# SECTION-02 - BE-02

## APTITUDE TEST (MATHEMATICS & SOFT SKILL)

### MATHEMATICS

1. Determinant and Matrices
2. Trigonometry
3. Vectors
4. Co-ordinate Geometry
5. Function & Limit
6. Differentiation and its Applications
7. Integration
8. Logarithm
9. Statistics

### ENGLISH

10. Comprehension of Unseen Passage
11. Theory of Communication
12. Grammar
13. Correction of Incorrect Words and Sentences

# 7. Integration

## [1] Introduction :

In the chapter on differentiation, we have already learnt that how to find the derivative $f'$ of the given function $f$. We have learnt some applications of derivatives also. Now, in this chapter, we shall study an operation which is 'inverse' to differentiation. For example, we know that $\frac{d}{dx}x^5 = 5x^4$. But if we rais the question that derivative of which function is $5x^4$ ? Then it is difficult to find the answer.

Let us frame a general question, "Is there a function whose derivative is the given function $f$ ?" The process to find answer to this question is called 'Antiderivation'. It is possible that this question has no answer or it may have more than one answer. For example $\frac{d}{dx}x^5 = 5x^4$, $\frac{d}{dx}(x^5 + 4) = 5x^4$ and in general $\frac{d}{dx}(x^5 + 4) = 5x^4$, where $c$ is any constant.

**Defintion :** If for the function $g(x)$ defined on the interval $I \subset R$, $\frac{d}{dx} g(x) = f(x)$, $\forall\, x \in I$, then $g(x)$ is called a **Primitive or antiderivative or indefinite integral** of $f(x)$. It is denoted by $\int f(x)\, dx$. Here, the process of finding $g(x)$ for given $f(x)$ is called antiderivation or indefinite integration.

- $\int f(x)\, dx$ denotes, integral of $f(x)$ with respect to $x$.
- In $\int f(x)\, dx$, $f(x)$ is called integrand.
- In $\int f(x)\, dx$, $\int \ldots .\, dx$ indicates the process of integration with respect to $x$.

## [2] Some results of Antiderivative :

(1) If $f$ and $g$ both are differentiable functions on $(a, b)$ and $f'(x) = g'(x)$, $\forall\, x \in (a, b)$, then $f(x) = g(x) + c$, where $c$ is a constant.

Thus if $\frac{d}{dx} f(x) = \frac{d}{dx} g(x) = h(x)$, then $\int h(x)\, dx = g(x)$ and $\int h(x)\, dx = f(x)$.

But $f(x) = g(x) + c$, so $\int h(x)\, dx = g(x) + c$

Hence if one integral of $h(x)$ is $g(x)$, then any other integral of $h(x)$ is $g(x) + c$. Here, c is called an arbitrary constant.

**Note :** Let us perform the operation of differentiation and integration successively in any order.

By defintion of antiderivative, we know that

$$\frac{d}{dx} g(x) = f(x) \Leftrightarrow \int f(x)\, dx = g(x) + c$$

$$\therefore \quad \frac{d}{dx}\left[\int f(x)\, dx\right] = \frac{d}{dx}[g(x) + c] = \frac{d}{dx} g(x) = f(x)$$

$$\text{and } \int \left[\frac{d}{dx} g(x)\right] dx = \int f(x)\, dx = g(x) + c$$

(2) If $f$ and $g$ are integrable functions on $(a, b)$, then

$$\int [f(x) + g(x)]\, dx = \int f(x)\, dx + \int g(x)\, dx$$

In general,

$$\int [f_1(x) + f_2(x) + \ldots . + f_n(x)]\, dx =$$
$$\int f_1(x)\,(x)\, dx + \int f_2(x)\,(x)\, dx + \ldots . + \int f_n(x)\,(x)\, dx$$

(3) If $f$ is an integrable function on $(a, b)$, and $k \in R$, then

$$\int kf(x)\, dx = k \int f(x)\, dx$$

(4) If $f$ and $g$ are integrable functions on $(a, b)$, then

$$\int [f(x) - g(x)]\, dx = \int f(x)\, dx - \int g(x)\, dx.$$

In general,

$$\int [k_1 f_1(x) + k_2 f_2(x) + \ldots . + k_n f_n(x)]\, dx =$$
$$k_1 \int f_1(x)\, dx + k_2 \int f_2(x)\, dx + \ldots . + k_n \int f_n(x)\, dx$$

Results (2), (3) and (4) are known as working rules of integration.

## [3] Some Standard Integrals :

(1) $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c,\ n \in R - \{-1\},\ x \in R^+$

Note : For $n = 0$, $\int 1\, dx = x + c$

For $n = 1$, $\int x\, dx = \frac{x^2}{2} + c$

(2) $\int \frac{1}{x}\, dx = \log|x| + c,\ x \in R - \{0\}$

(3) $\int \cos x\, dx = \sin x + c,\ x \in R$

(4) $\int \sin x\, dx = -\cos x + c,\ x \in R$

(5) $\int \sec^2 x\, dx = \tan x + c,\ x \in R - \left\{(2k+1)\frac{\pi}{2} / k \in Z\right\}$

(6) $\int \operatorname{cosec}^2 x\, dx = -\cot x + c,\ x \in R - \{k\pi / k \in Z\}$

(7) $\int \sec x \cdot \tan x\, dx = \sec x + c,$

$x \in R - \left\{(2k+1)\frac{\pi}{2} / k \in Z\right\}$

(8) $\int \operatorname{cosec} x \cdot \cot x\, dx = -\operatorname{cosec} x + c,$

$x \in R - \{k\pi / k \in Z\}$

(9) $\int a^x\, dx = \frac{a^x}{\log_e a} + c,\ a \in R^+ - \{1\},\ x \in R$

$\therefore \int e^x\, dx = e^x + c$

(10) $\int \frac{1}{x^2 + a^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c,\ a \in R - \{0\},$

$x \in R$

$= -\frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + c,\ a \in R - \{0\},\ x \in R$

(11) $\int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c,\ a \in R - \{0\}$

(12) $\int \frac{1}{a^2 - x^2}\, dx = \frac{1}{2a}\log\left|\frac{x+a}{x-a}\right| + c,\ a \in R - \{0\}$

(13) $\int \frac{1}{\sqrt{x^2 \pm a^2}}\, dx = \log\left|x + \sqrt{x^2 \pm a^2}\right| + c,\ x \in R$

(14) $\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ x \in (-a, a), a > 0$

$= -\cos^{-1}\left(\frac{x}{a}\right) + c,\ x \in (-a, a), a > 0$

(15) $\int \frac{1}{|x|\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c,\ |x| > |a| > 0$

$= -\frac{1}{a}\operatorname{cosec}^{-1}\left(\frac{x}{a}\right) + c$

**Example-1 : Obtain the following integrals :**

(1) $\int \left(4x^3 - \frac{1}{x} + \sin x - e^x\right) dx$

(2) $\int \left(x + \frac{1}{x}\right)^2 dx$

(3) $\int \frac{3x^2 + 2x + 5}{x}\, dx$

(4) $\int \frac{1}{x^2 + 25}\, dx$

(5) $\int \frac{1}{\sqrt{4 - x^2}}\, dx$

**Solution :**

(1) $I = \int \left(4x^3 - \frac{1}{x} + \sin x - e^x\right) dx$

$= 4\int x^3\, dx - \int \frac{1}{x}\, dx + \int \sin x\, dx - \int e^x\, dx$

$= 4\frac{x^4}{4} - \log|x| - \cos x - e^x + c$

(2) $I = \int \left(x + \frac{1}{x}\right)^2 dx$

$= \int \left(x^2 + 2 + \frac{1}{x^2}\right) dx$

$= \int \left(x^2 + 2 + x^{-2}\right) dx$

$$= \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1} + c$$

$$= \frac{x^3}{3} + 2x - \frac{1}{x} + c$$

(3) $I = \int \frac{3x^2 + 2x + 5}{x}\, dx$

$$= \int \left(3x + 2 + \frac{5}{x}\right) dx$$

$$= \frac{3x^2}{2} + 2x + 5 \log |x| + c$$

(4) $I = \int \frac{1}{x^2 + 25}\, dx = \int \frac{1}{x^2 + 5^2}\, dx = \frac{1}{5} \tan^{-1}\left(\frac{x}{5}\right) + c$

(5) $I = \int \frac{dx}{\sqrt{4 - x^2}}$

$$= \int \frac{dx}{\sqrt{2^2 - x^2}}$$

$$= \sin^{-1}\left(\frac{x}{2}\right) + c$$

## [4] Method of Substitution for Integration :

Sometimes if the integrand $f(x)$ is not in one of the standard forms or cannot be easily converted in one such form, then we may use a very useful method of substitution.

(1) **Substitution (Change of variable) rule :** If $g$ is continuous and differentiable function, $g'(t)$ is continous and $g'(t) \neq 0$ and a function $f$, defined on the range of $g$ is continous, then $x = g(t)$ gives $\int f(x)\, dx = \int f(g(t))\, g'(t)\, dt$

(2) **If $\int f(x)\, dx = F(x)$, then $\int f(ax + b)\, dx = \frac{1}{a} F(ax + b)$**

From the above result, the general forms of all the standard forms of integration stated in the section 3.3 can be written as follows.

(i) $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c \Rightarrow$

$$\int (ax + b)^n\, dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c$$

(ii) $\int \frac{1}{x}\, dx = \log |x| + c \Rightarrow$

$$\int \frac{1}{ax + b}\, dx = \frac{1}{a} \log |ax + b| + c$$

(iii) $\int \cos x\, dx = \sin x + c \Rightarrow$

$$\int \cos (ax + b)\, dx = \frac{1}{a} \sin (ax + b) + c$$

(iv) $\int \sin x\, dx = -\cos x + c \Rightarrow$

$$\int \sin (ax + b)\, dx = \frac{1}{a} \cos (ax + b) + c$$

(v) $\int \sec^2 x\, dx = \tan x + c \Rightarrow$

$$\int \sec^2 (ax + b)\, dx = \frac{1}{a} \tan (ax + b) + c$$

(vi) $\int \operatorname{cosec}^2 x\, dx = \cot x + c \Rightarrow$

$$\int \operatorname{cosec}^2 (ax + b)\, dx = -\frac{1}{a} \cot (ax + b) + c$$

(vii) $\int a^x\, dx = \frac{a^x}{\log a} + c \Rightarrow$

$$\int a^{px + q}\, dx = \frac{a^{px + q}}{p \log a} + c$$

(viii) $\int e^x\, dx = e^x + c \Rightarrow \int e^{ax + b}\, dx = \frac{1}{a} e^{ax + b} + c$

(ix) $\int \frac{1}{x^2 + a^2}\, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c \Rightarrow$

$$\int \frac{1}{(\alpha x + \beta)^2 + a^2}\, dx = \frac{1}{\alpha a} \tan^{-1}\left(\frac{\alpha x + \beta}{a}\right) + c$$

(x) $\int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a} \log \left|\frac{x - a}{x + a}\right| + c \Rightarrow$

$$\int \frac{1}{(\alpha x + \beta)^2 - a^2}\, dx = \frac{1}{2a\alpha} \log \left|\frac{\alpha x + \beta - a}{\alpha x + \beta + a}\right| + c$$

(xi) $\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c \Rightarrow$

$$\int \frac{1}{\sqrt{(\alpha x+\beta)^2 \pm a^2}}\,dx$$

$$= \frac{1}{\alpha}\log\left|\alpha x+\beta+\sqrt{(\alpha x+\beta)^2 \pm a^2}\right| + c$$

(xii) $\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c \Rightarrow$

$$\int \frac{1}{\sqrt{a^2 - (\alpha x+\beta)^2}}\,dx = \frac{1}{\alpha}\sin^{-1}\left(\frac{\alpha x+\beta}{a}\right) + c$$

(3) $\int [f(x)]^n f'(x)\,dx = \int \frac{[f(x)]^{n+1}}{n+1} + c,\ n \neq -1$

For example,

$$\int \sin^3 x \cdot \cos x\,dx = \int (\sin x)^3 \cdot \left(\frac{d}{dx}\sin x\right) dx$$

$$= \frac{\sin^4 x}{4} + c$$

(4) $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$

For example,

$$\int \frac{2x}{x^2+1}\,dx = \int \frac{\frac{d}{dx}(x^2+1)}{x^2+1}\,dx = \log|x^2+1| + c$$

(5) $\int \tan x\,dx = \log|\sec x| + c$

(6) $\int \cot x\,dx = \log|\sin x| + c$

(7) $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + c$

$$= \log\left|\tan\frac{x}{2}\right| + c$$

(8) $\int \sec x\,dx = \log|\sec x + \tan x| + c$

$$= \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$$

**Example-2 : Obtain the following integrals :**

(1) $\int \frac{dx}{9+4x^2}$

(2) $\int \frac{dx}{16-25x^2}$

(3) $\int \frac{1}{\sqrt{16-9x^2}}\,dx$

**Solution :**

(1) $I = \int \frac{dx}{9+4x^2} = \int \frac{dx}{(3)^2+(2x)^2}$

$$= \frac{1}{2}\frac{1}{3}\tan^{-1}\left(\frac{2x}{3}\right) + c$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right) + c$$

(2) $I = \int \frac{dx}{16-25x^2} = \int \frac{dx}{(4)^2-(5x)^2}$

$$= \frac{1}{5}\frac{1}{2(4)}\log\left|\frac{5x+4}{5x-4}\right| + c$$

$$= \frac{1}{40}\log\left|\frac{5x+4}{5x-4}\right| + c$$

(3) $I = \int \frac{dx}{\sqrt{16-9x^2}} = \int \frac{dx}{\sqrt{(4)^2-(3x)^2}}$

$$= \frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right) + c$$

## [5] Rule of Integration by Parts :

We know the rule of differentiating the product of two functions. Now we will learn a method to find integal of product of two functions. It is known as rule of integration by parts.

**"If $f(x)$ and $g(x)$ are differntiable functions on interval $(a, b)$ and $f'(x)$ and $g'(x)$ are continous on $(a, b)$, then**

$$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx,\ x \in (a, b)"$$

This rule is known as the rule of integration by parts.

Let us represent the above rule in the different form. Suppose $f(x) = u$ and $g'(x) = v$

$\therefore\ f'(x) = \frac{du}{dx}$ and $g(x) = \int v\,dx$

By substituting these values in the rule of integration by parts, $\int uv\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$

For the choice of $u$ and $v$ in the above rule, we will keep the following order in mind.

L : Logarithmic Function

e.g. $\log_e x$

I : Inverse Function

e.g. $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, ... etc.

A : Algebraic function

e.g. $x^3$, $x^2 + 2x + 3$, $\sqrt{x}$, ... etc.

T : Trigonometric function

e.g. $\sin x$, $\cos x$, $\tan x$ ... etc.

E : Exponential function

e.g. $e^x$, $2^x$, ... etc.

First letters of above functions generates LIATE. The first function appearing in LIATE order is taken as $u$. For example,

(1) In the product $x \log x$, $x$ is algebraic function and $\log x$ is logarithmic function. Now in LIATE rule, logarithmic function precedes algebraic function. Hence we take $u = \log x$ and $v = x$.

(2) In the product $x^2 \sin x$, $x^2$ is algebraie function and $\sin x$ is trigonometric function. Now in LIATE rule, algebraic function precedes trigonometric function. Hence we take $u = x^2$ and $v = \sin x$.

(3) Some times if we have to use this rule for the function like $\tan^{-1}x$, $\log x$, $\sin^{-1}x$ etc., we take these function as u and 1 as v. For example in $I = \int \tan^{-1}x\,dx = \int \tan^{-1}x \cdot 1\,dx$, take $u = \tan^{-1}x$ and $v = 1$.

**Example-3 : Obtain the following integrals :**

$\int xe^x\,dx$

**Solution :**

$$\begin{aligned}
I &= \int xe^x\,dx \\
&= x\int e^x\,dx - \int\left(\frac{d}{dx}x \int e^x\,dx\right)dx \qquad (\because u = x,\ v = e^x) \\
&= x\,e^x - \int 1\,(e^x)\,dx \\
&= xe^x - \int e^x\,dx \\
&= xe^x - e^x + c \\
&= (x-1)\,e^x + c
\end{aligned}$$

## [6] Some more standard forms of Integration :

(1) $\int [f(x) + f'(x)]\,e^x\,dx = e^x f(x) + c$

(2) $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$

(3) $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

(4) $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

(5) $\int e^{ax}\sin(bx + k)\,dx = \frac{e^{ax}}{a^2 + b^2}[a\sin(bx + k) - b\cos(bx + k)] + c$

(6) $\int e^{ax}\cos(bx + k)\,dx = \frac{e^{ax}}{a^2 + b^2}[a\cos(bx + k) + b\sin(bx + k)] + c$

## Definite Integration

### [7] Fundamental Principal of Definite Integration :

If function $f$ is continous on $[a, b]$ and F is differentiable function on $(a, b)$ such that $\forall\, x \in [a, b]$, $\frac{d}{dx} F(x) = f(x)$, then $\int_a^b f(x)\, dx = [F(x)]_a^b = F(b) - F(a)$.

This principal is known as the fundamental principal of definite integration.

Here, $F'(x) = f(x)$. So, $\int f(x)\, dx = F(x) + c$, where $c$ is an arbitrary constant. But

$$\int_a^b f(x)\, dx = [F(x) + c]_a^b$$

$$= (F(b) + c) - (F(a) + c)$$
$$= F(b) - F(a)$$

Thus, in definite integration arbitrary constant is eliminated and so there is no constant of integration.

### [8] Properties of Definite Integration :

(1) $\int_a^a f(x)\, dx = 0$

(2) $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

(3) The value of definite integral does not depend upon variable $x$. i.e. $\int_a^b f(x)\, dx = \int_a^b f(t)\, dt$

(4) $\int_a^b [f(x) + g(x)]\, dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$

(5) $\int_a^b k\, f(x)\, dx = k \int_a^b f(x)\, dx$, where $k$ is constant.

(6) If $a < c < b$, then

$$\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$$

(7) Rule of substitution for definite integration : If $x = g(t)$, $a = g(\alpha)$ and $b = g(\beta)$, then

$$\int_a^b f(x)\, dx = \int_\alpha^\beta f(g(t))\ g'(t) + dt$$

(8) Rule of substitution for definite integration :

$$\int_a^b f(x)\, g'(x)\, dx = [f(b)\, g(b) - f(a)\, g(a)]$$

$$- \int_a^b f'(x)\, g(x)\, dx$$

In another form

$$\int_a^b uv\, dx = \left[u \int v\, dx\right]_a^b - \int_a^b \left(\frac{du}{dx} \int v\, dx\right) dx$$

### [9] Some useful results for Definite Integration :

(1) $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$

(2) $\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$

(3) $\int_0^{2a} f(x)\, dx = \int_0^a f(x)\, dx + \int_0^a f(2a - x)\, dx$

(4) $\int_0^{2a} f(x)\, dx = 2\int_0^a f(x)\, dx$, if $f(2a - x) = f(x)$

$= 0$, if $f(2a - x) = -f(x)$

(5) $\int_{-a}^a f(x)\, dx = 2\int_0^a f(x)\, dx$, if $f$ is an even function

$= 0$, if $f$ is an odd function

(**Note :** If $f(-x) = f(x)$, $\forall x$, then $f$ is called an even function.

If $f(-x) = -f(x)$, $\forall\, x$, then $f$ is called an odd function.)

**Example-4 : Obtain the following integrals :**

(1) $\int_{-2}^{2} x^3\, dx$

(2) $\int_{-\pi}^{\pi} \sin^3 x\, dx$

**Solution :**

(1) $I = \int_{-2}^{2} x^3\, dx$

Here, $f(x) = x^3 \quad \therefore f(-x) = (-x)^3 = -x^3 = -f(x)$

$\therefore$ $f$ is an odd function

$\therefore$ $I = 0$

(2) $I = \int_{-\pi}^{\pi} \sin^3 x\, dx$

Here, $f(x) = \sin^3 x$

$\therefore f(-x) = \sin^3(-x)$

$= (-\sin x)^3$

$= -\sin^3 x$

$= -f(x)$

$\therefore$ $f$ is an odd function.

$\therefore$ $I = 0$

## Multiple Choice Questions (MCQs) (Solution with Explanation)

(1) $\int x^7\, dx = \text{........} + c$

(A) $\frac{x^8}{8}$ (B) $\frac{x^6}{6}$

(C) $7x^6$ (D) $7 \log x$

**Ans. : (A)** $\frac{x^8}{8}$

**Explanation :**

$I = \int x^7\, dx$

$= \frac{x^8}{8} + c \quad \left(\because \int x^n\, dx = \frac{x^{n+1}}{n+1} + c\right)$

(2) $\int \frac{1}{x^2}\, dx = \text{.......} + c$

(A) $\frac{1}{x}$ (B) $-\frac{1}{x}$

(C) $-\frac{1}{3x^2}$ (D) $\frac{1}{3x^2}$

**Ans. : (B)** $-\frac{1}{x}$

**Explanation :**

$I = \int \frac{1}{x^2}\, dx = \int x^{-2}\, dx = \frac{x^{-2+1}}{-2+1} + c$

$= \frac{x^{-1}}{-1} + c$

$= -\frac{1}{x} + c$

(3) $\int e^{x \log a}\, dx = \text{.......} + c$

(A) $e^{a \log x}$ (B) $a^x$

(C) $\frac{a^x}{\log a}$ (D) $\log a$

**Ans. : (C)** $\frac{a^x}{\log a}$

**Explanation :** $I = \int e^{x \log a}\, dx = \int e^{\log a^x}\, dx = \int a^x\, dx$

$= \frac{a^x}{\log a} + c$

(4) $\int (\sin^2 x + \cos^2 x)\, dx = \text{.......} + c$

(A) $x$ (B) $2x$

(C) $\sin 2x$ (D) 1

**Ans. : (A)** $x$

**Explanation :**

$I = \int (\sin^2 x + \cos^2 x)\, dx = \int 1\, dx = x + c$

(5) $\int e^{-\log \sec x}\, dx = \ldots\ldots + c$

(A) $\sec x \tan x$ (B) $\cos x$

(C) $\sin x$ (D) $\tan x$

**Ans. : (C) $\sin x$**

**Explanation :** $I = \int e^{-\log \sec x}\, dx$

$= \int e^{\log (\sec x)^{-1}}\, dx$

$= \int e^{\log \cos x}\, dx$

$= \int \cos x\, dx \quad \left(\because a^{\log_a x} = x\right)$

$= \sin x + c$

(6) $\int \tan^2 x\, dx = \ldots\ldots + c$

(A) $2 \tan x \sec^2 x$ (B) $\tan x - x$

(C) $\tan x + x$ (D) $\sec x \tan x$

**Ans. : (B) $\tan x - x$**

**Explanation :** $I = \int \tan^2 x\, dx$

$= \int (\sec^2 x - 1)\, dx$

$(\because \sec^2 x - \tan^2 x = 1)$

$= \tan x - x + c$

(7) $\int \sqrt{1 + \sin 2x}\, dx = \ldots\ldots + c$

(A) $\cos x + \sin x$ (B) $\cos x - \sin x$

(C) $\sin x - \cos x$ (D) $-\sin x - \cos x$

**Ans. : (C) $\sin x - \cos x$**

**Explanation :** $I = \int \sqrt{1 + \sin 2x}\, dx$

$= \int \sqrt{\cos^2 x + \sin^2 x + 2 \sin x \cos x}\, dx$

$(\because \cos^2 x + \sin^2 x = 1,\ \sin 2x = 2 \sin x \cos x)$

$= \int \sqrt{(\cos x + \sin x)^2}\, dx$

$= \int (\cos x + \sin x)\, dx$

$= \sin x - \cos x + c$

(8) $\int \frac{\sin x \cos x}{1 + \sin^2 x}\, dx = \ldots\ldots + c$

(A) $\frac{1}{2} \log |1 + \sin^2 x|$ (B) $\frac{1}{2} (1 + \sin^2 x)^2$

(C) $\log \cos x$ (D) $\log |1 + \sin^2 x|$

**Ans. : (A) $\frac{1}{2} \log |1 + \sin^2 x|$**

**Explanation :** $I = \frac{\sin x \cos x}{1 + \sin^2 x}\, dx$

$= \frac{1}{2} \int \frac{2 \sin x \cos x}{1 + \sin^2 x}\, dx$

$= \frac{1}{2} \int \frac{\frac{d}{dx}(1 + \sin^2 x)}{1 + \sin^2 x}\, dx$

$(\because \frac{d}{dx}(1 + \sin^2 x) = 0 + 2 \sin x \cos x$

$= 2 \sin x \cos x)$

$= \frac{1}{2} \log |1 + \sin^2 x| + c$

(9) $\int \log x\, dx = \ldots\ldots + c$

(A) $x \log x + x$ (B) $x \log x - x$

(C) $x \log x$ (D) $\frac{1}{x}$

**Ans. : (B) $x \log x - x$**

**Explanation :** $I = \int \log x\, dx$

$= \log x \int 1\, dx - \int \left(\frac{d}{dx} \log x \int 1\, dx\right) dx$

$(\because u = \log x,\ v = 1)$

$= x \log x - \int \frac{1}{x} \cdot x\, dx$

$= x \log x - \int 1\, dx$

$= x \log x - x + c$

(10) $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$ = ........ + c

(A) $\frac{e^x}{x}$ (B) $\frac{e^x}{x^2}$

(C) $xe^x$ (D) $(x-1)e^x$

**Ans. : (A)** $\frac{e^x}{x}$

**Explanation :** $I = \int\left(\frac{x-1}{x^2}\right)e^x\,dx$

$$= \int\left(\frac{1}{x}-\frac{1}{x^2}\right)e^x\,dx$$

$$= \int\left(\frac{1}{x}+\left(-\frac{1}{x^2}\right)\right)e^x\,dx$$

$$= \frac{e^x}{x}+c \quad \left(\because \frac{d}{dx}\frac{1}{x}=-\frac{1}{x^2}\right)$$

(11) $\int_1^8 \frac{dx}{x}$ = ........

(A) log 1 (B) – log 8

(C) log 8 (D) 1

**Ans. : (C) log 8**

**Explanation :** $I = \int_1^8 \frac{dx}{x} = [\log|x|]_1^8$

$$= \log 8 - \log 1$$

$$= \log 8 \quad (\because \log 1 = 0)$$

(12) $\int_0^1 \frac{4}{1+x^2}dx$ = ........ .

(A) $\pi$ (B) $2\pi$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

**Ans. : (A)** $\pi$

**Explanation :** $I = \int_0^1 \frac{4}{1+x^2}dx = 4[\tan^{-1}x]_0^1$

$$= 4[\tan^{-1}1 - \tan^{-1}0]$$

$$= 4\left(\frac{\pi}{4}-0\right)$$

$$= \pi$$

(13) $\int_{-2}^{2} x^3\,dx$ = ........ .

(A) 4 (B) 0

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

**Ans. : (B) 0**

**Explanation :** $I = \int_{-2}^{2} x^3\,dx$

Here, $f(x) = x^3$

$\therefore$ $f(-x) = (-x)^3 = -x^3 = -f(x)$

$\therefore$ $f$ is an odd function

$\therefore$ I = 0

(14) $\int_{-1}^{1} x^6\,dx$ = ........ .

(A) $-\frac{2}{7}$ (B) $\frac{7}{2}$

(C) $-\frac{7}{2}$ (D) $\frac{2}{7}$

**Ans. : (D)** $\frac{2}{7}$

**Explanation :** $I = \int_{-1}^{1} x^6\,dx$

$$= 2\int_{-1}^{1} x^6\,dx$$

($\because f(x) = x^6$ $\therefore f(-x) = (-x)^6 = x^6 = f(x)$

$\therefore f$ is an even function)

$$= 2\left[\frac{x^7}{7}\right]_0^1$$

$$= \frac{2}{7}(1 - 0) = \frac{2}{7}$$

**(15)** $\int_{-1}^{1} (x^2 + 1)\, dx =$ ........

(A) $\frac{8}{3}$ (B) $\frac{3}{8}$

(C) 0 (D) none of these

**Ans. : (B) $\frac{8}{3}$**

**Explanation :** $I = \int_{-1}^{1} (x^2 + 1)\, dx$

Here, $f(x) = x^2 + 1$

$\therefore$ $f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x)$

$\therefore$ $f$ is an even function

$$\therefore \quad I = 2\int_0^1 (x^2 + 1)\, dx$$

$$= 2\left[\frac{x^3}{3} + x\right]_0^1$$

$$= 2\left[\left(\frac{(1)^3}{3} + 1\right) - \left(\frac{0}{3} + 0\right)\right]$$

$$= 2\left(\frac{1}{3} + 1\right) = 2\left(\frac{1+3}{3}\right) = \frac{8}{3}$$

**(16)** $\int_{-2}^{2} x^5 (1 - x^2)^{\frac{3}{2}}\, dx =$ ........

(A) 0 (B) 4

(C) $\frac{3}{2}$ (D) 5

**Ans. : (A) 0**

**Explanation :** $I = \int_{-2}^{2} x^5 (1 - x^2)^{\frac{3}{2}}\, dx$

Here, $f(x) = x^5 \left(1 - x^2\right)^{\frac{3}{2}}$

$$\therefore \quad f(-x) = (-x)^5 \left(1 - (-x)^2\right)^{\frac{3}{2}}$$

$$= -x^5 \left(1 - x^2\right)^{\frac{3}{2}}$$

$$= -f(x)$$

$\therefore$ $f$ is an odd function

$\therefore$ $I = 0$

**(17)** $\int_1^e \log x\, dx =$ ........

(A) 0 (B) $e$ (C) 1 (D) $e - 1$

**Ans. : (C) 1**

**Explanation :**

$$I = \int_1^e \log x\, dx$$

$$= \left[\log x \int 1\, dx\right]_1^e - \int_1^e \left(\frac{d}{dx}\log x \int 1\, dx\right) dx$$

($\because$ Rule of integration by parts)

$$= [x \log x]_1^e - \int_1^e \frac{1}{x} \cdot x\, dx$$

$$= (e \log e - 1 \cdot \log 1) - \int_1^e 1\, dx$$

$$= (e - 1) - [x]_1^e$$

($\because \log_e = 1, \log 1 = 0$)

$$= e - (e - 1)$$

$$= 1$$

(18) $\int_{-1}^{1} \sin^3 x \cos^4 x \, dx =$ ......... .

(A) 1 (B) −1 (C) 0 (D) $\frac{1}{2}$

**Ans. : (C) 0**

**Explanation :** $I = \int_{-1}^{1} \sin^3 x \cos^4 x \, dx$

Here, $f(x) = \sin^3 x \cos^4 x$

$\therefore \; f(-x) = \sin^3(-x) \cos^4(-x)$

$= (-\sin x)^3 \cos^4 x$

$= -\sin^3 x \cos^4 x$

$= -f(x)$

$\therefore$ $f$ is an odd function.

$\therefore$ I = 0

**(19)** Area of the region bounded by the curve $x^2 + y^2 = 9$ is .........

(A) $9\pi$ (B) $4\pi$ (C) $81\pi$ (D) 9

**Ans. : (A) $9\pi$**

**Explanation :** Here, $x^2 + y^2 = 9$ represent a circle with radius 3.

$\therefore$ Required area $= \pi r^2$

$= \pi(3)^2$

$= 9\pi$

## Multiple Choice Questions (MCQ's) with (Final Answers)

(1) $\int x^2 \, dx =$ ....... + c

(A) $2x$ (B) $\frac{x^3}{3}$

(C) $x$ (D) 2

(2) $\int x^3 \, dx =$ ....... + c

(A) $3x^2$ (B) $\frac{x^4}{4}$

(C) $x^4$ (D) $3x^2$

(3) $\int x^4 \, dx =$ ....... + c

(A) $\frac{x^5}{5}$ (B) $4x^3$

(C) $\frac{x^3}{3}$ (D) $4 \log x$

(4) $\int \frac{1}{x} \, dx =$ ....... + c

(A) $x^{-1}$ (B) $x^{-2}$

(C) $-x^{-2}$ (D) $\log |x|$

(5) $\int \frac{1}{1+x^2} \, dx =$ ....... + c

(A) $\sin^{-1}x$ (B) $\cos^{-1}x$

(C) $\tan^{-1}x$ (D) $\cot^{-1}x$

(6) $\int \frac{1}{x^2+25} \, dx =$ ....... + c

(A) $\tan^{-1}\left(\frac{x}{5}\right)$ (B) $\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right)$

(C) $\frac{1}{5}\tan^{-1}\left(\frac{5}{x}\right)$ (D) $\tan^{-1}\left(\frac{5}{x}\right)$

(7) $\int \frac{1}{x^2+a^2} \, dx =$ ....... + c

(A) $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$ (B) $\frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right)$

(C) $\tan^{-1}\left(\frac{x}{a}\right)$ (D) $\cot^{-1}\left(\frac{x}{a}\right)$

(8) $\int \frac{1}{\sqrt{a^2-x^2}} \, dx =$ ....... + c

(A) $\frac{1}{a}\cos^{-1}\left(\frac{x}{a}\right)$ (B) $\cos^{-1}\left(\frac{x}{a}\right)$

(C) $\frac{1}{a}\sin^{-1}\left(\frac{x}{a}\right)$ (D) $\sin^{-1}\left(\frac{x}{a}\right)$

(9) $\int \sin x\, dx = ....... + c$

(A) $\cos x$ (B) $-\cos x$

(C) $\sin x$ (D) $-\sin x$

(10) $\int \cos x\, dx = ....... + c$

(A) $\cos x$ (B) $-\cos x$

(C) $\sin x$ (D) $-\sin x$

(11) $\int \tan x\, dx = ........ + c$

(A) $\log |\sec x|$ (B) $\log |\cos x|$

(C) $\log |\sin x|$ (D) $\log |\operatorname{cosec} x|$

(12) $\int \cos (ax + b)\, dx = ........ + c$

(A) $\sin (ax + b)$ (B) $-\frac{1}{a} \sin (ax + b)$

(C) $\frac{1}{a} \sin (ax + b)$ (D) $\frac{1}{b} \sin (ax + b)$

(13) $\int \frac{dx}{a^2 - x^2} = ........ + c$

(A) $\sin^{-1}\left(\frac{x}{a}\right)$ (B) $\frac{1}{2} \log \left|\frac{x+a}{x-a}\right|$

(C) $\frac{1}{2a} \log \left|\frac{x-a}{x+a}\right|$ (D) $\frac{1}{2a} \log \left|\frac{x+a}{x-a}\right|$

(14) $\int \frac{dx}{\sqrt{x^2 - a^2}} = ....... + c$

(A) $\log \left|x + \sqrt{x^2 - a^2}\right|$ (B) $\log \left|x - \sqrt{x^2 - a^2}\right|$

(C) $\sin^{-1}\left(\frac{x}{a}\right)$ (D) $\frac{1}{2a} \log \left|\frac{x-a}{x+a}\right|$

(15) $\int_0^1 x\, dx = .......... .$

(A) 1 (B) $-1$

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

(16) $\int_0^1 2x\, dx = .......$

(A) 0 (B) 1

(C) 2 (D) 3

(17) $\int_0^1 e^x\, dx = ........$

(A) $e - 1$ (B) $e + 1$

(C) $1 - e$ (D) $e$

(18) $\int_1^e \frac{dx}{x} = ....... .$

(A) 1 (B) 0 (C) $e$ (D) $e - 1$

(19) $\int_0^1 \frac{1}{1+x^2}\, dx = ....... .$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

(20) $\int_{-1}^1 x^3\, dx = ........ .$

(A) $-1$ (B) 0 (C) 1 (D) $\frac{1}{4}$

(21) $\int_{-2}^2 x\, dx = ........ .$

(A) 4 (B) 0 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

(22) $\int_0^2 \frac{dx}{x^2 + 4} = ........$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{8}$ (D) $\pi$

(23) $\int_1^2 \frac{2x}{x^2 + 1}\, dx = ........$

(A) $\log 5$ (B) $\log 2$

(C) $\log \frac{2}{5}$ (D) $\log \frac{5}{2}$

## Answers

| | | | | |
|---|---|---|---|---|
| (1) B | (2) B | (3) A | (4) D | (5) C |
| (6) B | (7) A | (8) D | (9) B | (10) C |
| (11) A | (12) C | (13) D | (14) A | (15) C |
| (16) B | (17) A | (18) A | (19) D | (20) B |
| (21) B | (22) C | (23) D | | |

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