SECTION-02 - BE-02

APTITUDE TEST (MATHEMATICS & SOFT SKILL)

MATHEMATICS

1. Determinant and Matrices
2. Trigonometry
3. Vectors
4. Co-ordinate Geometry
5. Function & Limit
6. Differentiation and its Applications
7. Integration
8. Logarithm
9. Statistics

ENGLISH

10. Comprehension of Unseen Passage
11. Theory of Communication
12. Grammar
13. Correction of Incorrect Words and Sentences

8. Logarithm

[1] Introduction :

Sometimes multiplication, division and powers of large numbers and small decimal numbers becomes necessary in practical life, science and engineering. There are a number of ways to make such calculations short. One of them is 'Logarithm'.

In the beginning of the 17th century, Jhon Napier, a great mathematician introduced the concept of logarithm. Later, Henery Brigs prepared a table of logarithm with base 10. This table made large calculation easier and faster. But today this kind of calculations have become easier and faster with the advent of calculator and computer. But the use of logarithm with base e (e = 2.718281 ...) is applied in many branches of mathematics, science and engineering.

It is very important to know the laws of exponents before studying logarithm. In the previous standards, we have learned the rules of exponents, which are as follows.

For $a, b \in R^+$ (set of positive real numbers)

$x, y \in R$ (set of real numbers)

(1) $a^x \cdot a^y = a^{x+y}$ (2) $\frac{a^x}{a^y} = a^{x-y}$

(3) $(a^x)^y = a^{xy}$ (4) $(ab)^x = a^x \cdot b^x$

(5) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$ (6) $a^{-x} = \frac{1}{a^x}$.

[2] Logarithm :

Before defining logarithm, let us try to understand the meaning of logarithm by a simple example.

For example, take $2^3 = 8$. Here 2 is called base and 3 is called index or exponent.

In this example, we see the relation between three numbers 2, 3 and 8. Here we have represented the number 8 in the form of numbers 2 and 3. i.e.

$8 = 2^3$ ...(1)

But if we want to represent the number 2 in the form of number 8 and 3 then ?

Here $8 = 2^3$

$8^{\frac{1}{3}} = (2^3)^{\frac{1}{3}}$ (Taking power $\frac{1}{3}$ both sides)

$\therefore \quad 8^{\frac{1}{3}} = 2^{3\times\frac{1}{3}} = 2$

Thus, $2 = 8^{\frac{1}{3}}$ ...(2)

Here we have represented the number 2 in the form of numbers 8 and 3.

Now if we want to represent the number 3 in the form of numbers 8 and 2, then how can it be represented ? The answer to this question is logarithm, which can be given as follows :

3 = logarithm of 8 to the base 2

$3 = \log_2 8$

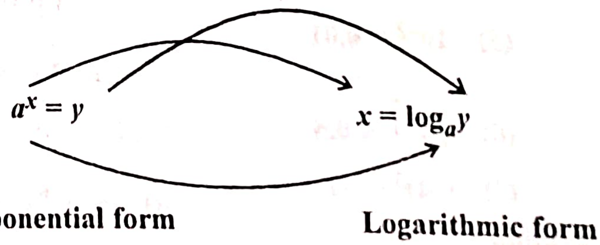
Thus, if $a^x = y$ then x can be written in the form of y and a as $x = \log_a y$

Definition :

If $a \in R^+ - \{1\}$ (the set of positive real numbers except 1), $y \in R^+$, $x \in R$ and $a^x = y$, then x is called the logarithm of y to the base a. It is denoted by $x = \log_a y$.

Thus if $a^x = y$ if and only if $x = \log_a y$.

Here $a^x = y$ is called the exponential form and $x = \log_a y$ is called its logarithmic form.



Some clear conclusions can be deirved from the defintion of logarithm.

(1) Logarithm of only positive real numbers exists. Logarithm of zero or negative real numbers does not exists. For example, $\log_a 0$, $\log_a(-5)$ does not exist.

(2) For any $a \in R^+ - \{1\}$

$a^0 = 1$ if and only if $\log_a 1 = 0$

$a^1 = a$ if and only if $\log_a a = 1$

(3) For all $x, y \in R^+$

$\log_a x = \log_a y$ if and only if $x = y$

(4) $\log_a x$ takes all the values in R.

(5) Negative real numbers, 0 and 1 can not be the base of logarithm.

For example, $\log_{-3} x$, $\log_0 x$, $\log_1 x$ ... etc. do not exist.

(6) Logarithm is a function. For any $a \in R^+ - \{1\}$, $f : R^+ \to R$, $f(x) = \log_a x$ is called logarithm function.

There are mainly two types of logarithm in practice : (1) Common logarithm (2) Natural logarithm.

(1) **Common Logarithm :**

Logarithm with base 10 is called common logarithm. Common logarithm is used in physics, calculation of interest ...etc. The Table of common logarithm is given at the end of most of the books.

(2) **Natural Logarithm :**

Logarithm with base e ($e = 2.718281...$ is an irrational number) is called natural logarithm. This type of logarithm is widely used in calculus and in the branches of science and engineering.

Example-1 : Convert the following exponential forms into logarithmic forms :

(1) $3^4 = 81$

(2) $2^3 = 8$

(3) $9^0 = 1$

(4) $9^{\frac{3}{2}} = 27$

(5) $10^{-2} = 0.01$

(6) $16^{-\frac{1}{4}} = 0.5$

(7) $(-2)^3 = -8$

Solution :

(1) Here by comparing $3^4 = 81$ with $a^x = y$, $a = 3$, $x = 4$ and $y = 81$.

Now the logarithmic form of $a^x = y$ is $\log_a y = x$

By substituting the values of a, x and y in $\log_a y = x$, the logarithmic form of $3^4 = 81$ is $\log_3 81 = 4$.

(2) In $2^3 = 8$, $a = 2$, $x = 3$ and $y = 8$

According to $\log_a y = x$, logarithmic form is $\log_2 8 = 3$

(3) $9^0 = 1$ $\quad\therefore \log_9 1 = 0$

(4) $9^{\frac{3}{2}} = 27$ $\quad\therefore \log_9 27 = \frac{3}{2}$

(5) $10^{-2} = 0.01$ $\quad\therefore \log_{10} 0.01 = -2$

(6) $9^{\frac{3}{2}} = 0.5$ $\quad\therefore \log_{16} 0.5 = -\frac{1}{4}$

(7) In $(-2)^3 = -8$, $a = -2$, $x = 3$, $y = -8$

Without focusing on the numbers, according to $\log_a y = x$, we write the logarithmic form of $(-2)^3 = -8$ as $\log_{-2}(-8) = 3$

But $\log_{-2}(-8) = 3$ does not exist as mentioned earlier $a \in R^+ - \{1\}$ and $y \in R^+$, while in this example $a = -2 \notin R^+ - \{1\}$ and $y = -8 \notin R^+$.

$\therefore$ The logarithmic form of $(-2)^3 = -8$ does not exist.

Example-2 : Convert the following logarithmic forms into exponential forms :

(i) $\log_2 32 = 5$ (ii) $\log_5 125 = 3$

(iii) $\log_{12}\left(\frac{1}{144}\right) = 2$ (iv) $\log_{10} 0.001 = 3$

(v) $\log_5 \sqrt[3]{5} = \frac{1}{3}$ (vi) $\log_{\frac{4}{3}}\left(\frac{9}{16}\right) = -2$

Solution :

(i) By comparing $\log_2 32 = 5$ with $\log_a y = x$,

$x = 5$, $a = 2$ and $y = 32$

Now the exponential form of $\log_a y = x$ is $a^x = y$.

$\therefore$ By substituting the values of a, x and y in $a^x = y$, the exponential form of $\log_2 32 = 5$ will be $2^5 = 32$.

(ii) In $\log_5 125 = 3$, $x = 3$, $a = 5$, $y = 125$

$\therefore$ According to $a^x = y$, exponential form is $5^3 = 125$.

(iii) $\log_{12}\left(\frac{1}{144}\right) = -2$ $\quad\therefore 12^{-2} = \frac{1}{144}$

(iv) $\log_{10} 0.001 = -3$ $\quad\therefore 10^{-3} = 0.001$

(v) $\log_5 \sqrt[3]{5} = \frac{1}{3}$ $\quad\therefore 5^{\frac{1}{3}} = \sqrt[3]{5}$

(vi) $\log_{\frac{4}{3}}\left(\frac{9}{16}\right) = -2$ $\quad\therefore \left(\frac{4}{3}\right)^{-2} = \frac{9}{16}$

Example-3 : Solve the following equations (Find the value of x) :

(i) $\log_x 32 = 5$

(ii) $\log_2 x = 5$

(iii) $\sqrt{\log_2 x} = 3$

(iv) $\log_x\left(\frac{9}{16}\right) = -2$

Solution :

(i) By converting $\log_x 32 = 5$ into exponential form

$x^5 = 32 = 2^5 \quad \therefore \quad x = 2$

(ii) By converting $\log_2 x = 5$ into exponential form

$x = 2^5 = 32 \quad \therefore \quad x = 32$

(iii) $\sqrt{\log_2 x} = 3 \quad \therefore \quad \log_2 x = 9$

$\therefore x = 2^9 = 512 \quad \therefore \quad x = 512$

(iv) $\log_x\left(\frac{9}{16}\right) = -2$

$$\therefore \quad x^{-2} = \frac{9}{16} = \left(\frac{3}{4}\right)^2 = \left(\frac{4}{3}\right)^{-2}$$

$$\therefore \quad x = \frac{4}{3}$$

[3] Properties of Logarithm :

(1) For all $a \in R^+ - \{1\}$, $\log_a 1 = 0$

For example, $\log_{\sqrt{2}} 1 = 0$, $\log_{\left(\frac{3}{4}\right)} 1 = 0$, $\log_e 1 = 0$

(2) For all $a \in R^+ - \{1\}$, $\log_a a = 1$

For example,

$$\log_3 3 = 1, \quad \log_{\sqrt{5}} \sqrt{5} = 1, \quad \log_{\left(\frac{5}{2}\right)}\left(\frac{5}{2}\right) = 1$$

(3) If $a \in R^+ - \{1\}$ and $x \in R^+$, then $a^{\log_a x} = x$.

For example, $5^{\log_5 2} = 2$

(4) **Multiplication Law :**

If $a \in R^+ - \{1\}$ and $x, y \in R^+$, then

$\log_a(xy) = \log_a x + \log_a y$

Corollary : For $x_i \in R^+$ ($i = 1, 2, 3, ..., n$) and $a \in R^+ - \{1\}$

$$\log_a(x_1 x_2 x_3 \dots x_n) = \log_a x_1 + \log_a x_2 + \log_a x_3 + \dots + \log_a x_n$$

For example,

$\log_a 6 = \log_a(2 \times 3) = \log_a 2 + \log_a 3$ and

$\log_a 5 + \log_a 2 = \log_a(5 \times 2) = \log_a 10$

(5) **Division Law :**

If $a \in R^+ - \{1\}$ and $x, y \in R^+$, then

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Corollary : $\log_a\left(\frac{1}{x}\right) = -\log_a x$

For example, $\log_a\left(\frac{7}{5}\right) = \log_a 7 - \log_a 5$

$\log_a 5 - \log_a 3 = \log_a\left(\frac{5}{3}\right)$ and

$\log_a\left(\frac{1}{2}\right) = -\log_a 2$

(6) **Law of Power :**

If $a \in R^+ - \{1\}$, $x \in R^+$ and $n \in R$, then

$$\log_a x^n = n \cdot \log_a x$$

For example, $\log_a 3^4 = 4\log_a 3$ and

$5\log_a 2 = \log_a 2^5$

(7) **Law of change of base :**

If $a, b \in R^+ - \{1\}$ and $x \in R^+$, then

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Corollary : $\log_a x = \dfrac{1}{\log_x a}$

For example, $\log_3 7 = \dfrac{\log_{10} 7}{\log_{10} 3}$, $\dfrac{\log_3 8}{\log_3 5} = \log_5 8$ and

$\log_2 10 = \dfrac{1}{\log_{10} 2}$ and $\dfrac{1}{\log_2 6} = \log_6 2$

(Note : Now onwards, we will assume that the bases are same where the bases are not shown.)

Example-4 : Evaluate :

(i) $\log_2 8$

(ii) $\log_5 125$

(iii) $\log_4 64$

(iv) $\log_8 2$

(v) $\log_{\sqrt{5}} \sqrt[3]{25}$

(vi) $\log_\alpha \left(\dfrac{1}{\alpha}\right)$

(vii) $\log_4 \left(\dfrac{1}{2}\right)$

(viii) $\log_5 4 \cdot \log_4 3 \cdot \log_3 2 \cdot \log_2 1$

Solution :

(i) $\log_2 8 = \log_2 2^3 = 3\log_2 2 = 3 \cdot (1) = 3$

(ii) $\log_5 125 = \log_5 5^3 = 3\log_5 5 = 3 \cdot (1) = 3$

(iii) $\log_4 64 = \log_4 4^3 = 3\log_4 4 = 3 \cdot (1) = 3$

(iv) Using the law of change of base makes the calculation easier when the number in the base is large.

Method-1 : (Using $\log_a x = \dfrac{\log_b x}{\log_b a}$)

$$\log_8 2 = \frac{\log_a 2}{\log_a 8} = \frac{\log_a 2}{\log_a 2^3} = \frac{\log_a 2}{3\log_a 2} = \frac{1}{3}$$

Method-2 : (Using $\log_a x = \dfrac{1}{\log_x a}$)

$$\log_8 2 = \frac{1}{\log_2 8} = \frac{1}{\log_2 2^3} = \frac{1}{3\log_2 2} = \frac{1}{3}$$

(v) $\log_{\sqrt{5}} \sqrt[3]{25} = \dfrac{\log_a \sqrt[3]{25}}{\log_a \sqrt{5}}$ (Law of change of base)

$$= \frac{\log_a (25)^{1/3}}{\log_a 5^{\frac{1}{2}}}$$

$$= \frac{\log_a (5^2)^{\frac{1}{3}}}{\log_a 5^{\frac{1}{2}}} = \frac{\log_a 5^{\frac{2}{3}}}{\log_a 5^{\frac{1}{2}}}$$

$$= \frac{\frac{2}{3}\log_a 5}{\frac{1}{2}\log_a 5}$$

$$= \frac{2}{3} \times \frac{2}{1} = \frac{4}{3}$$

(vi) $\log_\alpha \left(\dfrac{1}{\alpha}\right) = \log_\alpha \alpha^{-1} = (-1)\log_\alpha \alpha = -1$

(vii) $\log_4 \left(\dfrac{1}{2}\right) = \log_4 2^{-1} = \dfrac{\log_a 2^{-1}}{\log_a 4}$

$$= \frac{\log_a 2^{-1}}{\log_a 2^2} = \frac{-\log_a 2}{2\log_a 2} = -\frac{1}{2}$$

(viii) $\log_5 4 \cdot \log_4 3 \cdot \log_3 2 \cdot \log_2 1$

$= \log_5 4 \cdot \log_4 3 \cdot \log_3 2 \cdot (0)$

$= 0$ $(\because \log_a 1 = 0)$

Multiple Choice Questions (MCQs) (Solution with Explanation)

1. $a^{\log_a b} = ...$.

(A) 0 (B) a (C) b (D) 1

Ans. : (C)

Explanation : If $a \in R^+ - \{1\}$ and $x \in R^+$, then

$a^{\log_a x} = x$.

2. $\log m - \log n = ...$.

(A) $\log mn$ (B) $\log \frac{m}{n}$

(C) $\log \frac{n}{m}$ (D) $\log (m - n)$

Ans. : (B)

Explanation :

Division Law :

If $a \in R^+ - \{1\}$ and $x, y \in R^+$, then

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

3. If $f(x) = \log x$, then $f(x) - f(y) = ...$.

(A) $f(x + y)$ (B) $f(x - y)$

(C) $f(xy)$ (D) $f\left(\frac{x}{y}\right)$

Ans. : (D)

Explanation :

$f(x) - f(y) = \log x - \log y$

$= \log \left(\frac{x}{y}\right)$ (Division law)

$= f\left(\frac{x}{y}\right)$

4. If $\sqrt{\log_3 x} = 2$, then $x = ...$.

(A) 9 (B) 81 (C) $\sqrt{3}$ (D) 6

Ans. : (B)

Explanation : $\sqrt{\log_3 x} = 2$ $\therefore \log_3 x = 4$

$\therefore x = 3^4 = 81$

5. If $\log_x \left(\frac{9}{16}\right) = -2$, then $x = ...$.

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$

(C) $\frac{81}{256}$ (D) $\frac{256}{81}$

Ans. : (B)

Explanation : $\log_x \left(\frac{9}{16}\right) = -2$

$\therefore x^{-2} = \frac{9}{16} = \left(\frac{3}{4}\right)^2 = \left(\frac{4}{3}\right)^{-2}$

$\therefore x = \frac{4}{3}$

6. If $\log (x^2 + 1) = \log 2x$, then $x = ...$.

(A) 2 (B) 1

(C) −2 (D) −1

Ans. : (B)

Explanation :

$\log (x^2 + 1) = \log 2x$

$\therefore x^2 + 1 = 2x$

$\therefore x^2 - 2x + 1 = 0$

$\therefore (x - 1)^2 = 0$ $\therefore x - 1 = 0$ $\therefore x = 1$

7. If $a^{\log x} = b^{\log y}$, then $\log_y x = ...$.

(A) $\log \left(\frac{a}{b}\right)$ (B) $\log \left(\frac{b}{a}\right)$

(C) $\log_b a$ (D) $\log_a b$

Ans. : (D) $a^{\log x} = b^{\log y}$

Explanation :

$\therefore \log (a^{\log x}) = \log (b^{\log y})$

(Applying log on both sides)

$\therefore \log x \cdot \log a = \log y \cdot \log b$

$\therefore \frac{\log x}{\log y} = \frac{\log b}{\log a}$

$\therefore \log_y x = \log_a b$

8. If $\log(a+b) = \log a + \log b$, then $\frac{1}{a} + \frac{1}{b} =$ ....

(A) 1 (B) $\frac{1}{ab}$ (C) ab (D) 0

Ans. : (A)

Explanation :

$\therefore \log(a+b) = \log a + \log b$

$\therefore \log(a+b) = \log ab$

$\therefore a + b = ab \quad \therefore \frac{1}{a} + \frac{1}{b} = 1$ (Dividing by ab)

9. $\log_3\left(\frac{1}{45}\right) + \frac{1}{\log_5 3} =$ ....

(A) −3 (B) 3

(C) 2 (D) −2

Ans. : (D)

Explanation :

$$\log_3\left(\frac{1}{45}\right) + \frac{1}{\log_5 3} = \log_3\left(\frac{1}{45}\right) + \log_3 5$$

$$= \log_3\left(\frac{1}{45} \times 5\right)$$

$$= \log_3\left(\frac{1}{9}\right) = \log_3 3^{-2} = -2\log_3 3 = -2$$

10. If $\log\frac{a}{b} + \log\frac{b}{a} = \log(a+b)$, then ....

(A) $a - b = 1$ (B) $a + b = 1$

(C) $a = b$ (D) $a^2 - b^2 = 1$

Ans. : (B)

Explanation : $\log\frac{a}{b} + \log\frac{b}{a} = \log(a+b)$

$\therefore \log\left(\frac{a}{b} \times \frac{b}{a}\right) = \log(a+b)$ (Multiplication law)

$\therefore \log 1 = \log(a+b)$

$\therefore a + b = 1$

Multiple Choice Questions (MCQ's) with (Final Answers)

1. The value of cofactor of 5 in $\begin{vmatrix} -1 & 6 & -2 \\ 5 & 0 & 7 \\ 4 & 1 & -3 \end{vmatrix}$ is ..... .

(A) 16 (B) −16

(C) 20 (D) −20

2. If $f(x) = x^2 - 3x + 1$, then $f(2) + f(3) =$ ....

(A) 1 (B) 0

(C) 2 (D) −2

3. If $f(x) = x^3 - 3$, then $\frac{f(2) - f(-2)}{f(-1)} =$ ....

(A) $\frac{3}{4}$ (B) 4

(C) −4 (D) $-\frac{3}{4}$

4. If $f(x) = x^2 - 5x + 7$, then $f(3) =$ ....

(A) $f(-2)$ (B) $f(-3)$

(C) $f(2)$ (D) $f(1)$

5. $\log x^4 =$ ....

(A) $\log_4 x$ (B) $\log 4x$

(C) $(\log x)^4$ (D) $4\log x$

6. If $f(x) = \log_x 1$, then $f(100) =$ ....

(A) 0 (B) 1 (C) 100 (D) x

7. If $f(x) = \log_2 x$, then $f(2) =$ ....

(A) 0 (B) 1

(C) 4 (D) None of these

8. If $f(x) = \log_2 x$, then $f(4) =$ ....

(A) 0 (B) 1 (C) 2 (D) 4

9. If $f(x) = \log_3 x$, then $f(1) =$ ....

(A) 1 (B) 11 (C) 3 (D) 0

10. If $f(x) = \log_e e^x$, then $f(0) =$ ....

(A) 0 (B) 1 (C) 2 (D) e

11. If $f(x) = \log_e e^x$, then $f(-1) =$ ....

(A) 0 (B) 1 (C) -1 (D) e

12. Which of the following statement is not correct ?

(A) $\log_6 6 = 1$

(B) $\log (2 + 3) = \log 2 \times \log 3$

(C) $\log_6 1 = 0$

(D) $\log (1 + 2 + 3) = \log 1 + \log 2 + \log 3$

13. $\log_4 9 =$ ....

(A) $\log_9 4$ (B) $2\log_2 9$

(C) $\log_2 3$ (D) $2\log_2 3$

14. $2^{\log_2 5} - 3^{\log_3 2} =$ ....

(A) 2 (B) 3

(C) 4 (D) 5

Answers

(1) A	(2) B	(3) C	(4) C	(5) D
(6) A	(7) B	(8) C	(9) D	(10) A
(11) C	(12) B	(13) C	(14) B	

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