GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – MID TERM – S26 – ANSWER BANK

Subject Name & Code:

FLUID MECHANICS - BE04000161

Important Question For Mid Sem Exam - Answer Bank

3 Marks Questions

Q1. State Pascal's law and briefly explain its real-world application in automotive hydraulic brakes.

- **Pascal's Law:** It states that **pressure applied to an enclosed fluid (at rest) is transmitted equally in all directions throughout the fluid and acts at right angles to the containing surfaces.** This pressure is transmitted with undiminished intensity.
- **Real-World Application in Automotive Hydraulic Brakes:**
  1. When the driver presses the brake pedal, it pushes a piston in the **master cylinder**, creating pressure in the brake fluid.
  2. According to Pascal's law, this pressure is transmitted instantly and equally through the brake fluid in the pipes to all the wheel cylinders (or calipers).
  3. This uniform pressure forces the pistons in the wheel cylinders to move, pushing the **brake shoes or pads against the drums or discs**. This frictional force slows down or stops the vehicle.
  4. The system ensures that an equal braking force is applied to all wheels simultaneously.

Q2. Explain the concept of the fluid continuum. Why is this assumption necessary in macroscopic fluid mechanics?

- **Concept of Fluid Continuum:** In reality, fluids are made of discrete molecules with empty spaces between them. The **continuum assumption** ignores this molecular structure and treats the fluid as a **continuous, infinitely divisible substance** with no empty spaces. Properties like density, pressure, and velocity are assumed to be defined at every point in space, representing an average of the molecular behavior in a small volume around that point.
- **Necessity in Macroscopic Fluid Mechanics:**
  1. It allows us to describe fluid properties as **smooth, continuous functions** of space and time using calculus.
  2. Without this assumption, analyzing fluid flow by tracking every single molecule would be impossibly complex.
  3. It enables the use of differential equations (like the Navier-Stokes equations) to model and predict fluid behavior at the scale relevant to engineering applications (pipes, pumps, vehicles).

Q3. Define compressibility and bulk modulus. How are these two fluid properties related to each other?

- **Compressibility:** It is the measure of the relative change in the volume of a fluid (or solid) in response to a change in pressure. It indicates how easily a fluid can be squeezed into a smaller volume. It is the reciprocal of the bulk modulus.

- **Bulk Modulus (K):** It is a material property that quantifies the substance's resistance to uniform compression. It is defined as the ratio of the infinitesimal pressure increase to the resulting relative decrease in volume. Mathematically, **K = -V (dP/dV)**. A high bulk modulus means the fluid is incompressible.
- **Relationship:** Compressibility and Bulk Modulus are **inversely proportional** to each other.
    - **Compressibility = 1 / K**
    - If a fluid has a high bulk modulus (e.g., water), it has low compressibility. If it has a low bulk modulus (e.g., air), it has high compressibility.

Q4. What is an inclined manometer? State its primary advantage over a standard vertical U-tube manometer for pressure measurement.

- **Inclined Manometer:** It is a precision instrument used to measure small pressure differences. It consists of a transparent tube inclined at a known angle (θ) relative to the horizontal. One end is connected to a large reservoir, and the other is the inclined measuring tube.
- **Primary Advantage:** Its main advantage is **increased sensitivity and accuracy for measuring very small pressures.**
    - When the tube is inclined, a small vertical rise (h) of the manometric liquid is stretched into a much longer movement (L) along the tube, according to the relation **h = L sin θ**.
    - This amplification of the length scale (L) allows for more precise reading of small pressure changes that would result in an imperceptible vertical movement in a standard U-tube manometer.

Q5. A sample of a Jatropha oil biodiesel blend has a mass of 850 kg and occupies a volume of 0.95 m³. Calculate its density, specific weight, and specific gravity.

Step 1: Given Data

- Mass, $m = 850\ \text{kg}$
- Volume, $V = 0.95\ \text{m}^3$
- Acceleration due to gravity, $g = 9.81\ \text{m/s}^2$ (Standard value)
- Density of water, $\rho_w = 1000\ \text{kg/m}^3$ (Standard value)

Step 2: Calculate Density (ρ)

$$\rho = \frac{m}{V} = \frac{850}{0.95}$$
$$\rho = 894.74\ \text{kg/m}^3$$

Step 3: Calculate Specific Weight (γ)

$$\gamma = \rho \cdot g = 894.74 \times 9.81$$
$$\gamma = 8777.4\ \text{N/m}^3 \text{or} 8.777\ \text{kN/m}^3$$

Step 4: Calculate Specific Gravity (s)

$$s = \frac{\rho}{\rho_w} = \frac{894.74}{1000}$$
$$s = 0.895$$

Final Answer:

$$\boxed{\rho = 894.74\,\text{kg/m}^3,\ \gamma = 8777.4\,\text{N/m}^3,\ s = 0.895}$$

Q6. Calculate the capillary rise of a fuel in a glass tube of a fuel injector with a 2.5 mm diameter. The surface tension of the fuel is 0.025 N/m, its specific gravity is 0.82, and the contact angle is 0°.

Step 1: Given Data

- Diameter, $d = 2.5\text{ mm} = 0.0025\text{ m}$
- Radius, $r = \frac{d}{2} = 0.00125\text{ m}$
- Surface tension, $\sigma = 0.025\text{ N/m}$
- Specific Gravity, $s = 0.82$
- Contact angle, $\theta = 0° \Rightarrow \cos\ \theta = 1$
- Density of water, $\rho_w = 1000\text{ kg/m}^3$
- $g = 9.81\text{ m/s}^2$

Step 2: Calculate Density of Fuel

$$\rho = s \cdot \rho_w = 0.82 \times 1000 = 820\text{ kg/m}^3$$

Step 3: Apply Capillary Rise Formula

$$h = \frac{2\sigma\cos\ \theta}{\rho g r}$$

Substitute the values:

$$h = \frac{2 \times 0.025 \times 1}{820 \times 9.81 \times 0.00125}$$

Step 4: Simplify the Expression

$$h = \frac{0.05}{820 \times 9.81 \times 0.00125}$$

First, calculate the denominator:

$$820 \times 9.81 = 8044.2$$
$$8044.2 \times 0.00125 = 10.05525$$

Thus:

$$h = \frac{0.05}{10.05525} = 0.004972\text{ m}$$

Step 5: Convert to Millimeters

$$h = 0.004972 \times 1000 = 4.97\text{ mm}$$

Final Answer:

$$\boxed{h = 4.97\text{ mm}}$$

4 Marks Questions

Q1. Differentiate between dynamic viscosity and kinematic viscosity. Discuss their specific significance when selecting lubricants for machinery.

- **Differentiation:**

Feature	Dynamic Viscosity (μ)	Kinematic Viscosity (ν)
Definition	Measure of a fluid's internal resistance to flow when an external force is applied. It is the ratio of shear stress to shear rate.	Measure of a fluid's resistance to flow under the influence of gravity. It is the ratio of dynamic viscosity to density.
Symbol	μ (mu)	ν (nu)
Formula	$\mu = \tau / (du/dy)$	$\nu = \mu / \rho$
Units (SI)	Pascal-second (Pa·s) or $N \cdot s/m^2$	m^2/s
Nature	Represents absolute viscosity, independent of fluid density.	Represents viscous effect coupled with inertial effect (density).

- **Significance in Lubricant Selection:**
  - **Dynamic Viscosity (μ)** is significant for determining the **load-carrying capacity** of the lubricant. It directly relates to the shear stress developed in the oil film. For high-load, high-speed machinery, a lubricant with sufficient dynamic viscosity is chosen to ensure a thick enough film to prevent metal-to-metal contact.
  - **Kinematic Viscosity (ν)** is more practical for comparing lubricants and is the standard value reported in oil specifications. It is crucial for **flow characteristics**. It determines how easily the oil will pump, circulate, and flow into narrow clearances (like in engine bearings) at different temperatures. It is the primary factor in selecting an oil grade (e.g., SAE 30, 10W-40).

Q2. Describe the concept of vapor pressure. Explain the mechanism by which it leads to cavitation in pumps and cooling systems.

- **Concept of Vapor Pressure:**
  Vapor pressure is the pressure exerted by the vapor molecules above a liquid surface when the liquid and its vapor are in **dynamic equilibrium** in a closed container. It is a measure of a liquid's tendency to evaporate. A liquid with high vapor pressure evaporates easily. Vapor pressure is **strongly dependent on temperature**, increasing as temperature increases.
- **Mechanism of Cavitation:**
  1. In pumps and cooling systems, the fluid's velocity increases at specific points (e.g., at the impeller eye or on the suction side), causing a significant drop in local pressure, as per Bernoulli's principle.
  2. If this local pressure falls **below the vapor pressure** of the liquid at that operating temperature, the liquid starts to boil at that point, forming small vapor bubbles (cavities).
  3. These bubbles are carried along by the flow to a region of higher pressure

downstream.

4. In this high-pressure region, the vapor bubbles **collapse or implode violently** back into liquid.
5. This implosion releases intense shockwaves that can erode material from the pump impeller and casing surfaces, causing pitting, vibration, noise, and a significant drop in pump performance and efficiency.

Q3. Discuss the phenomenon of surface tension and capillarity. How do these properties influence the performance of fuel injectors and brake fluids?

- **Surface Tension:**
  It is a property of a liquid that allows its surface to resist an external force, due to the cohesive forces between liquid molecules. Molecules at the surface have higher energy than those in the bulk, creating a "skin-like" effect. It is measured in N/m.
- **Capillarity:**
  It is the ability of a liquid to flow in narrow spaces (tubes or porous materials) without the assistance of, or even in opposition to, external forces like gravity. It occurs due to the combined effects of cohesion and adhesion. A liquid rises in a tube if adhesive forces are stronger than cohesive forces (e.g., water in glass), and it depresses if cohesion is stronger (e.g., mercury in glass).
- **Influence on Fuel Injectors:**
  - **Surface tension** affects the atomization of fuel. Low surface tension allows fuel to break up into finer droplets, leading to better mixing with air and more efficient combustion.
  - **Capillarity** can cause fuel to wick or seep through small clearances in the injector tip, leading to dribbling or deposit formation, which can affect the spray pattern and engine performance.
- **Influence on Brake Fluids:**
  - Brake fluids are designed to have low surface tension to ensure good wetting of seals and internal components, preventing air bubbles from adhering to surfaces.
  - **Capillarity** is undesirable. Brake fluid must not be drawn by capillary action past seals or through microscopic gaps, as this would lead to leakage. Seals are designed to prevent this.

Q4. Differentiate between absolute pressure, gauge pressure, and vacuum pressure. Provide the mathematical relationship between them.

- **Definitions and Differentiation:**
  - **Absolute Pressure (P_abs):** It is the pressure measured relative to a perfect **absolute vacuum** (zero pressure). It is the true, total pressure exerted by a fluid.
  - **Gauge Pressure (P_gauge):** It is the pressure measured relative to the **local atmospheric pressure**. It is the pressure indicated by most pressure gauges. It can be positive or negative.
  - **Vacuum Pressure (P_vac):** When the absolute pressure is less than atmospheric pressure, the gauge pressure becomes negative. This negative gauge pressure is often termed vacuum pressure. It is the amount by which the atmospheric pressure exceeds the absolute pressure.

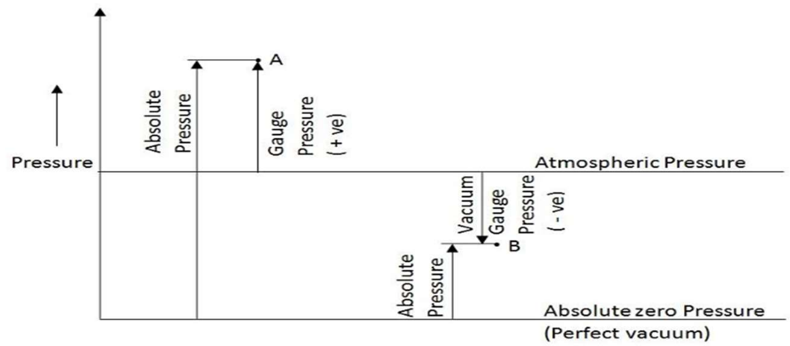


Fig.1.14 – Relationship between pressures

- **Mathematical Relationships:**

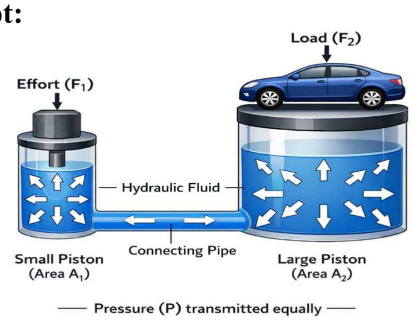
Absolute pressure = Atmospheric pressure + Gauge pressure

$$p_{abs} = p_{atm} + p_{gauge}$$

Vacuum pressure = Atmospheric pressure - Absolute pressure.

$$p_{vac} = p_{atm} - p_{abs}$$

Q5. With the help of a neat sketch, explain the working principle of a hydraulic lift based on Pascal's law.
Diagram Generation Prompt:



- **Working Principle (Pascal's Law):**
  Step 1: Understanding the Principle
  A hydraulic lift works on **Pascal's law**, which states that pressure applied to an enclosed fluid is transmitted equally in all directions without diminution.
  Step 2: Define the Variables
  Let:
- A_1 = Area of the smaller piston (input piston)
- A_2 = Area of the larger piston (output piston)
- F_1 = Force applied on the smaller piston (Effort)
- F_2 = Force obtained on the larger piston (Load)
- P = Pressure transmitted through the fluid
  Step 3: Apply Pascal's Law
  When a force F_1 is applied on the smaller piston, the pressure developed in the fluid is:

$$P = \frac{F_1}{A_1}$$

According to Pascal's law, this same pressure P acts on the larger piston.
Step 4: Calculate the Force on the Larger Piston
The force exerted on the larger piston due to this pressure is:

$$F_2 = P \times A_2$$

Substitute the expression for P:

$$F_2 = \frac{F_1}{A_1} \times A_2$$

Step 5: Obtain the Force Multiplication Ratio

$$\boxed{F_2 = F_1 \times \frac{A_2}{A_1}}$$

Since $A_2 > A_1$, the ratio $\frac{A_2}{A_1}$ is greater than 1. Therefore, the output force F_2 is magnified by this factor compared to the input force F_1.

Q6. A flat plate 0.05 mm distant from a fixed plate moves at 1.2 m/s and requires a shear stress of 2.5 N/m² to maintain this velocity. Determine the dynamic and kinematic viscosity of the lubricant between the plates if its specific gravity is 0.85.

Step 1: Given Data

- Distance between plates, $dy = 0.05\,\text{mm} = 0.05 \times 10^{-3} = 5 \times 10^{-5}\,\text{m}$
- Velocity of moving plate, $u = 1.2\,\text{m/s}$
- Since the fixed plate is stationary, velocity gradient, $du = u - 0 = 1.2\,\text{m/s}$
- Shear stress, $\tau = 2.5\,\text{N/m}^2$
- Specific Gravity, $s = 0.85$
- Density of water, $\rho_w = 1000\,\text{kg/m}^3$

Step 2: Calculate Velocity Gradient

$$\frac{du}{dy} = \frac{1.2}{5 \times 10^{-5}} = 24000\,\text{s}^{-1}$$

Step 3: Apply Newton's Law of Viscosity to Find Dynamic Viscosity (μ)

Newton's law states:

$$\tau = \mu \frac{du}{dy}$$

Rearranging for μ:

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)} = \frac{2.5}{24000}$$

$$\mu = 1.04167 \times 10^{-4}\,\text{Pa} \cdot \text{s}$$

Step 4: Calculate Density of Lubricant (ρ)

$$\rho = s \cdot \rho_w = 0.85 \times 1000 = 850\,\text{kg/m}^3$$

Step 5: Calculate Kinematic Viscosity (ν)

The relationship is:

$$\nu = \frac{\mu}{\rho} = \frac{1.04167 \times 10^{-4}}{850}$$

$$\nu = 1.2255 \times 10^{-7}\,\text{m}^2/\text{s}$$

Final Answer:

$$\boxed{\mu = 1.042 \times 10^{-4}\,\text{Pa}\cdot\text{s},\ \nu = 1.226 \times 10^{-7}\,\text{m}^2/\text{s}}$$

7 Marks Questions

Q1. Explain the working principle of a differential U-tube manometer. Derive the hydrostatic equation used to determine the pressure difference between two points in a pipe containing a fluid under pressure.

- **Working Principle:**
  A differential U-tube manometer is a device used to measure the **difference in pressure** between two points in a pipe or between two different pipes. It works on the principle of **hydrostatic equilibrium**.
  1. It consists of a U-shaped tube filled with a manometric liquid (e.g., mercury) which is heavier and immiscible with the fluid whose pressure difference is to be measured.
  2. The two limbs of the manometer are connected to the two points (A and B) where the pressure difference is desired.
  3. The pressure difference causes the manometric liquid to be displaced. The heavier liquid will be at a lower level on the side with higher pressure and at a higher level on the side with lower pressure.
  4. By measuring the difference in the levels of the manometric liquid (h), and knowing the specific gravities of the manometric liquid and the working fluid, the pressure difference can be calculated.
- **Derivation of Hydrostatic Equation:**

Diagram:

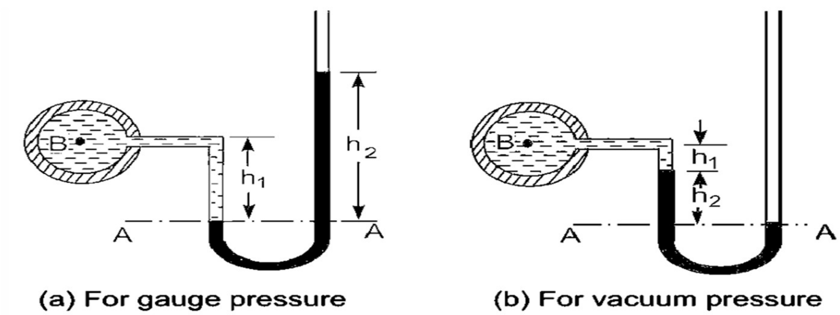


Fig.1.16 – U-tube Manometer

Consider a differential manometer connected between two points A and B in pipes carrying fluids of specific weights γ_1 and γ_2. The manometer contains a heavier liquid (e.g., mercury) of specific weight γ_m. Let the difference in the levels of the manometric liquid be h.

Step 1: Choose a Datum

Select the lower level of the manometric liquid as the datum line (say, the level in the left limb).

Step 2: Write Pressure Equation from Left Side (A to Datum)

Starting from point A, moving down through the fluid of specific weight γ_1 to the datum:

$$P_{\text{datum}} = P_A + \gamma_1 \cdot h_1$$

where h_1 is the vertical distance from the center of pipe A to the datum.

Step 3: Write Pressure Equation from Right Side (B to Datum)

Starting from point B, move down to the manometric liquid level in the right limb, and then through the manometric liquid down to the datum.

First, pressure at the manometric liquid surface in the right limb:

$$P_{\text{right surface}} = P_B + \gamma_2 \cdot h_2$$

where h_2 is the vertical distance from the center of pipe B down to the manometric liquid surface in the right limb.
Now, move down from this surface to the datum through the manometric liquid of height h:

$$P_{\text{datum}} = P_{\text{right surface}} + \gamma_m \cdot h$$

Substitute the expression for $P_{\text{right surface}}$:

$$P_{\text{datum}} = P_B + \gamma_2 h_2 + \gamma_m h$$

Step 4: Equate the Two Datum Pressure Equations
Since the pressure at the same horizontal datum line is equal in a static fluid:

$$P_A + \gamma_1 h_1 = P_B + \gamma_2 h_2 + \gamma_m h$$

Step 5: Rearrange for Pressure Difference

$$P_A - P_B = \gamma_2 h_2 + \gamma_m h - \gamma_1 h_1$$

This is the general expression for the pressure difference. If the same fluid flows in both pipes ($\gamma_1 = \gamma_2 = \gamma_f$), and considering the vertical elevations z_A and z_B, the equation simplifies to the commonly used form:

$$\boxed{P_A - P_B = (\gamma_m - \gamma_f)h + \gamma_f(z_B - z_A)}$$

Q2. Discuss the conditions of equilibrium (stable, unstable, and neutral) for floating and submerged bodies. Explain how buoyancy and stability principles dictate the design of automotive fuel tanks and float gauges.

- **Conditions of Equilibrium for Submerged Bodies:**
  For a body completely submerged in a fluid, its stability is determined solely by the relative positions of its **center of gravity (G)** and its **center of buoyancy (B)**.
  1. **Stable Equilibrium:** If a small angular displacement is given to the body, it returns to its original position. This occurs when **G is directly below B**. The restoring moment created by the weight and buoyancy force brings it back.
  2. **Unstable Equilibrium:** If a small angular displacement causes the body to move further away from its original position. This occurs when **G is directly above B**. The weight and buoyancy force create an overturning moment.
  3. **Neutral Equilibrium:** If a body, when displaced, remains in the new position. This occurs when **G and B coincide**.
- **Conditions of Equilibrium for Floating Bodies:**
  For a body partially submerged (floating), stability is governed by the **meta-center (M)** .
  1. **Stable Equilibrium:** When tilted, the body returns to its original position. This occurs when the **meta-center (M) is above the center of gravity (G)** (i.e., M > G). The restoring couple is generated.
  2. **Unstable Equilibrium:** When tilted, the body overturns. This occurs when **M is below G** (i.e., M < G), creating an overturning couple.
  3. **Neutral Equilibrium:** When tilted, the body remains in the tilted position. This occurs when **M coincides with G** (i.e., M = G).
- **Application in Automotive Design:**
  - **Automotive Fuel Tanks:**
    1. **Low Center of Gravity:** Fuel tanks are typically designed to be wide and flat and are placed low in the vehicle, under the rear seat or below

the floorpan. This ensures that the overall **center of gravity (G) of the fuel tank assembly is as low as possible.**

2. **Stability:** As the fuel is consumed, the center of gravity of the liquid fuel (and thus the tank's CG) changes. Designers ensure that even with varying fuel levels, the meta-centric height is positive (M > G), preventing the sloshing fuel from creating an unstable condition that could affect the vehicle's handling.
3. **Baffles:** Internal baffles are used to reduce the dynamic movement of fuel (sloshing). This prevents rapid shifts in the center of mass of the fuel, which could cause a momentary unstable condition during cornering.

- **Float Gauges (Fuel Level Sensor):**
  1. The float itself must be **stable** in the fuel. It is designed to have a density lower than the fuel and is shaped (often a hollow sphere or donut) so that its **center of buoyancy (B) is above its center of gravity (G)** . This ensures the float remains upright and stable on the fuel surface.
  2. As the fuel level changes, the float's position changes, moving the arm and the variable resistor. The principle of **stable equilibrium** for a floating body is essential for the float to sit correctly on the surface and provide an accurate reading without tilting and binding on its arm.

Q3. Derive an expression for the total hydrostatic force and the position of the center of pressure for a vertically submerged plane surface. Explain why the center of pressure always lies below the center of gravity.

Derivation for a Vertically Submerged Plane Surface:
Consider a plane vertical surface submerged in a static fluid. The fluid has a specific weight $\gamma = \rho g$. Let the surface make an angle $\theta = 90°$ with the horizontal.
Step 1: Expression for Total Hydrostatic Force (F)
Consider a small horizontal strip of area dA at a depth h from the free surface.
The pressure intensity on this strip is:

$$p = \rho g h$$

The force dF on this elemental strip is:

$$dF = p \cdot dA = \rho g h \, dA$$

The total force F on the entire surface is obtained by integrating over the whole area:

$$F = \int dF = \int \rho g h \, dA$$

Since ρ and g are constants:

$$F = \rho g \int h \, dA$$

From the definition of the centroid, the integral $\int h \, dA$ is the first moment of the area about the free surface, which is equal to the product of the total area A and the depth of its centroid h_c:

$$\int h \, dA = A \cdot h_c$$

Substituting this, we get the expression for total hydrostatic force:

$$\boxed{F = \rho g A h_c}$$

Step 2: Expression for the Position of Center of Pressure (h*)
The center of pressure is the point where the resultant force F acts. To find its depth h^*, we take moments of the elemental forces about an axis on the free surface (line O-O).
Let y be the distance of the elemental strip from the free surface along the plane. For a vertical surface, $h = y$. The moment of the elemental force dF about the free surface is:

$$dM = dF \cdot y = (\rho g h \, dA) \cdot y$$

Since $h = y$:

$$dM = \rho g y^2 \, dA$$

The total moment of the resultant force F about the free surface is $F \cdot h^*$. Equating the total moment to the sum of the elemental moments:

$$F \cdot h^* = \int \rho g y^2 \, dA = \rho g \int y^2 \, dA$$

The integral $\int y^2 \, dA$ is the second moment of area (moment of inertia) of the surface about the free surface axis, denoted as I_o. Therefore:

$$F \cdot h^* = \rho g I_o$$

Substitute $F = \rho g A h_c$:

$$(\rho g A h_c) \cdot h^* = \rho g I_o$$

$$h^* = \frac{I_o}{A h_c}$$

Now, apply the **parallel axis theorem** to relate I_o to the moment of inertia about the centroidal axis (I_c):

$$I_o = I_c + A h_c^2$$

Substitute this into the equation for h^*:

$$h^* = \frac{I_c + A h_c^2}{A h_c}$$

$$\boxed{h^* = h_c + \frac{I_c}{A h_c}}$$

Step 3: Why the Center of Pressure Lies Below the Center of Gravity
From the derived formula:

$$h^* = h_c + \frac{I_c}{A h_c}$$

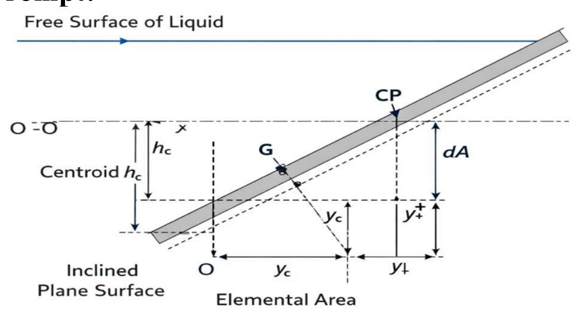
The terms I_c, A, and h_c are all positive quantities (since I_c is always positive). Therefore, the term $\frac{I_c}{A h_c}$ is always positive. This means:

$$h^* > h_c$$

This inequality mathematically proves that the center of pressure (CP) is always located at a greater depth than the centroid (CG) of the surface. This occurs physically because pressure increases with depth. The lower parts of the surface experience higher pressure, shifting the resultant force below the centroid, which is the point of average pressure.

Q4. Derive an expression for the total hydrostatic force and the center of pressure for an inclined plane surface submerged in a static fluid.

Diagram Generation Prompt:



Derivation:

Consider a plane surface of arbitrary shape, submerged in a static fluid and inclined at an angle θ to the horizontal. Let the fluid have a specific weight $\gamma = \rho g$.

Step 1: Define Geometric Relationships

Let the line of intersection of the plane containing the surface (extended) with the free surface be the axis O-O.

- Let y = distance of an elemental area dA from the axis O-O, measured along the inclined plane.
- Let h = vertical depth of the elemental area dA from the free surface.

From the geometry of the inclined plane:

$$h = y\sin\,\theta$$

Let:

- y_c = distance of the centroid G of the surface from the axis O-O along the inclined plane.
- h_c = vertical depth of the centroid G from the free surface.

Therefore:

$$h_c = y_c\sin\,\theta$$

Step 2: Expression for Total Hydrostatic Force (F)

The pressure intensity on the elemental area dA is:

$$p = \rho g h = \rho g(y\sin\,\theta)$$

The force dF on this elemental area is:

$$dF = p \cdot dA = \rho g\sin\,\theta\, y\, dA$$

The total force F on the entire surface is obtained by integrating over the whole area:

$$F = \int dF = \int \rho g\sin\,\theta\, y\, dA$$

Since ρ, g, and $\sin\theta$ are constants:

$$F = \rho g\sin\,\theta \int y\, dA$$

From the definition of the centroid, the integral $\int y\, dA$ is the first moment of the area about the axis O-O, which equals $A \cdot y_c$:

$$\int y\, dA = A \cdot y_c$$

Substituting this:

$$F = \rho g \sin\ \theta \cdot A \cdot y_c$$

Since $h_c = y_c \sin\ \theta$, we can also write:

$$\boxed{F = \rho g A h_c}$$

This shows that the total hydrostatic force on any submerged plane surface is equal to the product of the pressure at its centroid and the area of the surface.

Step 3: Expression for the Position of Center of Pressure (y and h*)**

The center of pressure is the point where the resultant force F acts. To find its distance y^* from the axis O-O along the inclined plane, we take moments of the elemental forces about the axis O-O.

The moment of the elemental force dF about the axis O-O is:

$$dM = dF \cdot y = (\rho g \sin\ \theta\, y\, dA) \cdot y = \rho g \sin\ \theta\, y^2\, dA$$

The total moment of the resultant force F about the axis O-O is $F \cdot y^*$. Equating the total moment to the sum of the elemental moments:

$$F \cdot y^* = \int \rho g \sin\ \theta\, y^2\, dA = \rho g \sin\ \theta \int y^2\, dA$$

The integral $\int y^2\, dA$ is the second moment of area (moment of inertia) of the surface about the axis O-O, denoted as I_o. Therefore:

$$F \cdot y^* = \rho g \sin\ \theta\, I_o$$

Substitute $F = \rho g \sin\ \theta\, A y_c$:

$$(\rho g \sin\ \theta\, A y_c) \cdot y^* = \rho g \sin\ \theta\, I_o$$

Cancel $\rho g \sin\ \theta$ from both sides (since $\sin\ \theta \neq 0$):

$$y^* = \frac{I_o}{A y_c}$$

Step 4: Apply the Parallel Axis Theorem

The parallel axis theorem relates I_o to the moment of inertia about a centroidal axis parallel to O-O, denoted as I_c:

$$I_o = I_c + A y_c^2$$

Substitute this into the equation for y^*:

$$y^* = \frac{I_c + A y_c^2}{A y_c}$$

Step 5: Simplify the Expression

$$\boxed{y^* = y_c + \frac{I_c}{A y_c}}$$

Step 6: Determine the Vertical Depth of Center of Pressure (h*)

The vertical depth h^* of the center of pressure is related to y^* by:

$$h^* = y^* \sin\ \theta$$

Substitute the expression for y^*:

$$h^* = \left(y_c + \frac{I_c}{A y_c}\right)\sin\,\theta$$

Since $h_c = y_c \sin\,\theta$:

$$\boxed{h^* = h_c + \frac{I_c \sin^2\theta}{A h_c}}$$

Step 7: Special Cases

1. **For a vertical surface** ($\theta = 90°$, $\sin\,\theta = 1$, and $y_c = h_c$):
$$h^* = h_c + \frac{I_c}{A h_c}$$
2. **For a horizontal surface** ($\theta = 0°$, $\sin\,\theta = 0$):
   The pressure is uniform, so the center of pressure coincides with the centroid:
$$h^* = h_c$$

Final Result:
The total hydrostatic force on an inclined submerged plane surface is:

$$\boxed{F = \rho g A h_c}$$

The distance of the center of pressure from the free surface along the inclined plane is:

$$\boxed{y^* = y_c + \frac{I_c}{A y_c}}$$

The vertical depth of the center of pressure is:

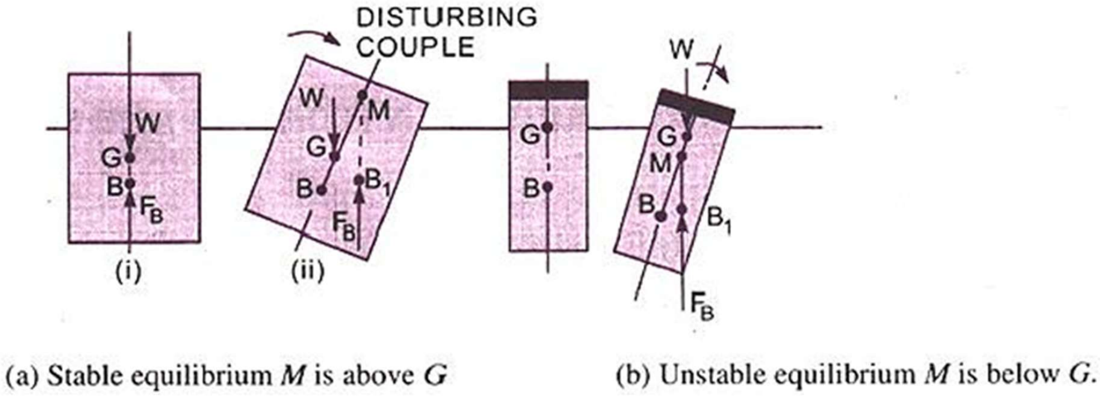
$$\boxed{h^* = h_c + \frac{I_c \sin^2\theta}{A h_c}}$$

Q5. Define the terms 'Center of Buoyancy', 'Meta-center', and 'Meta-centric height'. Explain with diagrams how the relative positions of the center of gravity and the meta-center determine the stability of a floating body.

- **Definitions:**
  - **Center of Buoyancy (B):** It is the point through which the resultant force of buoyancy (the upward force exerted by the fluid) acts on a submerged or floating body. It is essentially the **centroid of the volume of fluid displaced** by the body. Its position changes as the body tilts.
  - **Meta-center (M):** When a floating body is given a small angular displacement (tilt), the center of buoyancy shifts from its original position (B) to a new position (B_1). The **meta-center** is defined as the point about which the body starts oscillating for small angles of tilt. More precisely, it is the **intersection point of the line of action of the buoyant force** (passing through B_1) with the **normal axis of the body** (the original vertical line passing through the center of gravity G and the original center of buoyancy B).
  - **Meta-centric Height (GM):** It is the distance between the center of gravity (G) and the meta-center (M) of a floating body. It is a critical measure of **initial static stability**.
- **Stability Determination (using diagrams):**

The stability of a floating body is determined by the relative positions of G and M.

Diagram Generation Prompts:



- **Explanation:**
    1. **Stable Equilibrium (M > G):** As shown in the first diagram, when M is above G, the weight force (W) acting downwards at G and the buoyant force (F_B) acting upwards at B_1 form a **restoring couple**. This couple acts in the direction opposite to the tilt, trying to bring the body back to its original upright position. The meta-centric height GM is **positive**.
    2. **Unstable Equilibrium (M < G):** In the second diagram, M is below G. The weight and buoyant force now form an **overturning couple**. This couple acts in the same direction as the tilt, causing the body to heel further and eventually capsize. The meta-centric height GM is **negative**.
    3. **Neutral Equilibrium (M = G):** In the third diagram, M and G coincide. In this case, no couple is formed. The body will remain in whatever angular position it is displaced to. The meta-centric height GM is **zero**.

Q6. A differential U-tube manometer containing mercury is used to measure the pressure difference between two pipes, A and B, containing liquids of specific gravities 0.8 and 1.0 respectively. The center of pipe A is 0.5 m above the center of pipe B. The mercury level in the limb connected to pipe A is 0.4 m below the center of pipe B, and the difference in mercury levels is 0.3 m. Calculate the pressure difference between pipes A and B.

Step 1: Given Data

- Specific gravity of fluid in A, $s_1 = 0.8$
- Specific gravity of fluid in B, $s_2 = 1.0$
- Specific gravity of mercury, $s_m = 13.6$ (Standard value)
- Height of pipe A above pipe B, $z_A - z_B = +0.5$ m (So $z_B - z_A = -0.5$ m)
- Specific weight of water, $\gamma_w = 9810$ N/m^3

Step 2: Determine the Geometric Parameters (with reference to Pipe B's center as elevation 0 m)

- **Elevation of Pipe A:** $+0.5$ m
- **Elevation of left mercury meniscus (connected to A):** It is 0.4 m *below* pipe B.

$$\text{Elevation}_{\text{left}} = 0 - 0.4 = -0.4 \text{ m}$$

- **Height of fluid column from Pipe A to left meniscus (h_1):**

$$h_1 = \text{Elevation}_A - \text{Elevation}_{\text{left}} = 0.5 - (-0.4) = 0.9 \text{ m}$$

- **Difference in mercury levels (h):** 0.3 m. Since the left meniscus is lower, the right meniscus is higher.

$$\text{Elevation}_{\text{right}} = \text{Elevation}_{\text{left}} + h = -0.4 + 0.3 = -0.1 \text{ m}$$

- **Height of fluid column from right meniscus to Pipe B (h_2):** (Moving *up* from right meniscus to B)

$$h_2 = \text{Elevation}_B - \text{Elevation}_{\text{right}} = 0 - (-0.1) = 0.1 \text{ m}$$

Step 3: Calculate Specific Weights

$$\gamma_1 = s_1 \cdot \gamma_w = 0.8 \times 9810 = 7848 \text{ N/m}^3$$
$$\gamma_2 = s_2 \cdot \gamma_w = 1.0 \times 9810 = 9810 \text{ N/m}^3$$
$$\gamma_m = s_m \cdot \gamma_w = 13.6 \times 9810 = 133416 \text{ N/m}^3$$

Step 4: Apply the Hydrostatic Equation (Equating pressures at the datum - the lower left meniscus)

From the left limb side (A to Datum):

$$P_{\text{datum}} = P_A + \gamma_1 h_1$$
$$P_{\text{datum}} = P_A + (7848 \times 0.9)$$

From the right limb side (B to Datum):
First, pressure at the right meniscus:

$$P_{\text{right}} = P_B + \gamma_2 h_2 = P_B + (9810 \times 0.1)$$

Now, go down through the mercury from the right meniscus to the datum:

$$P_{\text{datum}} = P_{\text{right}} + \gamma_m h$$
$$P_{\text{datum}} = [P_B + (9810 \times 0.1)] + (133416 \times 0.3)$$

Step 5: Equate the Two Expressions and Solve for $P_A - P_B$

$$P_A + (7848 \times 0.9) = P_B + (9810 \times 0.1) + (133416 \times 0.3)$$

Rearrange to isolate $P_A - P_B$:

$$P_A - P_B = (9810 \times 0.1) + (133416 \times 0.3) - (7848 \times 0.9)$$

Calculate each term:

- $9810 \times 0.1 = 981$
- $133416 \times 0.3 = 40024.8$
- $7848 \times 0.9 = 7063.2$

$$P_A - P_B = 981 + 40024.8 - 7063.2$$
$$P_A - P_B = 41005.8 - 7063.2$$
$$P_A - P_B = 33942.6 \text{ N/m}^2$$

Final Answer:

$$\boxed{P_A - P_B = 33942.6 \text{ Pa (or 33.94 kPa)}}$$

Unit 5: Dimensional Analysis & Similitude

3 & 4-Marks Questions

Q.1 What do you mean by fundamental units and derived units?
Step 1: Definition of Fundamental Units

- **Fundamental Units** (also called Base Units) are the basic units of measurement that are **independently defined** and cannot be expressed in terms of other units.
- They form the foundation upon which all other units are built.
- In the **MLT system** (Mass, Length, Time) commonly used in fluid mechanics, the fundamental units are:
  - **Mass (M)** - measured in kilograms (kg)
  - **Length (L)** - measured in meters (m)
  - **Time (T)** - measured in seconds (s)
- In the **FLT system** (Force, Length, Time), the fundamental units are Force (F), Length (L), and Time (T).

Step 2: Definition of Derived Units

- **Derived Units** are units that can be expressed in terms of fundamental units using mathematical relationships.
- They are obtained by combining fundamental units according to the physical laws defining the quantity.

Step 3: Examples

Quantity	Relationship	Dimension (MLT)	Derived Unit
Velocity	Displacement / Time	LT^{-1}	m/s
Acceleration	Velocity / Time	LT^{-2}	m/s²
Force	Mass × Acceleration	MLT^{-2}	Newton (N) or kg·m/s²
Pressure	Force / Area	$ML^{-1}T^{-2}$	Pascal (Pa) or N/m²
Energy	Force × Distance	$ML^{2}T^{-2}$	Joule (J) or N·m

Q.2 Determine the dimension of the following quantities: (i) Angular Acceleration, (ii) Discharge (iii) Force (iv) Specific Weight
Step 1: General Approach
We use the **MLT system** (Mass, Length, Time) to determine dimensions. The dimension of a quantity is expressed as $M^a L^b T^c$.
Step 2: (i) Angular Acceleration (α)

- Angular velocity (ω) = Angle / Time = $\frac{1}{T} = T^{-1}$ (Angle is dimensionless)
- Angular acceleration (α) = Rate of change of angular velocity = $\frac{\omega}{T} = \frac{T^{-1}}{T}$

$$\boxed{\alpha = T^{-2}}$$

- **Dimension:** $M^0 L^0 T^{-2}$

Step 3: (ii) Discharge (Q)

- Discharge (also called flow rate or volumetric flow rate) = Volume / Time
- Volume = L^3
- Time = T

$$Q = \frac{L^3}{T} = L^3T^{-1}$$
$$\boxed{Q = L^3T^{-1}}$$

- **Dimension:** $M^0L^3T^{-1}$

Step 4: (iii) Force (F)

- From Newton's Second Law: Force = Mass × Acceleration
- Mass = M
- Acceleration = Velocity / Time = $\frac{LT^{-1}}{T} = LT^{-2}$

$$F = M \times LT^{-2} = MLT^{-2}$$
$$\boxed{F = MLT^{-2}}$$

- **Dimension:** $M^1L^1T^{-2}$

Step 5: (iv) Specific Weight (γ)

- Specific weight = Weight / Volume
- Weight = Mass × Acceleration due to gravity = $M \times LT^{-2} = MLT^{-2}$
- Volume = L^3

$$\gamma = \frac{MLT^{-2}}{L^3} = ML^{-2}T^{-2}$$
$$\boxed{\gamma = ML^{-2}T^{-2}}$$

- **Dimension:** $M^1L^{-2}T^{-2}$

Final Answer Table:

Quantity	Symbol	Dimension (MLT)
Angular Acceleration	α	$M^0L^0T^{-2}$
Discharge	Q	$M^0L^3T^{-1}$
Force	F	$M^1L^1T^{-2}$
Specific Weight	γ	$M^1L^{-2}T^{-2}$

Q.3 What is meant by dimensional analysis? What are its limitations and advantages?

Step 1: Definition of Dimensional Analysis

- **Dimensional analysis** is a mathematical technique used in engineering and physics to study the relationships between physical quantities by examining their **dimensions** (M, L, T, etc.).
- It is based on the fundamental principle of **dimensional homogeneity**, which states that every term in a valid physical equation must have the same dimensions.
- It helps in:
  - Deriving relationships between variables without complete theory

- Converting units from one system to another
- Planning experiments and reducing the number of variables
- Checking the dimensional correctness of equations

Step 2: Advantages of Dimensional Analysis

Advantage	Explanation
1. Simplification	It reduces the number of variables by grouping them into dimensionless numbers (e.g., from 7 variables to 3 dimensionless groups).
2. Experimental Design	It guides experimenters by indicating which variables are important and how to present results using dimensionless parameters.
3. Scale Modeling	It enables the design of scaled models (prototype testing) using similitude principles.
4. Equation Verification	It helps check the dimensional correctness of derived or empirical equations.
5. Missing Variable Detection	It can sometimes reveal if a relevant variable has been omitted from analysis.

Step 3: Limitations of Dimensional Analysis

Limitation	Explanation
1. No Constant Information	It does not provide the value of dimensionless constants (e.g., the factor $\pi/4$ in discharge formula). These must be found experimentally.
2. Requires Complete Variable List	The analysis is only as good as the list of variables considered. If a significant variable is omitted, the result will be incorrect.
3. Cannot Differentiate	It cannot distinguish between quantities with the same dimensions (e.g., energy and torque both have dimension ML^2T^{-2}).
4. No Directional Information	It provides only the form of the relationship, not the physical mechanism or directional sense.
5. Selection of Repeating Variables	The choice of repeating variables is arbitrary and requires physical insight.

Q.4 Explain the dimensionless numbers: (i) Reynold's Number (ii) Mach Number

Step 1: Definition of Dimensionless Numbers

- **Dimensionless numbers** are pure numbers that have no units. They represent the ratio of two forces or effects and are used to characterize the flow regime in fluid mechanics.

Step 2: (i) Reynold's Number (Re)

Definition:

Reynold's number is the ratio of **inertia force** to **viscous force** in a flowing fluid.

Mathematical Expression:

$$Re = \frac{\text{Inertia Force}}{\text{Viscous Force}} = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$$

Where:

- ρ = Density of fluid (ML^{-3})
- V = Characteristic velocity (LT^{-1})
- L = Characteristic length (L) (e.g., pipe diameter)
- μ = Dynamic viscosity ($ML^{-1}T^{-1}$)
- ν = Kinematic viscosity (L^2T^{-1})

Physical Significance:

- **Low Re (<2000):** Viscous forces dominate. Flow is **laminar** (smooth, orderly).
- **High Re (> 4000):** Inertia forces dominate. Flow is **turbulent** (chaotic, with eddies).
- **Transitional Re ($2000 < Re < 4000$):** Flow is unstable and switches between laminar and turbulent.

Applications:

- Determining flow regime in pipes
- Scaling models in ship and aircraft design
- Analyzing boundary layer behavior

Step 3: (ii) Mach Number (M)

Definition:

Mach number is the ratio of **inertia force** to **elastic force** in a fluid flow. More commonly, it is defined as the ratio of the **velocity of flow** to the **velocity of sound** in the same fluid medium.

Mathematical Expression:

$$M = \frac{\text{Inertia Force}}{\text{Elastic Force}} = \frac{V}{C}$$

Where:

- V = Velocity of flow (LT^{-1})
- C = Velocity of sound in the fluid (LT^{-1}) = $\sqrt{\frac{K}{\rho}}$ (where K is bulk modulus)

Physical Significance:

Mach number classifies flow regimes based on compressibility effects:

Mach Number Range	Regime	Characteristics
$M < 0.8$	Subsonic	Compressibility effects negligible; density constant
$0.8 < M < 1.2$	Transonic	Mixed flow; shock waves begin to form
$1.2 < M < 5.0$	Supersonic	Compressibility effects dominant; shock waves present
$M > 5.0$	Hypersonic	Very high speeds; extreme temperatures

Applications:

- Aircraft and missile design
- Wind tunnel testing at high speeds
- Compressible flow analysis in nozzles and diffusers

7 Marks Questions

Ex. 1. Fluid of density ρ and viscosity μ flows at an average velocity ν through a circular pipe of diameter D. Show by dimensional analysis, that the shear stress of the pipe wall is $\tau_0 = \rho\nu^2\Phi\left(\frac{\rho\nu D}{\mu}\right)$

Step 1: Identify the Variables and Their Dimensions

The wall shear stress τ_0 depends on:

- Diameter of pipe, D [L]
- Average velocity, V [LT^{-1}] *(Note: The question uses ν for velocity, but we'll use V to avoid confusion with kinematic viscosity)*
- Fluid density, ρ [ML^{-3}]
- Fluid viscosity, μ [$ML^{-1}T^{-1}$]
- (Possibly) Roughness height, but for a smooth pipe it's not considered.

Let the functional relationship be:

$$\tau_0 = f(D, V, \rho, \mu)$$

Step 2: Write Dimensions of Each Variable

Variable	Symbol	Dimension
Wall shear stress	τ_0	$ML^{-1}T^{-2}$ (Force/Area)
Diameter	D	L
Velocity	V	LT^{-1}
Density	ρ	ML^{-3}
Dynamic viscosity	μ	$ML^{-1}T^{-1}$

Number of variables, $n = 5$

Number of fundamental dimensions, $m = 3$ (M, L, T)

According to Buckingham's π-theorem, number of dimensionless π-terms = $n - m = 5 - 3 = 2$.

Step 3: Select Repeating Variables

Choose $m = 3$ repeating variables that together contain all fundamental dimensions (M, L, T). Common choices are:

- ρ (contains M)
- V (contains L and T)
- D (contains L)

These three variables are independent and cannot themselves form a dimensionless group.

Step 4: Form the First π-Term (π_1)

Let:

$$\pi_1 = \rho^a V^b D^c \cdot \tau_0$$

Write the dimensional equation:

$$[M^0L^0T^0] = [ML^{-3}]^a[LT^{-1}]^b[L]^c \cdot [ML^{-1}T^{-2}]$$

Equate exponents for M, L, and T:

For M: $a + 1 = 0 \Rightarrow a = -1$

For T: $-b - 2 = 0 \Rightarrow -b = 2 \Rightarrow b = -2$

For L: $-3a + b + c - 1 = 0$
Substitute $a = -1$ and $b = -2$:

$$-3(-1) + (-2) + c - 1 = 0$$
$$3 - 2 + c - 1 = 0$$
$$c + 0 = 0 \Rightarrow c = 0$$

Therefore:

$$\pi_1 = \rho^{-1}V^{-2}D^0 \cdot \tau_0 = \frac{\tau_0}{\rho V^2}$$

Step 5: Form the Second π-Term (π_2)
Let:

$$\pi_2 = \rho^a V^b D^c \cdot \mu$$

Write the dimensional equation:

$$[M^0L^0T^0] = [ML^{-3}]^a[LT^{-1}]^b[L]^c \cdot [ML^{-1}T^{-1}]$$

Equate exponents:
For M: $a + 1 = 0 \Rightarrow a = -1$
For T: $-b - 1 = 0 \Rightarrow -b = 1 \Rightarrow b = -1$
For L: $-3a + b + c - 1 = 0$
Substitute $a = -1$ and $b = -1$:

$$-3(-1) + (-1) + c - 1 = 0$$
$$3 - 1 + c - 1 = 0$$
$$1 + c = 0 \Rightarrow c = -1$$

Therefore:

$$\pi_2 = \rho^{-1}V^{-1}D^{-1} \cdot \mu = \frac{\mu}{\rho V D}$$

Step 6: Form the Functional Relationship
According to Buckingham's theorem:

$$\pi_1 = \Phi(\pi_2)$$

Substitute the expressions:

$$\frac{\tau_0}{\rho V^2} = \Phi\left(\frac{\mu}{\rho V D}\right)$$

Step 7: Rearrange to Obtain the Desired Form
The second π-term is the reciprocal of the Reynolds number:

$$\frac{\mu}{\rho V D} = \frac{1}{Re} \text{ where} Re = \frac{\rho V D}{\mu}$$

Therefore:

$$\frac{\tau_0}{\rho V^2} = \Phi\left(\frac{1}{Re}\right)$$

Or more commonly written as:

$$\frac{\tau_0}{\rho V^2} = \Phi(Re)$$

Thus:

$$\boxed{\tau_0 = \rho V^2\, \Phi\left(\frac{\rho V D}{\mu}\right)}$$

This shows that the wall shear stress is proportional to ρV^2 times some function of the Reynolds number.

Ex. 2. Find the expression for drag force on a smooth sphere of diameter D, moving with a uniform velocity V in a fluid of density ρ and dynamic viscosity μ.

Step 1: Identify the Variables and Their Dimensions

The drag force F_D depends on:

- Diameter of sphere, D [L]
- Velocity of sphere relative to fluid, V [LT^{-1}]
- Fluid density, ρ [ML^{-3}]
- Fluid viscosity, μ [$ML^{-1}T^{-1}$]

Let the functional relationship be:

$$F_D = f(D, V, \rho, \mu)$$

Step 2: Write Dimensions of Each Variable

Variable	Symbol	Dimension
Drag Force	F_D	MLT^{-2}
Diameter	D	L
Velocity	V	LT^{-1}
Density	ρ	ML^{-3}
Dynamic viscosity	μ	$ML^{-1}T^{-1}$

Number of variables, $n = 5$

Number of fundamental dimensions, $m = 3$ (M, L, T)

According to Buckingham's π-theorem, number of dimensionless π-terms = $n - m = 5 - 3 = 2$.

Step 3: Select Repeating Variables

Choose $m = 3$ repeating variables:

- ρ (contains M)
- V (contains L and T)
- D (contains L)

These variables are independent and together contain all fundamental dimensions.

Step 4: Form the First π-Term (π_1)

Let:

$$\pi_1 = \rho^a V^b D^c \cdot F_D$$

Write the dimensional equation:

$$[M^0L^0T^0] = [ML^{-3}]^a[LT^{-1}]^b[L]^c \cdot [MLT^{-2}]$$

Equate exponents for M, L, and T:

For M: $a + 1 = 0 \Rightarrow a = -1$

For T: $-b - 2 = 0 \Rightarrow -b = 2 \Rightarrow b = -2$

For L: $-3a + b + c + 1 = 0$

Substitute $a = -1$ and $b = -2$:

$$-3(-1) + (-2) + c + 1 = 0$$
$$3 - 2 + c + 1 = 0$$
$$2 + c = 0 \Rightarrow c = -2$$

Therefore:

$$\pi_1 = \rho^{-1}V^{-2}D^{-2} \cdot F_D = \frac{F_D}{\rho V^2 D^2}$$

Step 5: Form the Second π-Term (π_2)
Let:

$$\pi_2 = \rho^a V^b D^c \cdot \mu$$

Write the dimensional equation:

$$[M^0 L^0 T^0] = [ML^{-3}]^a [LT^{-1}]^b [L]^c \cdot [ML^{-1}T^{-1}]$$

Equate exponents:
For M: $a + 1 = 0 \Rightarrow a = -1$
For T: $-b - 1 = 0 \Rightarrow -b = 1 \Rightarrow b = -1$
For L: $-3a + b + c - 1 = 0$
Substitute $a = -1$ and $b = -1$:

$$-3(-1) + (-1) + c - 1 = 0$$
$$3 - 1 + c - 1 = 0$$
$$1 + c = 0 \Rightarrow c = -1$$

Therefore:

$$\pi_2 = \rho^{-1}V^{-1}D^{-1} \cdot \mu = \frac{\mu}{\rho V D}$$

Step 6: Identify the Dimensionless Numbers

- $\pi_1 = \frac{F_D}{\rho V^2 D^2}$ is a form of the **Drag Coefficient** (C_D) multiplied by a constant. In standard form, the drag coefficient is defined as:

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = \frac{F_D}{\frac{1}{2}\rho V^2 \left(\frac{\pi D^2}{4}\right)} = \frac{8F_D}{\pi \rho V^2 D^2}$$

So π_1 is proportional to C_D.

- $\pi_2 = \frac{\mu}{\rho V D} = \frac{1}{Re}$, where $Re = \frac{\rho V D}{\mu}$ is the **Reynolds Number**.

Step 7: Form the Functional Relationship
According to Buckingham's theorem:

$$\pi_1 = \Phi(\pi_2)$$

Substitute the expressions:

$$\frac{F_D}{\rho V^2 D^2} = \Phi\left(\frac{\mu}{\rho V D}\right)$$

Or in terms of Reynolds number:

$$\frac{F_D}{\rho V^2 D^2} = \Phi\left(\frac{1}{Re}\right) = \Psi(Re)$$

Step 8: Rearrange to Obtain the Expression for Drag Force

$$F_D = \rho V^2 D^2 \cdot \Psi(Re)$$

In standard fluid mechanics, this is written as:

$$\boxed{F_D = C_D \cdot \frac{1}{2}\rho V^2 A}$$

where $A = \frac{\pi D^2}{4}$ is the projected area of the sphere, and $C_D = f(Re)$ is the drag coefficient obtained from experiments.
The final expression shows that the drag force depends on:

- Dynamic pressure $\frac{1}{2}\rho V^2$
- Projected area $\frac{\pi D^2}{4}$
- Drag coefficient C_D, which is a function of the Reynolds number

ANSWER BANK: COMPRESSIBLE FLUID FLOW

---

1. Define compressible fluid flow.

- **Definition:** Compressible fluid flow is the branch of fluid mechanics that deals with fluids where the **density changes (ρ ≠ constant)** within the flow field are significant enough to influence the solution.
- **Key Criteria:** Density changes are primarily driven by high-pressure variations caused by high-velocity flows, typically when the flow velocity approaches or exceeds the speed of sound.
- **Governing Factors:** Temperature and pressure variations lead to considerable density variations, requiring the use of thermodynamic principles (equations of state) alongside fluid mechanics equations.
- **Examples:** Flow through nozzles, turbines, compressors, and high-speed aircraft.

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2. What is Mach number?

- **Definition:** The Mach number (M) is a dimensionless quantity in fluid dynamics representing the ratio of the **velocity of an object or flow (V)** relative to a fluid to the **local speed of sound (a)** in that fluid.
- **Formula:** It is mathematically expressed as:

$$M = \frac{V}{a}$$

- **Significance:** It is the single most important parameter in compressible flow, used to:
  - Classify flow regimes (subsonic, supersonic, etc.)
  - Quantify the importance of compressibility effects
  - Determine whether density changes are negligible
- **Physical Meaning:** It represents the ratio of **inertia forces to elastic forces** in the fluid.

---

3. What is stagnation pressure?

- **Definition:** Stagnation pressure (also known as **total pressure** or **pitot pressure**) is the pressure a fluid attains when it is brought to rest **isentropically** (i.e., without friction or heat transfer) from its actual flow condition.
- **Physical Meaning:** It represents the sum of the static pressure and the dynamic pressure due to the fluid's kinetic energy.
- **For Incompressible Flow:** The relationship is:

$$P_0 = P + \frac{1}{2}\rho V^2$$

where:

- P_0 = Stagnation pressure
- P = Static pressure
- ρ = Density
- V = Flow velocity
- **For Compressible Flow:** The relationship is more complex and depends on Mach number:

$$P_0 = P\left[1 + \frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}$$

where k is the specific heat ratio.

---

4. What is stagnation temperature?

- **Definition:** Stagnation temperature (or **total temperature**) is the temperature a fluid attains when it is brought to rest **adiabatically** (i.e., without any heat transfer) from its actual flow condition.
- **Physical Meaning:** It represents the **total energy content** of the fluid per unit mass. It is the sum of the static enthalpy $\left(c_p T\right)$ and the kinetic energy $\left(\frac{V^2}{2}\right)$.
- **Mathematical Expression:**

$$T_0 = T + \frac{V^2}{2c_p}$$

or in dimensionless form:

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

where:

- T_0 = Stagnation temperature
- T = Static temperature
- c_p = Specific heat at constant pressure
- k = Specific heat ratio
- M = Mach number
- **Key Property:** Stagnation temperature remains constant in an adiabatic flow, even if the velocity changes.

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5. Define subsonic, sonic and supersonic flow.

Subsonic Flow:

- **Definition:** Flow where the fluid velocity is **less than** the local speed of sound.
- **Mach Number Range:** $M < 1$
- **Characteristics:**
  - Density changes are negligible at low subsonic speeds ($M < 0.3$)
  - Density changes become significant as M approaches 1
  - Pressure disturbances can propagate upstream
  - Streamlines are smooth and continuous

Sonic Flow:

- **Definition:** Flow where the fluid velocity is **exactly equal** to the local speed of sound.
- **Mach Number Range:** $M = 1$
- **Characteristics:**
  - This is a critical condition in gas dynamics
  - Occurs at the throat (minimum area) of a nozzle
  - Mass flow rate is maximum at this condition

Supersonic Flow:

- **Definition:** Flow where the fluid velocity is **greater than** the local speed of sound.
- **Mach Number Range:** $M > 1$
- **Characteristics:**
  - Flow behavior is fundamentally different from subsonic flow
  - Area increase causes velocity to increase (opposite of subsonic flow)
  - Shock waves and expansion fans form
  - Pressure disturbances cannot propagate upstream

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6. Define Mach cone.

- **Definition:** A Mach cone is a pressure shock wave pattern formed in the air when an object (like an aircraft) travels at **supersonic speeds** ($M > 1$).
- **Formation:** As the object moves faster than the sound waves it generates, the waves cannot propagate forward and instead coalesce into a cone-shaped shock front.
- **Diagram Prompt:** Create a diagram showing a supersonic aircraft moving from left to right. Draw concentric circles representing sound waves emitted from the aircraft at different times. Show how these circles overlap to form a V-shaped cone behind the aircraft. Label the cone boundary, the aircraft, and indicate the direction of motion.
- **Key Elements:**
  - **Mach Angle (μ):** The half-angle of the cone, measured from the direction of motion. It is given by:

$$\sin \mu = \frac{a}{V} = \frac{1}{M}$$

  - **Zone of Action:** The region **inside** the cone, where the disturbances (sound) are present and can be heard.
  - **Zone of Silence:** The region **outside** the cone, which remains undisturbed by the object's passage. An observer in this zone will not hear the object until after it has passed.
- **Sonic Boom:** The intersection of the Mach cone with the ground creates a sonic boom.

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7. Derive the expression for velocity of sound in compressible fluid.

- **Given:** Consider a plane pressure wave (sound wave) propagating through a stationary compressible fluid in a constant area duct.
- **To Find:** Expression for the velocity of sound, a.
- **Assumptions:**
  1. Frictionless flow
  2. No heat transfer (adiabatic)
  3. The disturbance is small ($dP, d\rho$ are infinitesimal)
  4. The process is **reversible and adiabatic**, hence **isentropic**
- **Solution:**

Step 1: Setup Control Volume

Imagine the wave is stationary and the fluid is flowing from right to left into the wave at velocity a. The fluid ahead of the wave is undisturbed at (P, ρ). The fluid behind the wave is at $(P + dP, \rho + d\rho)$ and has a velocity dV relative to the wave.

Step 2: Apply Continuity Equation

Mass flow into the control volume = Mass flow out

$$\rho A a = (\rho + d\rho)A(a - dV)$$

Simplifying:

$$\rho a = (\rho + d\rho)(a - dV)$$
$$\rho a = \rho a - \rho\, dV + a\, d\rho - d\rho\, dV$$

Neglecting the product of small terms $d\rho\, dV$:

$$0 = -\rho\, dV + a\, d\rho$$

$$\rho\, dV = a\, d\rho \text{...(Equation 1)}$$

Step 3: Apply Momentum Equation

Net force in the direction of flow = Rate of change of momentum

$$PA - (P + dP)A = \dot{m}(V_{out} - V_{in})$$
$$-dP \cdot A = (\rho a A)[(a - dV) - a]$$
$$-dP = \rho a(-dV)$$
$$dP = \rho a\, dV \text{...(Equation 2)}$$

Step 4: Combine Equations

From Equation 1: $dV = \frac{a\, d\rho}{\rho}$

Substitute into Equation 2:

$$dP = \rho a \left(\frac{a\, d\rho}{\rho}\right)$$
$$dP = a^2\, d\rho$$

Step 5: Final Expression

Therefore, the velocity of sound is:

$$a^2 = \frac{dP}{d\rho}$$
$$a = \sqrt{\frac{dP}{d\rho}}$$

Step 6: Apply Isentropic Relation

For an isentropic process, $\frac{P}{\rho^k} = \text{constant}$

Taking natural log and differentiating:

$$\ln\, P - k\ln\, \rho = \text{constant}$$
$$\frac{dP}{P} - k\frac{d\rho}{\rho} = 0$$
$$\frac{dP}{d\rho} = \frac{kP}{\rho}$$

Using the ideal gas law, $\frac{P}{\rho} = RT$:

$$\frac{dP}{d\rho} = kRT$$

Step 7: Final Answer

The velocity of sound in a compressible fluid is:

$$\boxed{a = \sqrt{\frac{dP}{d\rho}}}$$

For a perfect gas under isentropic conditions, this becomes:

$$\boxed{a = \sqrt{kRT}}$$

where:

- k = Specific heat ratio $\left(\frac{c_p}{c_v}\right)$
- R = Specific gas constant
- T = Absolute temperature

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8. Explain Mach number and classify flows based on Mach number.
Explanation of Mach Number:
The Mach number (M) is a dimensionless parameter defined as the ratio of the **local flow velocity (V)** to the **local speed of sound (a)** in the fluid.

$$M = \frac{V}{a}$$

It is the fundamental governing parameter in compressible flow because it quantifies the significance of compressibility effects. It represents the ratio of **inertia forces to elastic forces** in the fluid.
Physical Significance:

- Low Mach number ($M < 0.3$): Fluid's kinetic energy is small compared to its internal energy, allowing density to be considered constant.
- High Mach number ($M > 0.3$): Density variations become significant, fundamentally altering flow behavior.

Classification of Flows Based on Mach Number:

Flow Regime	Mach Number Range	Characteristics
1. Incompressible Flow	$M < 0.3$	• Density changes are negligible (<5%) • Fluid is treated as having constant density • Example: Flow of water in pipes, low-speed air flow
2. Subsonic Flow	$0.3 \leq M < 0.8$	• Density changes are significant and must be considered • Flow velocity is less than the speed of sound • Streamlines are smooth • Pressure changes can propagate upstream • Example: Flow over commercial aircraft during takeoff and landing
3. Transonic Flow	$0.8 \leq M \leq 1.2$	• Mixed flow regime with both subsonic and supersonic regions • Shock waves begin to form on the body surface • Sharp increase in drag • Example: Flow over modern airliners at cruising speed
4. Supersonic Flow	$1.2 < M < 5.0$	• Flow velocity is greater than the speed of sound • Velocity increases with increasing area (opposite to subsonic flow) • Shock waves and expansion fans are prominent • Example: Flow over a supersonic fighter jet

Flow Regime	Mach Number Range	Characteristics
5. Hypersonic Flow	$M \geq 5.0$	• Extremely high-speed flow • High kinetic energy causes significant aerodynamic heating • Gas dissociates and ionizes • Example: Re-entry of space vehicle, ICBMs

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9. Derive stagnation temperature and pressure relation.

- **Given:** A flowing fluid with static properties (T, P) and velocity V. The fluid is brought to rest isentropically.
- **To Find:** The relationship between stagnation properties (T_0, P_0) and static properties (T, P, M).
- **Assumptions:**
  1. Steady, one-dimensional flow
  2. Perfect gas with constant specific heats
  3. Adiabatic and reversible (isentropic) deceleration process

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Part A: Derivation of Stagnation Temperature (T_0)

Step 1: Apply Steady Flow Energy Equation (SFEE)
For an adiabatic process (no heat transfer) with no work done, the SFEE states that stagnation enthalpy (h_0) is constant.

$$h_0 = h + \frac{V^2}{2}$$

For a perfect gas, $h = c_p T$, so:

$$c_p T_0 = c_p T + \frac{V^2}{2}$$

where T_0 is the stagnation temperature.

Step 2: Rearrange the equation

$$c_p T_0 - c_p T = \frac{V^2}{2}$$
$$c_p (T_0 - T) = \frac{V^2}{2}$$
$$T_0 - T = \frac{V^2}{2c_p}$$
$$\frac{T_0}{T} = 1 + \frac{V^2}{2c_p T}$$

Step 3: Express in terms of Mach Number
We know:

- Speed of sound: $a = \sqrt{kRT}$
- Mach number: $M = \frac{V}{a}$

- For a perfect gas: $c_p = \frac{kR}{k-1}$

Substitute c_p and a:

$$\frac{V^2}{2c_pT} = \frac{V^2}{2\left(\frac{kR}{k-1}\right)T}$$

$$\frac{V^2}{2c_pT} = \frac{V^2(k-1)}{2kRT}$$

Since $a^2 = kRT$, then $\frac{V^2}{kRT} = \left(\frac{V}{a}\right)^2 = M^2$:

$$\frac{V^2}{2c_pT} = \frac{k-1}{2}M^2$$

Step 4: Final Relation for Stagnation Temperature

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

$$\boxed{T_0 = T\left[1 + \frac{k-1}{2}M^2\right]}$$

Part B: Derivation of Stagnation Pressure (P_0)

Step 1: Apply Isentropic Relation

For an isentropic process, the relationship between pressure and temperature is:

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}}$$

Step 2: Substitute Temperature Ratio

Substitute the temperature ratio derived above:

$$\frac{P_0}{P} = \left[1 + \frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}$$

Step 3: Final Relation for Stagnation Pressure

$$\boxed{P_0 = P\left[1 + \frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}}$$

Summary of Final Results:

Property	**Relation**
Stagnation Temperature	$T_0 = T\left[1 + \frac{k-1}{2}M^2\right]$

Property	Relation
Stagnation Pressure	$P_0 = P\left[1 + \frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}$

where:

- T_0, P_0 = Stagnation (total) properties
- T, P = Static properties
- M = Mach number
- k = Specific heat ratio $\left(\frac{c_p}{c_v}\right)$

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