Subject Name & Code:

MATHEMATICS I- BE01R00041

Assignment – 8

Topic: Infinite series, Power series

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Question 1: Define Alternating series. Using Leibnitz test discuss convergence of the series.

$\frac{1}{1\cdot 2}+\frac{1}{3\cdot 4}+\frac{1}{5\cdot 6}-\frac{1}{7\cdot 8}+\cdots$

Solution:

Step 1: The series seems to have a sign error in given (likely all positive). If alternating:

General term $a_n=\frac{1}{(2n-1)(2n)}$ for n odd, and negative for n even? But given shows +, +, +, -, so not standard alternating. Possibly a typo — if alternating: $\sum(-1)^{n-1}\frac{1}{(2n-1)(2n)}$.

Step 2: Check Leibniz Test conditions:

(i) $a_n=\frac{1}{(2n-1)(2n)}>0$

(ii) a_n decreasing:

$a_{n+1}=\frac{1}{(2n+1)(2n+2)}$, compare with a_n:

$(2n-1)(2n)=4n^2-2n$, $(2n+1)(2n+2)=4n^2+6n+2$, denominator increases $\Rightarrow a_n$ decreases.

(iii) $\lim_{n\to\infty} a_n=0$

So by Leibniz Test, converges.

$$\boxed{\text{Converges}}$$

Question 2: Define Absolutely convergent series and Conditionally convergent series. Give an example of conditionally convergent series.

Solution:

Step 1: Absolutely convergent: $\sum \mid a_n \mid$ converges.
Conditionally convergent: $\sum a_n$ converges but $\sum \mid a_n \mid$ diverges.

Step 2: Example: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges conditionally.

$$\boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}}$$

Question 3: Which of the following series is conditionally convergent?
(1) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3+1}$
(2) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{1/3}}$

Solution:

(1) $\sum \frac{(-1)^n n^2}{n^3+1}$

Step 1: Check absolute convergence:
$\mid a_n \mid \sim \frac{1}{n}$, so $\sum \mid a_n \mid$ diverges (harmonic).
Check convergence of alternating series:
a_n decreasing? Derivative check or ratio:
$\frac{n^2}{n^3+1}$ decreasing for large n, limit 0 ⇒ converges by Leibniz.
So conditionally convergent.

(2) $\sum \frac{(-1)^{n+1}}{n^{1/3}}$

Step 1: $\mid a_n \mid = \frac{1}{n^{1/3}}$, p-series p=1/3 < 1 ⇒ $\sum \mid a_n \mid$ diverges.
Alternating series: $1/n^{1/3}$ decreasing, limit 0 ⇒ converges by Leibniz.
So conditionally convergent.

Both are conditionally convergent.

$$\boxed{\text{Both (1) and (2)}}$$

Question 4: Discuss convergence of following power series. Also find radius and interval of convergence.

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(1) $\sum_{n=1}^{\infty} \frac{n+1}{n} x^{n-1}$

Step 1: Use Ratio Test for radius:

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{\frac{n+2}{n+1} | x |^n}{\frac{n+1}{n} | x |^{n-1}} = \lim_{n\to\infty} \frac{n(n+2)}{(n+1)^2} | x | = | x |$$

Set $| x | < 1 \Rightarrow$ radius $R = 1$.

Step 2: Check endpoints:

$x = 1$: $\sum \frac{n+1}{n}$ diverges (terms → 1).

$x = -1$: $\sum \frac{n+1}{n} (-1)^{n-1}$ fails nth term test (limit not 0) ⇒ diverges.

Interval of convergence: $(-1,1)$.

$$\boxed{R = 1, (-1,1)}$$

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(2) $\sum_{n=1}^{\infty} \frac{(-3)^n}{\sqrt{n+1}} x^n$

Step 1: Ratio Test:

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{3^{n+1}}{\sqrt{n+2}} \cdot \frac{\sqrt{n+1}}{3^n} | x | = 3 | x |$$

Set $3 | x | < 1 \Rightarrow | x | < 1/3$, so $R = 1/3$.

Step 2: Endpoints:

$x = 1/3$: $\sum \frac{(-1)^n}{\sqrt{n+1}}$ converges by Leibniz.

$x = -1/3$: $\sum \frac{1}{\sqrt{n+1}}$ diverges (p-series p=1/2 < 1).

Interval: $(-1/3,1/3]$.

$$\boxed{R = \frac{1}{3}, (-\frac{1}{3}, \frac{1}{3}]}$$

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(3) $1 - \frac{1}{2}(x-2) + \frac{1}{4}(x-2)^2 - \frac{1}{8}(x-2)^3 + \cdots$

Step 1: General term: $a_n = (-\frac{1}{2})^n (x-2)^n$.
Ratio Test:

$$\lim_{n\to\infty} \mid \frac{a_{n+1}}{a_n} \mid = \frac{1}{2} \mid x-2 \mid$$

Set $\frac{1}{2} \mid x-2 \mid < 1 \Rightarrow \mid x-2 \mid < 2 \Rightarrow R = 2$.

Step 2: Endpoints:
$x = 0$: $\sum(-\frac{1}{2})^n (-2)^n = \sum 1^n$ diverges.
$x = 4$: $\sum(-\frac{1}{2})^n (2)^n = \sum(-1)^n$ diverges.

Interval: $(0,4)$.

$$\boxed{R = 2, (0,4)}$$