GOVERNMENT ENGINEERING COLLEGE BHUJ
BE01R00041
TWA-4: Applications of Partial Derivatives
Term Work Assignment

Department of Mathematics

September 11, 2025

Instructions: Show all steps. Unless stated otherwise, justify answers using appropriate calculus tools (gradients, tangent planes, Lagrange multipliers, etc.).

Problem 1 (Tangent plane). *Find the equation of the tangent plane to the surface* $z = x^2 + xy + 2y^2$ *at the point* $(1, -1, f(1, -1))$.

Problem 2 (Steepest ascent). *For* $f(x, y) = x^2 + 3y^2 - 4x + 12y$*, find the direction of maximal increase and the maximal directional derivative at* $(1, -2)$.

Problem 3 (Critical points and classification). *Find and classify all critical points of* $f(x, y) = x^3 - 3xy^2$.

Problem 4 (Laplace's equation). *Suppose* u *has continuous second partials and satisfies* $u_{xx} + u_{yy} = 0$*. If* $u_{xx}(1, 2) = 4$*, find* $u_{yy}(1, 2)$.

Problem 5 (Lagrange on a line). *Use Lagrange multipliers to find the extrema of* $f(x, y) = x^2 + y^2$ *subject to the constraint* $x + y = 1$.

Problem 6 (Normal vector and normal line). *For the surface* $z = xy^2$*, find (i) a normal vector and (ii) parametric equations of the normal line at* $(2, 1, 2)$.

Problem 7 (Differentials error estimate). *A rectangle has measured sides* $x = 10, \text{cm}$*,* $y = 5, \text{cm}$ *with errors* $\Delta x = 0.1, \text{cm}$*,* $\Delta y = -0.05, \text{cm}$*. Using differentials, estimate the change in area* $A = xy$.

Problem 8 (Tangent plane: level surface). *Find the tangent plane and the normal line to the surface* $x^2 + y^2 + z - 9 = 0$ *at the point* $(1, 2, 4)$.

Problem 9 (Normal to a sphere). *Find the equations of the normal line to the sphere* $x^2 + y^2 + z^2 = 6$ *at* (a, b, c) *and show it passes through the origin.*

Problem 10 (Distance to quadratic surface). *Find the minimum distance from the surface* $x^2 + y^2 - z^2 = 1$ *to the origin using Lagrange multipliers.*

Problem 11 (Box in first octant under a plane). *A rectangular box lies in the first octant with one corner at the origin and the opposite corner on the plane* $x + 2y + 3z = 3$*. Find the maximum possible volume.*

Problem 12. *Extremize* $x^2 + y^2 + z^2 + xy + yz + zx$ *subject to* $x + y + z = 1$ *and* $x + 2y + 3z = 1$.

Problem 13 (Unconstrained)**.** *Find and classify the critical points of* $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.

Problem 14 (Unconstrained)**.** *Determine local maxima, minima, and saddle points of* $f(x, y) = x^4 + y^4 - 2x^2 + 4y$.

Problem 15 (Constrained: line)**.** *Use Lagrange multipliers to find extrema of* $f(x, y) = x^2 + y^2$ *subject to* $x + y = 1$.

Problem 16 (Constrained: quadratic)**.** *Find the maximum and minimum of* $f(x, y) = x^2 + y^2$ *subject to* $x^2 - 2x + y^2 - 4y = 0$.

Problem 17 (Distance)**.** *Find the minimum distance from the surface* $x^2 + y^2 - z^2 = 1$ *to the origin.*

Problem 18 (Inscribed box)**.** *Find the volume of the largest axis-aligned rectangular box that can be inscribed in the ellipsoid* $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

Problem 19 (Three-part split)**.** *Divide* 24 *into three parts* a, b, c *so that the product* a, b^2, c^3 *is maximized. Find the maximum value.*

Problem 20 (Quadratic form on disk)**.** *Find the maximum and minimum of* $f(x, y) = 4x^2 + 10y^2$ *on the disk* $x^2 + y^2 \leq 4$.