GOVERNMENT ENGINEERING COLLEGE BHUJ

B.E. 1st SEMESTER

Subject: MATHS–1 (BE01R00041)

TERM WORK ASSIGNMENT – 6

(Sequences and Basic Convergence Methods)

Q1. Convergence of Sequences

Determine convergence and, if applicable, the limit:

(a) $a_n = \dfrac{5n-2}{2n+7}$.

(b) $a_n = \left(1+\dfrac{3}{n}\right)^{1/n}$.

(c) $a_n = \dfrac{(-1)^n}{\sqrt{n}+1}$.

(d) $a_n = \sin\left(\dfrac{1}{n^2}\right)$.

Q2. Squeeze Theorem Problems

Use the Squeeze Theorem to show convergence of:

(a) $\{n\sin\left(\dfrac{1}{n^3}\right)\}$.

(b) $\{\dfrac{\tan(1/n)}{1+2n}\}$.

(c) $\{\dfrac{\cos(n)}{n^2}\}$.

Q3. Limits by Continuous Function Theorem

Let $a_n = (3n^2+1)^{1/n}$.

(a) Compute $\lim a_n$.

(b) Evaluate $\lim \ln(a_n)$ and interpret the result.

(c) Compute $\lim(e^{a_n}-1)$, with justification.

Q4. Telescoping Series

Determine convergence and sum wherever possible:

(a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n+3}-\frac{1}{n+4}\right)$.

(b) $\displaystyle\sum_{n=2}^{\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$.

Q5. Geometric-Type Series

Determine convergence and find sum (if convergent):

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{5^{n+2}}$.

(b) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{4}\right)^n$.

Q6. nth-Term Test for Divergence

Use $\lim a_n$ to conclude:

(a) $a_n=\dfrac{4n}{n+2}$.

(b) $a_n=n\cos\left(\frac{1}{n}\right)$.

(c) $a_n=\dfrac{3^n}{2^{n+1}}$.

Q7. Introductory Integral Test

Examine convergence of:

$$\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2},\qquad \sum_{n=2}^{\infty}\frac{1}{n\sqrt{\ln n}}.$$