Subject Name & Code:

MATHEMATICS II- BE02R00011

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Assignment – 1

1. Row Echelon and Reduced Row Echelon Form of the Matrix

Given:

$$A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 13 & 10 \end{bmatrix}$$

Row Echelon Form (REF):
We perform elementary row operations to obtain zeros below leading entries.

1. $R_3 \leftarrow R_3 - 2R_1$:

$$\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 0 & 3 & 2 \end{bmatrix}$$

2. $R_3 \leftarrow R_3 - R_2$:

$$\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

This matrix is in **Row Echelon Form**.

Reduced Row Echelon Form (RREF):
We further simplify to make leading entries 1 and zeros above them.

1. $R_2 \leftarrow \frac{1}{3}R_2$:

$$\begin{bmatrix} 1 & 5 & 4 \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{bmatrix}$$

2. $R_1 \leftarrow R_1 - 5R_2$:

$$\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{bmatrix}$$

Final Answer:

- **REF:** $\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

- **RREF:** $\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{bmatrix}$

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2. Rank of Matrices by Row Echelon Form

(i)
Given:

$$B = \begin{bmatrix} 1 & 5 & 3 & -2 \\ 2 & 0 & 4 & 1 \\ 4 & 8 & 9 & -1 \end{bmatrix}$$

Row Operations:

1. $R_2 \leftarrow R_2 - 2R_1$
2. $R_3 \leftarrow R_3 - 4R_1$

$$\begin{bmatrix} 1 & 5 & 3 & -2 \\ 0 & -10 & -2 & 5 \\ 0 & -12 & -3 & 7 \end{bmatrix}$$

3. $R_3 \leftarrow R_3 - \frac{6}{5}R_2$ (after simplifying)

$$\begin{bmatrix} 1 & 5 & 3 & -2 \\ 0 & -10 & -2 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = Number of non-zero rows in REF = **2**.

2. (ii) Rank of Matrix by Row Echelon Form

Given:

$$B = \begin{bmatrix} 0 & 6 & 7 \\ -5 & 4 & 2 \\ 1 & -2 & 0 \end{bmatrix}$$

To Find: Rank by reducing to row echelon form.

Row Operations:

Step 1: Swap R_1 and R_3 to bring a non-zero element to the (1,1) position.

$$\begin{bmatrix} 1 & -2 & 0 \\ -5 & 4 & 2 \\ 0 & 6 & 7 \end{bmatrix}$$

Step 2: $R_2 \leftarrow R_2 + 5R_1$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & -6 & 2 \\ 0 & 6 & 7 \end{bmatrix}$$

Step 3: $R_3 \leftarrow R_3 + R_2$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & -6 & 2 \\ 0 & 0 & 9 \end{bmatrix}$$

This matrix is now in **Row Echelon Form** (all zero rows, if any, are at the bottom; leading entries shift to the right).

Number of non-zero rows: 3

Rank of the matrix:

$$\boxed{3}$$

Answer Summary for Question 2:
(i) Rank = 2
(ii) Rank = 3

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3. Inverse by Gauss-Jordan Method

(i)
Given:

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

We form the augmented matrix $[C \mid I]$:

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{bmatrix}$$

Steps:

1. $R_2 \leftarrow R_2 - 2R_1$, $R_3 \leftarrow R_3 - R_1$
2. $R_2 \leftarrow \frac{1}{1}R_2$ (normalize pivot)
3. Eliminate above/below pivots to reach $[I \mid C^{-1}]$

After complete reduction:

$$C^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

Final Answer:

$$C^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

3. (ii) Inverse by Gauss-Jordan Method

Given:

$$A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$$

To Find: A^{-1} using Gauss-Jordan elimination.

Step 1: Form the augmented matrix $[A \mid I]$:

$$\begin{bmatrix} 2 & 6 & 6 & 1 & 0 & 0 \\ 2 & 7 & 6 & 0 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{bmatrix}$$

Step 2: $R_1 \leftarrow \frac{1}{2}R_1$ to make leading 1 in row 1:

$$\begin{bmatrix} 1 & 3 & 3 & \frac{1}{2} & 0 & 0 \\ 2 & 7 & 6 & 0 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{bmatrix}$$

Step 3: $R_2 \leftarrow R_2 - 2R_1$, $R_3 \leftarrow R_3 - 2R_1$:

$$\begin{bmatrix} 1 & 3 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{bmatrix}$$

Step 4: $R_3 \leftarrow R_3 - R_2$:

$$\begin{bmatrix} 1 & 3 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

Step 5: $R_1 \leftarrow R_1 - 3R_2$:

$$\begin{bmatrix} 1 & 0 & 3 & \frac{7}{2} & -3 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

Step 6: $R_1 \leftarrow R_1 - 3R_3$:

$$\begin{bmatrix} 1 & 0 & 0 & \frac{7}{2} & 0 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

Step 7: Adjust R_1: multiply by 2? Wait—check leading coefficient.
Actually R_1 first element is already 1. The entry $\frac{7}{2}$ is fine as it is.

Final inverse matrix:

$$A^{-1} = \begin{bmatrix} \frac{7}{2} & 0 & -3 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

We can write with integer entries by scaling R_1 by 2 if desired, but standard form allows fractions.

Check:
Multiply $A \times A^{-1}$ should give I:
First row: $2 \cdot \frac{7}{2} + 6 \cdot (-1) + 6 \cdot 0 = 7 - 6 + 0 = 1$ ✓
Second row: $2 \cdot 0 + 7 \cdot 1 + 6 \cdot (-1) = 0 + 7 - 6 = 1$ ✓
Third row: $2 \cdot (-3) + 7 \cdot 0 + 7 \cdot 1 = -6 + 0 + 7 = 1$ ✓
Off-diagonals also check to 0.

Final Answer:

$$A^{-1} = \begin{bmatrix} \frac{7}{2} & 0 & -3 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

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4. Consistency of System Using Rank Concept

A system $AX = B$ is **consistent** iff:

$$\text{Rank}(A) = \text{Rank}([A \mid B])$$

- If Rank(A) = number of variables → unique solution.
- If Rank(A) < number of variables → infinite solutions.
- If Rank(A) ≠ Rank([A|B]) → inconsistent (no solution).

This concept is applied in Gauss elimination by comparing ranks of coefficient and augmented matrices.

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5. Consistency and Solution Using Gauss Elimination

(1) Given:

$$\begin{cases} x + y + 2z = 9 & (1) \\ 2x + 4y - 3z = 1 & (2) \\ 3x + 6y - 5z = 0 & (3) \end{cases}$$

Augmented matrix:

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{bmatrix}$$

Row operations:

1. $R_2 \leftarrow R_2 - 2R_1$
2. $R_3 \leftarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 3 & -11 & -27 \end{bmatrix}$$

3. $R_3 \leftarrow R_3 - 1.5R_2$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 0 & -0.5 & -1.5 \end{bmatrix}$$

Since no row of the form $[0\ 0\ 0 \mid b]$ with $b \neq 0$, system is **consistent**.

Back substitution:

- From $-0.5z = -1.5 \rightarrow z = 3$

- From $2y - 7z = -17 \rightarrow y = 2$
- From $x + y + 2z = 9 \rightarrow x = 1$

Final Answer: Consistent; unique solution $x = 1, y = 2, z = 3$.

5. (2) Consistency and Solution Using Gauss Elimination

Given:

$$\begin{cases} x + 2y + z = 8 & (1) \\ 2x + 3y + z = 13 & (2) \\ x + y = 5 & (3) \end{cases}$$

Step 1: Write augmented matrix

$$\begin{bmatrix} 1 & 2 & 1 & 8 \\ 2 & 3 & 1 & 13 \\ 1 & 1 & 0 & 5 \end{bmatrix}$$

Step 2: Apply row operations

- $R_2 \leftarrow R_2 - 2R_1$:

$$\begin{bmatrix} 1 & 2 & 1 & 8 \\ 0 & -1 & -1 & -3 \\ 1 & 1 & 0 & 5 \end{bmatrix}$$

- $R_3 \leftarrow R_3 - R_1$:

$$\begin{bmatrix} 1 & 2 & 1 & 8 \\ 0 & -1 & -1 & -3 \\ 0 & -1 & -1 & -3 \end{bmatrix}$$

- $R_3 \leftarrow R_3 - R_2$:

$$\begin{bmatrix} 1 & 2 & 1 & 8 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 3: Check consistency
The last row is $[0\ 0\ 0 \mid 0]$, which does not create inconsistency.
Number of non-zero rows in coefficient matrix = 2.
Number of variables = 3.
Since Rank(A) = Rank([A|B]) = 2 < 3, the system is **consistent with infinitely many solutions**.

Step 4: Express solution
From REF:

$$\begin{cases} x + 2y + z = 8 \\ -y - z = -3 \Rightarrow y + z = 3 \end{cases}$$

Let $z = t$ (parameter).
Then:

$$y = 3 - t$$

$$x + 2(3 - t) + t = 8 \;\Rightarrow\; x + 6 - 2t + t = 8 \;\Rightarrow\; x - t = 2 \;\Rightarrow\; x = t + 2$$

General solution:

$$\boxed{(x, y, z) = (t + 2,\ 3 - t,\ t), \quad t \in \mathbb{R}}$$

Final Answer:
System is **consistent**, with infinitely many solutions as given above.

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6. Solution Using Gauss Elimination

Given:

$$\begin{cases} x - 2y + z = 4 & (1) \\ 3x + 5y + z = 6 & (2) \\ 6x - y + 4z = 2 & (3) \end{cases}$$

Augmented matrix:

$$\begin{bmatrix} 1 & -2 & 1 & 4 \\ 3 & 5 & 1 & 6 \\ 6 & -1 & 4 & 2 \end{bmatrix}$$

Row operations:

1. $R_2 \leftarrow R_2 - 3R_1$, $R_3 \leftarrow R_3 - 6R_1$

$$\begin{bmatrix} 1 & -2 & 1 & 4 \\ 0 & 11 & -2 & -6 \\ 0 & 11 & -2 & -22 \end{bmatrix}$$

2. $R_3 \leftarrow R_3 - R_2$

$$\begin{bmatrix} 1 & -2 & 1 & 4 \\ 0 & 11 & -2 & -6 \\ 0 & 0 & 0 & -16 \end{bmatrix}$$

Third row: $0 = -16 \rightarrow$ **inconsistent**.

Final Answer: No solution.