

Subject Name & Code:
MATHEMATICS II- BE02R00011

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Assignment – 10

1. Find the real and imaginary part of

(i) $f(z) = \sin z$

Let $z = x + iy$.

Using identity:

$$\sin(z) = \sin(x + iy) = \sin x \cosh y + i \cos x \sinh y.$$

Hence:

$$\operatorname{Re}(\sin z) = \sin x \cosh y, \operatorname{Im}(\sin z) = \cos x \sinh y.$$

(ii) $f(z) = \cosh z$

Recall $\cosh z = \cos(iz)$ or directly:

$$\cosh(z) = \cosh(x + iy) = \cosh x \cos y + i \sinh x \sin y.$$

Hence:

$$\operatorname{Re}(\cosh z) = \cosh x \cos y, \operatorname{Im}(\cosh z) = \sinh x \sin y$$

2. Solve: $\sin z = 6$

Let $z = x + iy$. Then

$$\sin z = \sin x \cosh y + i \cos x \sinh y = 6 + 0i.$$

Equate real and imaginary parts:

1. $\sin x \cosh y = 6$

2. $\cos x \sinh y = 0$

From (2):

- Case 1: $\sinh y = 0 \Rightarrow y = 0$. Then (1) gives $\sin x = 6$ impossible (since $|\sin x| \leq 1$), so no solution.
- Case 2: $\cos x = 0 \Rightarrow x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$.

Then $\sin x = \pm 1$.

Substitute into (1):

$$(\pm 1)\cosh y = 6 \Rightarrow \cosh y = 6.$$

Since $\cosh y \geq 1$ and is even,

$$y = \pm \cosh^{-1}(6) = \pm \ln(6 + \sqrt{36 - 1}) = \pm \ln(6 + \sqrt{35}).$$

Thus:

$$z = \frac{\pi}{2} + n\pi \pm i \ln(6 + \sqrt{35}), \quad n \in \mathbb{Z}$$

3. Show that $|\sin z|^2 = \sin^2 x + \sinh^2 y$

Let $z = x + iy$.

$$\sin z = \sin x \cosh y + i \cos x \sinh y.$$

Then

$$\begin{aligned} |\sin z|^2 &= (\sin x \cosh y)^2 + (\cos x \sinh y)^2 \\ &= \sin^2 x \cosh^2 y + \cos^2 x \sinh^2 y. \end{aligned}$$

Use identities $\cosh^2 y = 1 + \sinh^2 y$, $\cos^2 x = 1 - \sin^2 x$:

$$\begin{aligned} |\sin z|^2 &= \sin^2 x(1 + \sinh^2 y) + (1 - \sin^2 x)\sinh^2 y \\ &= \sin^2 x + \sin^2 x \sinh^2 y + \sinh^2 y - \sin^2 x \sinh^2 y \\ &= \sin^2 x + \sinh^2 y. \end{aligned}$$

$$\boxed{|\sin z|^2 = \sin^2 x + \sinh^2 y}$$

4. Show that $\text{Log}(i^3) \neq 3\text{Log}(i)$

Here Log denotes the principal logarithm ($-\pi < \text{Arg } z \leq \pi$).

First, $i^3 = -i$.

- Argument of $-i$ is $-\frac{\pi}{2}$ (since $-\pi < -\frac{\pi}{2} \leq \pi$).

$$\text{Log}(-i) = \ln |-i| + i \text{Arg}(-i) = \ln 1 + i \left(-\frac{\pi}{2}\right) = -\frac{i\pi}{2}.$$

Now $3\text{Log}(i)$:

$$\begin{aligned} \text{Log}(i) &= \ln |i| + i \text{Arg}(i) = \ln 1 + i \frac{\pi}{2} = \frac{i\pi}{2}. \\ 3\text{Log}(i) &= \frac{3i\pi}{2}. \end{aligned}$$

But $\frac{3i\pi}{2}$ is **not** the principal log of $-i$ because $\frac{3\pi}{2} > \pi$, so it is not equal to $-\frac{i\pi}{2}$.

Thus:

$$\text{Log}(i^3) = -\frac{i\pi}{2} \neq \frac{3i\pi}{2} = 3\text{Log}(i).$$

$$\boxed{\text{Log}(i^3) \neq 3\text{Log}(i)}$$

5. Solve: $e^z = -2$

Let $z = x + iy$. Then

$$e^x e^{iy} = -2 = 2e^{i\pi} \text{ (since } -2 = 2e^{i(\pi+2n\pi)}).$$

Equate moduli and arguments:

$$e^x = 2 \Rightarrow x = \ln 2,$$

$$y = \pi + 2n\pi, n \in \mathbb{Z}.$$

Thus:

$$z = \ln 2 + i(2n + 1)\pi, n \in \mathbb{Z}.$$

$$\boxed{z = \ln 2 + i(2n + 1)\pi, n \in \mathbb{Z}}$$

6. Solve: $\text{Log } z = 4 - 3i$

Given:

$$\text{Log } z = 4 - 3i.$$

Here $\text{Log } z = \ln |z| + i\text{Arg } z$ with $-\pi < \text{Arg } z \leq \pi$.

Equate real and imaginary parts:

$$\ln |z| = 4 \Rightarrow |z| = e^4,$$

$$\text{Arg } z = -3.$$

Check: $-\pi < -3 < \pi$ is true ($\pi \approx 3.1416$, so -3 is within the range).

Thus:

$$z = |z| e^{i\text{Arg } z} = e^4 e^{-3i} = e^{4-3i}.$$

$$\boxed{z = e^{4-3i}}$$