}{dx} x$$

$$= x \frac{1}{x} + \log x (1)$$

$$= 1 + \log x$$

$$\therefore \frac{d^2y}{dx^2} = 0 + \frac{1}{x} = \frac{1}{x}$$

(25) If $y = e^x + 4$, then $\frac{d^2y}{dx^2} = \ldots\ldots\ldots$

(A) e^x (B) e^{2x}

(C) e^{x^2} (D) e^{-x}

Ans. : (A) e^x

Explanation : Here, $y = e^x + 4$

$$\therefore \frac{dy}{dx} = e^x + 0 = e^x$$

$$\therefore \frac{d^2y}{dx^2} = e^x$$

(26) If $y = \log x$, then $\frac{d^2y}{dx^2} = \ldots\ldots\ .$

(A) $\frac{1}{x}$ (B) $-\frac{1}{x^2}$

(C) $\frac{1}{x^2}$ (D) $-\frac{1}{x}$

Ans. : (B) $-\frac{1}{x^2}$

Explanation : Here, $y = \log x$

$$\therefore \frac{dy}{dx} = \frac{1}{x}$$

$$\therefore \frac{d^2y}{dx^2} = -\frac{1}{x^2}$$

(27) If $y = x^2 + 2$, then $\frac{d^2y}{dx^2} = \ldots\ldots\ .$

(A) $2x$ (B) 3

(C) 2 (D) 4

Ans. : (C) 2

Explanation : Here, $y = x^2 + 2$

$$\therefore \frac{dy}{dx} = 2x + 0 = 2x$$

$$\therefore \frac{d^2y}{dx^2} = 2(1) = 2$$

(28) If $y = \sec\theta$ and $x = \tan\theta$, then $\frac{d^2y}{dx^2} = \text{.......}$.

(A) $\sin\theta$ (B) $\cos\theta$

(C) $\tan\theta$ (D) none of these

Ans. : (D) none of these

Explanation : Here, $y = \sec\theta$, $x = \tan\theta$

$$\therefore \frac{dy}{d\theta} = \sec\theta \tan\theta, \quad \frac{dx}{d\theta} = \sec^2\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\sec\theta \tan\theta}{\sec^2\theta} = \frac{\tan\theta}{\sec\theta}$$

$$= \frac{\sec\theta}{\cos\theta} \times \cos\theta$$

$$= \sin\theta$$

$$\therefore \frac{dy}{dx} = \sin\theta$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}(\sin\theta)$$

$$= \cos\theta \cdot \frac{d\theta}{dx}$$

$$= \cos\theta \cdot \frac{1}{\sec^2\theta}$$

$$\left(\because \frac{dx}{d\theta} = \sec^2\theta \ \therefore \frac{d\theta}{dx} = \frac{1}{\sec^2\theta}\right)$$

$$= \cos^3\theta$$

(29) If the equation of motion of a particle is $s = t^3 + 2t^2 - 3t + 5$, then its velocity at $t = 1$ second is ....

(A) 4 m/s (B) 10 m/s

(C) 4 m/s^2 (D) 10 m/s^2

Ans. : (A) 4 m/s

Explanation : Here, $s = t^3 + 2t^2 - 3t + 5$

$$\therefore \text{velocity } v = \frac{ds}{dt} = 3t^2 + 4t - 3$$

$$\therefore v_{t=1} = 3(1)^2 + 4(1) - 3 = 3 + 4 - 3 = 4 \text{ unit/s}$$

Multiple Choice Questions (MCQ's) with (Final Answers)

(1) $\frac{d}{dx}(uv) = \text{.........}$

(A) $\frac{du}{dx} \cdot \frac{dv}{dx}$ (B) $\frac{du}{dx} + \frac{dv}{dx}$

(C) $u\frac{dv}{dx} + v\frac{du}{dx}$ (D) $u\frac{dv}{dx} - v\frac{du}{dx}$

(2) $\frac{d}{dx}\left(\frac{u}{v}\right) = \text{.......}$

(A) $\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (B) $\frac{v\frac{du}{dx} + u\frac{dv}{dx}}{v^2}$

(C) $\frac{u\frac{dv}{dx} - v\frac{du}{dx}}{v^2}$ (D) $\frac{u\frac{dv}{dx} + v\frac{du}{dx}}{v^2}$

(3) If $y = 100000$, then $\frac{dy}{dx} = \text{.........}$.

(A) 1 (B) 10000

(C) 100000 (D) 0

(4) $\frac{d}{dx}(x^3 + 3^x + 3^3) = \text{.........}$.

(A) $3x^2 + x\, 3^{x-1} + 3 \cdot 3^2$

(B) $3x^2 + 3^x \log 3$

(C) $3x^2 + 3^x$

(D) none of these

(5) $\frac{d}{dx}(\tan^2 x - \sec^2 x) = \text{........}$.

(A) 1 (B) –1

(C) 0 (D) none of these

(6) $\frac{d}{dx}(\cot x) =$ ....

(A) $-\text{cosec}^2 x$ (B) $\text{cosec}^2 x$

(C) $\sec x \cdot \tan x$ (D) $-\text{cosec}\, x \cdot \cot x$

(7) $\frac{d}{dx}(x^n) =$ ....

(A) nx^n (B) nx^{n-1}

(C) $\frac{1}{n}x^n$ (D) $\frac{x^{n+1}}{n+1}$

(8) $\frac{d}{dx}(a^x) =$ ....

(A) a^x (B) $a^x \log_e a$

(C) $x^a \log_e a$ (D) $\log_e a$

(9) $\frac{d}{dx}\sec x =$ ....

(A) $\sec x \cdot \tan x$ (B) $\text{cosec}\, x \cdot \cot x$

(C) $-\text{cosec}\, x \cdot \cot x$ (D) $-\sec x \cdot \tan x$

(10) $\frac{d}{dx}\tan x =$ ....

(A) $\sec^2 x$ (B) $-\sec^2 x$

(C) $\sec x \cdot \tan x$ (D) $-\sec x \cdot \tan x$

(11) If $y = \sin^{99}\left(\frac{\pi}{2}\right)$, then $\frac{dy}{dx} =$ ....

(A) 1 (B) -1

(C) 0 (D) none of these

(12) $\frac{d}{dx}\sqrt{x} =$ ....

(A) $\frac{1}{2\sqrt{x}}$ (B) 0

(C) $\frac{1}{\sqrt{x}}$ (D) $x^{3/2}$

(13) $\frac{d}{dx}\text{cosec}\, x =$ ....

(A) $-\cot x$ (B) $\sec x \cdot \tan x$

(C) $\sec^2 x$ (D) $-\text{cosec}\, x \cdot \cot x$

(14) If $y = 5^x$, then $\frac{dy}{dx} =$ ....

(A) 5^x (B) $x \cdot 5^{x-1}$

(C) $5^x \log_e 5$ (D) $5^x \log_5 e$

(15) $\frac{d}{dx}e^x =$ ....

(A) x (B) e

(C) e^x (D) 0

(16) $\frac{d}{dx}(\tan^{-1}x) =$ ....

(A) $\frac{1}{1+x^2}$ (B) $\frac{1}{1-x^2}$

(C) $-\frac{1}{1+x^2}$ (D) $-\frac{1}{1-x^2}$

(17) If $f(x) = e^{3x}$, then $f'(0) =$ ....

(A) $3e$ (B) 3

(C) 1 (D) 0

(18) $\frac{d}{dx}(x \log x) =$ ....

(A) $1 + \log x$ (B) $1 - \log x$

(C) $x + \log x$ (D) none of these

(19) $\frac{d}{dx}\log(\sin x) =$ ....

(A) $\text{cosec}\, x$ (B) $-\text{cosec}\, x$

(C) $\cot x$ (D) $-\cot x$

(20) If $f(x) = \log_e \sin x$, then $f'\left(\frac{\pi}{4}\right) =$ ....

(A) 1 (B) -1

(C) $\sqrt{2}$ (D) $-\sqrt{2}$

(21) $\frac{d}{dx}\cos(2x + 3) =$ ....

(A) $\sin(2x + 3)$ (B) $-\sin(2x + 3)$

(C) $2\sin(2x + 3)$ (D) $-2\sin(2x + 3)$

(22) $\frac{d}{dx}\cos^2 x =$ ....

(A) $2\cos x$ (B) $\sin 2x$

(C) $-\sin 2x$ (D) $\cos 2x$

(23) If $\sqrt{x}+\sqrt{y}=1$, then $\frac{dy}{dx} =$ ....

(A) $\sqrt{\frac{x}{y}}$ (B) $-\sqrt{\frac{x}{y}}$

(C) $\sqrt{\frac{y}{x}}$ (D) $-\sqrt{\frac{y}{x}}$

(24) If $x = \sin\theta$ and $y = \cos\theta$, then $\frac{dy}{dx} =$ ....

(A) $\tan\theta$ (B) $-\tan\theta$

(C) $-\cot\theta$ (D) $\cot\theta$

(25) If $y = e^x$, then $\frac{d^2y}{dx^2} =$ ....

(A) e^x (B) e^{2x}

(C) e^{x^2} (D) e^{-x}

(26) $\frac{d}{dx}\sin^2 x =$ ....

(A) $\cos^2 x$ (B) $2\sin x$

(C) $2\sin x\cos x$ (D) $-2\sin x\cos x$

(27) If the function $y = f(x)$ is maximum of (a, b), then at (a, b) ....

(A) $\frac{d^2y}{dx^2} > 0$ (B) $\frac{dy}{dx} < 0$

(C) $\frac{d^2y}{dx^2} < 0$ (D) $\frac{dy}{dx} > 0$

(28) The necessary condition for the function $f(x)$ to be maximum at $x = a$ is ....

(A) $f''(a) > 0$ (B) $f''(a) < 0$

(C) $f''(a) = 0$ (D) none of these

(29) $\frac{d}{dx}\log(x^2) =$ ....

(A) $\frac{1}{x^2}$ (B) $\frac{2}{x}$

(C) $2x$ (D) $\frac{1}{x}$

(30) $\frac{d}{dx}\log_2 x =$ ....

(A) $\frac{1}{x}$

(B) $\frac{1}{x\log_e 2}$

(C) $\frac{\log_e 2}{x}$

(D) $x\log_e 2$

Answers

(1) C	(2) A	(3) D	(4) B	(5) C
(6) A	(7) B	(8) B	(9) A	(10) A
(11) C	(12) A	(13) D	(14) C	(15) C
(16) A	(17) B	(18) A	(19) C	(20) A
(21) D	(22) C	(23) C	(24) B	(25) A
(26) C	(27) C	(28) B	(29) B	(30) B

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