GOVERNMENT ENGINEERING COLLEGE BHUJ

Subject: MATHEMATICS-1 (BE01R00041)

Term Work Assignment-3(TWA-3)

Partial Derivatives and the Chain Rule

This assignment consists of problems on partial differentiation and the chain rule. The problems are designed to test your understanding of calculating partial derivatives of various functions and applying the chain rule in different scenarios.

Partial Derivatives

1. Find the first-order partial derivatives of the following functions:
   (a) $f(x,y) = x^3y^2 + 5x^2y^4$
   (b) $f(x,y) = e^{xy}\sin(x)$
   (c) $f(x,y) = \frac{x^2+y^2}{xy}$
   (d) $f(x,y,z) = x^2yz^3 + 2x\cos(y) - 5z^4$
2. If $z = (x^2+y^2)^{3/2}$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3z$.
3. Find all second-order partial derivatives ($f_{xx}, f_{yy}, f_{xy}, f_{yx}$) for the function $f(x,y) = \ln(x^2+y^2)$. Verify that $f_{xy} = f_{yx}$.
4. Given the function $u = e^{x/y}\sin(x/y)$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.
5. If $u(x,y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{1}{1+x^2} + \frac{1}{1+y^2}$.

Chain Rule of Partial Derivatives

6. Let $z = x^2\sin(y)$, where $x = t^2$ and $y = \ln(t)$. Use the chain rule to find $\frac{dz}{dt}$.
7. Suppose $w = \cos(x+y)$, with $x = r\cos(\theta)$ and $y = r\sin(\theta)$. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ using the chain rule.
8. If $z = e^{xy}$, and $x = u^2+v^2$ and $y = u^2-v^2$, use the chain rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.
9. Let $w = x^2+y^2+z^2$, where $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, and $z = \rho\cos(\phi)$. Find $\frac{\partial w}{\partial \rho}$ using the chain rule.
10. If $u = f(x^2+y^2)$, show that $y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = 0$.
11. A function $z = f(x,y)$ is given, where $x = r\cosh(t)$ and $y = r\sinh(t)$. Show that $\left(\frac{\partial z}{\partial r}\right)^2 - \frac{1}{r^2}\left(\frac{\partial z}{\partial t}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2$.
12. The pressure P of a gas is a function of its volume V and temperature T, given by the ideal gas law $PV = nRT$, where n and R are constants. If the volume and temperature are functions of time t, show that $\frac{dP}{dt} = \frac{nR}{V}\frac{dT}{dt} - \frac{nRT}{V^2}\frac{dV}{dt}$.
13. Given $z = f(u,v)$, where $u = x^2-y^2$ and $v = 2xy$. Prove that $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2(u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v})$.
14. If $u = f(y-z, z-x, x-y)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.
15. Let $u(x,y) = x^nf(y/x)$. Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$.