GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – MID TERM – S26 – ANSWER BANK

Subject Name & Code:

KINEMATICS AND THEORY OF MACHINES- BE04000171

Module 1: Fundamentals of Mechanisms

1. State the difference between machine and structure.
(2-3 Marks)

Feature	**Machine**	**Structure**
Primary Function	To transmit or modify **motion and force**.	To **support** loads and withstand forces.
Relative Motion	Consists of parts (**kinematic links**) with **relative motion** between them.	Consists of parts rigidly fixed, with **no relative motion**.
Example	Internal combustion engine, Lathe machine.	Bridge, Building frame, Crane structure.

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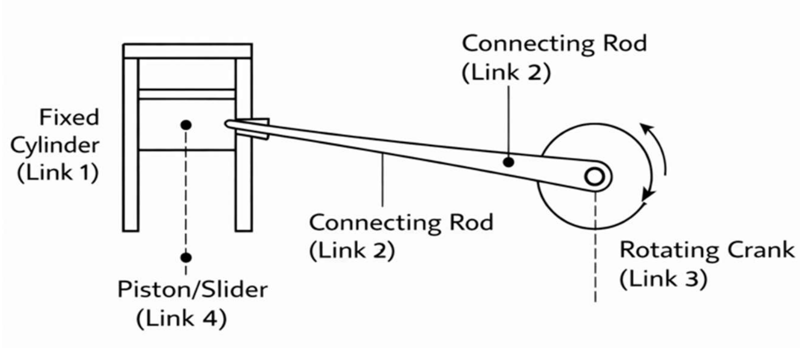
2. What is inversion? List inversion of single slider crank mechanism and explain any one in detail with neat sketch.
(5 Marks)

- **Inversion:** It is the process of obtaining different mechanisms by **fixing different links** of a kinematic chain, one at a time. The relative motions of the links remain the same, but their absolute motions (as seen from the fixed link) change, resulting in mechanisms with different applications.
- **Inversions of Single Slider Crank Mechanism:**
    1. **First Inversion (Link 2 - Connecting Rod fixed):** Whitworth Quick Return Mechanism, Rotary Engine.
    2. **Second Inversion (Link 1 - Cylinder fixed):** Reciprocating Engine (IC Engine), Compressor.
    3. **Third Inversion (Link 3 - Crank fixed):** Oscillating Cylinder Engine.
    4. **Fourth Inversion (Link 4 - Slider fixed):** Hand Pump, Crank and Slotted Lever Mechanism.
- **Detailed Explanation: Second Inversion - IC Engine (Reciprocating Engine)**
    - In this inversion, the **cylinder (link 1)** is fixed.
    - The **crank (link 3)** rotates about a fixed point.
    - The **connecting rod (link 2)** transmits the motion from the piston to the crank.
    - The **piston (link 4)** acts as a slider, reciprocating within the fixed cylinder.
    - **Working:** The pressure of expanding gases from combustion pushes the piston (slider). This reciprocating motion is converted into rotary motion of the crank by the connecting rod. This is the most common inversion used in

automobile engines.

Diagram:



IC Engine: Second Inversion of Single Slider Crank Mechanism

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3. Define the following terms:
(2-3 Marks Each)

- **(1) Lower pair:** A pair is called a lower pair when the two elements have **surface contact** when in motion. The nature of contact makes them more resistant to wear.
  - *Examples:* Pin joint in a hinge, sliding motion in a dovetail joint, shaft rotating in a bush.
- **(2) Higher pair:** A pair is called a higher pair when the two elements have **point or line contact** when in motion. The contact stresses are higher compared to lower pairs.
  - *Examples:* Ball bearing (point contact), Cam and follower (line contact), meshing of gear teeth (line contact).
- **(3) Successfully constrained motion:** This is a type of constrained motion where the constraint is **not achieved solely by the pairing of the elements**, but is aided by some external means, like the effect of gravity, spring force, or centrifugal force.
  - *Examples:* Footstep bearing where shaft is constrained by its own weight, valve in an engine kept on its seat by a spring.
- **(4) Completely constrained motion:** This is a type of constrained motion where the **relative motion between the two elements of a pair is constrained to a definite direction**, irrespective of the direction of the applied force. This is achieved by the geometry of the pairing itself.
  - *Examples:* Rectilinear motion of a piston and cylinder, motion of a square bar in a square hole.

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4. What is degree of freedom in mechanism? Explain Grubler's criteria.
(5 Marks)

- **Degree of Freedom (DOF):** The degree of freedom of a mechanism is the **number of independent inputs (or parameters) required to define the configuration or position of all its links** at any given instant. It is essentially the number of independent relative motions the mechanism can have.
- **Grubler's Criteria:** Grubler's criterion is a formula used to determine the **degree of freedom (n) of a planar mechanism** (a mechanism whose motion is confined to a single plane). The formula is derived from the basic DOF equation and applies to mechanisms with **lower pairs only** and **one degree of freedom**.

The general DOF equation for a planar mechanism is:

$$n = 3(l - 1) - 2j - h$$

Where:

- n = Degree of freedom
- l = Number of links
- j = Number of binary joints (lower pairs)
- h = Number of higher pairs

Grubler's Criterion is a special case of this equation for mechanisms that have:

1. Only **lower pairs** ($h = 0$).
2. A degree of freedom of **1** ($n = 1$).

Substituting $h = 0$ and $n = 1$ into the general equation:

$$1 = 3(l - 1) - 2j$$
$$3l - 3 - 2j - 1 = 0$$
$$3l - 2j - 4 = 0$$

This equation ($3l - 2j - 4 = 0$) is known as **Grubler's Criterion**. It gives the relationship between the number of links and the number of joints for a **constrained (DOF=1) planar mechanism with lower pairs**.

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5. Compare and discuss: kinematic link, kinematic pair, and kinematic chain. (5 Marks)

Feature	**Kinematic Link (or Element)**	**Kinematic Pair**	**Kinematic Chain**
Definition	A **resistant body** (or an assembly of parts rigidly connected) that forms part of a machine and has relative motion with respect to other parts.	A **joint** formed when two kinematic links are **connected in such a way that their relative motion is constrained**.	An **assembly of kinematic links** interconnected by kinematic pairs, where the relative motion between the links is **constrained**.
Constituents	A single part (e.g., a crank) or an assembly (e.g., an engine piston with rings).	Consists of **two links** and the nature of their contact (surface, point, or line).	Consists of **multiple links and multiple pairs**.
Analogy	It is like a **word** in a sentence.	It is like the **grammatical connection** between two words.	It is like a **complete clause or phrase** formed by connecting words.
Example	Crank, connecting rod, piston, cylinder, flywheel.	Crank and bearing (turning pair), Piston and cylinder (sliding pair).	Four-bar chain, Slider-crank chain.
Relation	The fundamental building block.	Formed by connecting two links.	Formed by connecting multiple links with multiple pairs.

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6. What are usual types of joints in a mechanism?
(2-3 Marks)
The usual types of joints (or kinematic pairs) in a mechanism, based on the type of contact and relative motion, are:

1. **Turning Pair (Revolute Joint):** When two links are connected such that one can only rotate or revolve about a fixed axis of the other. *Example: Elbow joint, hinge.*
2. **Sliding Pair (Prismatic Joint):** When two links are connected such that one can only slide along a fixed axis relative to the other. *Example: Piston and cylinder.*
3. **Rolling Pair:** When two links are connected such that one rolls over the other. *Example: Ball bearings, wheels on a track.*
4. **Screw Pair (Helical Joint):** When two links are connected such that one has both rotary and linear motion relative to the other, with a definite relationship between them. *Example: Nut and bolt.*
5. **Spherical Pair (Globular Joint):** When two links are connected such that one can swivel in any direction about a fixed point on the other. *Example: Ball and socket joint in the human shoulder.*

Gear and Gear Train

1. What are epicyclic gear trains? State its special advantages.
(5 Marks)

- **Epicyclic Gear Trains:** An epicyclic (or planetary) gear train is a system of gears in which one or more gears **(planet gears) rotate about a central axis**, just like planets in the solar system. These planet gears are carried on a rotating **arm (or carrier)** and mesh with a central **sun gear** and/or an outer **ring gear (annulus)** . The key characteristic is that the axes of some of the gears have **relative motion** (they revolve around the central axis).
- **Special Advantages:**
  - **High Speed Reduction in Compact Space:** They can achieve large speed reductions (or increases) in a small, lightweight package compared to simple or compound gear trains. This is their most significant advantage.
  - **High Torque-to-Weight Ratio:** Due to multiple planet gears sharing the load, they can transmit very high torques relative to their size.
  - **Coaxial Arrangement:** The input and output shafts are coaxial (aligned on the same axis), which simplifies mechanical design.
  - **Differential Action:** They can sum or split torques and speeds, which is essential in applications like automobile differentials.
  - **Higher Efficiency:** Despite their complexity, they are generally very efficient (e.g., 97% per stage).

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2. Explain simple, compound and reverted gear trains.
(5 Marks)

Feature	**Simple Gear Train**	**Compound Gear Train**	**Reverted Gear Train**
Definition	A gear train where **each shaft carries only one gear**. Motion is transmitted from one gear to the next.	A gear train where **at least one shaft carries two or more gears** (compound gear). This allows for a greater speed reduction in a smaller space.	A special type of compound gear train where the **first and last gears are coaxial** (input and output shafts are on the same axis).
Arrangement	Gears are arranged in series on separate shafts.	Gears are arranged in series, but intermediate shafts carry multiple gears fixed together.	The gear train is designed so that the distance between the centers of the first and last gears is the same, allowing their shafts to be aligned.
Advantage/ Use	Simple and easy to understand. Limited speed reduction.	Provides **large speed reductions** in a compact space. Widely used in machine tools and gearboxes.	Used where space is limited and the input and output shafts must be **coaxial**, e.g., in automobile gearboxes and clocks.
Condition	Speed ratio is independent of gear sizes.	Speed ratio depends on the teeth of all gears.	The center distances of all gear meshes must be **equal** for the shafts to be reverted.

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3. What is law of gearing? Derive the equation for sliding velocity in gears. (Do not Attempt- Try To Do by your Prompt)
(10 Marks)

- **Law of Gearing:** The law of gearing states that for two gears to maintain a **constant angular velocity ratio** during meshing, the **common normal** (line of action) at the point of contact must always **pass through a fixed point** (the pitch point) on the line joining the centers of the two gears.

In other words, to avoid fluctuations in the speed ratio (and thus vibration and noise), the shape of the gear tooth profiles (involute profile) must be such that this condition is satisfied.

- **Derivation of Sliding Velocity:**
    - **Given:**
        - Two gears 1 and 2 with angular velocities ω_1 and ω_2.
        - Radii of the base circles: r_{b1} and r_{b2}.
        - Point of contact is K.
        - KP is the distance from the pitch point P to the point of contact along the line of action (path of contact).
    - **Concept:** When two gear teeth are in mesh, there is both rolling and sliding action between them. The velocities of the two tooth surfaces along the common tangent at the point of contact are different. This difference is the **sliding velocity**.
    - **Diagram (Conceptual):** Imagine two gears meshing. At a point of contact K, the velocity of a point on gear 1 is perpendicular to the radius from its center O_1 to K. The same for gear 2. The components of these velocities along the common tangent are what we compare.
    - **Derivation Steps:**
        1. Let v_s be the sliding velocity.
        2. The velocity of point K on gear 1 relative to the point of contact along the common tangent is $v_1 = \omega_1 \times$ (distance from O_1 to the tangent point). A simpler way is to use the concept that the sliding velocity is the difference in velocities along the line of action? No, it's along the *tangent*.
        3. We can find the velocity of the point of contact along the common tangent for each gear. A standard result from gear theory is that the sliding velocity is proportional to the distance of the contact point from the pitch point.
        4. Consider the velocities of the two tooth profiles at the contact point K. These velocities can be resolved into two components: one along the common normal (line of action) and one along the common tangent. The components along the common normal must be equal (to maintain contact). The difference in the tangential components is the sliding velocity.
        5. From the geometry of involute gears, it can be shown that the tangential component of velocity for a point on a gear is $\omega \times$ (length of the perpendicular from the gear center to the tangent). This is complex to derive in text.
        6. A more straightforward derivation uses the concept of angular velocities and the distance from the pitch point. The linear velocity of a point on gear 1 along the line of action is $\omega_1 \times O_1K$? No, that's not correct for the tangential component.
        7. **Standard Result:** The sliding velocity v_s is the product of the angular

velocity difference and the distance from the pitch point.

$$v_s = (\omega_1 + \omega_2) \times q$$

Where:

- ω_1, ω_2 are the absolute angular velocities of gear 1 and 2.
- q is the distance of the point of contact from the pitch point P measured along the line of action.

8. **Proof Outline:** Let K be a point on the path of contact at a distance q from the pitch point P. The velocity of sliding is the difference between the tangential components of the velocities of the two teeth at K. These tangential components are proportional to the angular velocities and the distance from the respective centers to the tangent line. After trigonometric manipulation and using the property that O_1P and O_2P are the pitch circle radii, the expression simplifies to $(\omega_1 + \omega_2)q$. (The derivation is lengthy and involves geometry of the involute, but the final formula is the key takeaway).

- **Final Expression:**

$$\boxed{v_s = (\omega_1 + \omega_2) \cdot q}$$

Conclusion: The sliding velocity is zero at the pitch point ($q = 0$) and increases as the point of contact moves away from the pitch point along the line of action.

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4. Write a short note on the differential gear box.
(5 Marks)

A **differential gear box** is an epicyclic gear train used primarily in automobiles. Its function is to:

1. Transmit power from the engine to the two driving wheels.
2. Allow the two driving wheels to rotate at **different speeds** when the vehicle takes a turn.

- **Why is it needed?** When a car turns, the outer wheel has to travel a greater distance than the inner wheel. If both wheels were rigidly connected to a single axle, they would have to rotate at the same speed, causing one wheel to skid or slip, leading to tire wear and poor handling. The differential solves this by allowing speed differentiation while still transmitting torque to both wheels.
- **Main Components:**
    - **Crown Wheel (Ring Gear):** Receives power from the engine via the propeller shaft and bevel pinion.
    - **Differential Case (Carrier):** Rigidly attached to the crown wheel. It acts as the **arm** in the epicyclic train.
    - **Planet Pinions:** Two or more small bevel gears mounted on a cross-pin inside the differential case. They rotate with the case and can also spin on their own axis.
    - **Side (Sun) Gears:** Two bevel gears, one on each end, splined to the axle shafts connected to the wheels. They mesh with the planet pinions.
- **Working Principle:**
    - **Straight Motion:** The crown wheel rotates the case. The case carries the planet pinions around. Since both wheels have equal resistance, the planet pinions do not spin on their own axis; instead, they lock the two side gears together, causing both axles and wheels to rotate at the same speed as the case.
    - **Turning Motion:** When the vehicle turns, the inner wheel encounters more

resistance. This resistance causes the planet pinions to **spin on their own axis** while also revolving around the sun gears. This spinning action allows the outer side gear (and hence the outer wheel) to rotate **faster** than the case, and the inner side gear to rotate **slower** than the case. The sum of the speeds of the two wheels is always twice the speed of the case.

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5. Numerical Problem (Epicyclic Gear Train) (Easy way is given by sir) (10 Marks)

- **Given:**
  - Arm C (carrier) rotates at $N_A = 150$ rpm (anticlockwise)
  - Gear A (Sun), $T_A = 36$
  - Gear B (Planet), $T_B = 45$
  - Module is same for all gears.
- **(I) To Find:** Speed of gear B (N_B) when gear A is fixed ($N_A = 0$).
- **Formula & Solution:**

We use the **tabular method** for epicyclic gear trains.

Step 1: Assume the arm is fixed.

- Give gear A (+x) revolutions (say, anticlockwise).
- The gear B will rotate in the opposite direction. For two meshing gears, the speed ratio is inversely proportional to the number of teeth.

$$\frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

The negative sign indicates opposite direction.
So, if $N_A = +x$, then $N_B = -\frac{T_A}{T_B} \cdot x = -\frac{36}{45}x = -0.8x$.

Step 2: Create the table of motions.

Step No.	Conditions of Motion	Arm C	Gear A ($T_A =$ 36)	Gear B ($T_B =$ 45)
1.	Arm fixed, A gets +x rev	0	$+x$	$-\frac{36}{45}x$
2.	Add +y rev to the whole train	$+y$	$+y$	$+y$
3.	**Total Revolutions**	**+y**	**x + y**	$\mathbf{y - 0.8x}$

Step 3: Apply given conditions to find x and y.

- Arm C rotates at $+150$ rpm (anticlockwise is taken as positive).

$$y = 150$$

- Gear A is fixed.

$$x + y = 0 \implies x + 150 = 0 \implies x = -150$$

Step 4: Calculate speed of gear B.

- $N_B = y - 0.8x = 150 - 0.8(-150) = 150 + 120$
- $N_B = 270$ rpm.
- Since the result is positive, the direction is the same as the arm's initial direction.

- **Final Answer (I):**

$$\boxed{N_B = 270 \text{ rpm (Anticlockwise)}}$$

- **(II) To Find:** Speed of gear B (N_B) when gear A makes $N_A = -300$ rpm (clockwise is negative).
- **Solution:**

We use the same table from Part (I). The total revolutions are:

  - Arm: y
  - Gear A: $x + y$
  - Gear B: $y - 0.8x$

Apply new conditions:

  - Arm rotates at $+150$ rpm.

$$y = 150$$

  - Gear A rotates at -300 rpm.

$$x + y = -300 \implies x + 150 = -300 \implies x = -450$$

Calculate speed of gear B:

  - $N_B = y - 0.8x = 150 - 0.8(-450) = 150 + 360$
  - $N_B = 510$ rpm.
  - The result is positive, indicating the direction is anticlockwise.

- **Final Answer (II):**

$$\boxed{N_B = 510 \text{ rpm (Anticlockwise)}}$$

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6. Numerical Problem (Reverted Epicyclic Gear Train) (Easy way is given by sir) (10 Marks)

- **Given:**
  - Arm A (carrier) makes $N_A = -100$ rpm (clockwise).
  - Gear B is fixed ($N_B = 0$).
  - $T_B = 75, T_C = 30, T_D = 90$.
  - Compound gear D-E is carried by the arm. Gear E meshes with B, Gear D meshes with C.
- **Diagram Interpretation:** Based on the image coordinates [338, 564, 519, 650] and standard reverted train construction:
  - Arm A is the central carrier.
  - Gear B (fixed) is the sun gear (likely outer/ring? or inner? We'll solve based on meshing).
  - Compound gear (D-E) is mounted on the arm. E meshes with fixed gear B, and D meshes with gear C.
  - Gear C is the output gear.

To Find: Speed and direction of gear C (N_C).

- **Step 1: Find T_E.**

  Since it's a **reverted** train, the center distances must be equal.
  - Center distance between B and E = Center distance between D and C.
  - Assuming all gears have the same module (m), center distance $= \frac{m}{2}(T_1 + T_2)$.

$$\frac{m}{2}(T_B + T_E) = \frac{m}{2}(T_D + T_C)$$

- $T_B + T_E = T_D + T_C$
- $75 + T_E = 90 + 30$
- $75 + T_E = 120$
- $T_E = 120 - 75 = 45$

- **Step 2: Tabular Method.**

Step A: Fix the arm and give +x revolutions to the arm's link? We give +x to gear E, as it's the first driver in the fixed-arm scenario.

- Let gear E be given +x revolutions (assume anticlockwise).
- Gear E meshes with gear B. The direction of B will be opposite.

$$N_B(\text{re E}) = -\frac{T_E}{T_B} \cdot x = -\frac{45}{75}x = -0.6x$$

- The compound gear D-E has gears D and E fixed on the same shaft. So, if E gets +x, then D also gets **+x** revolutions.
- Gear D meshes with gear C. The direction of C will be opposite to D.

$$N_C(\text{re D}) = -\frac{T_D}{T_C} \cdot N_D = -\frac{90}{30} \cdot x = -3x$$

Step B: Create the table.

(Let Arm = A, Gear B, Gear E, Gear D, Gear C)

Step	Motion	Arm A	Gear B (Fixed)	Gear E (on comp)	Gear D (on comp)	Gear C (Output)
1.	Arm fixed, E gets +x	0	$-0.6x$	$+x$	$+x$	$-3x$
2.	Add +y to whole train	$+y$	$+y$	$+y$	$+y$	$+y$
3.	**Total Revolutions**	$\mathbf{y}$	$\mathbf{y - 0.6x}$	$\mathbf{x + y}$	$\mathbf{x + y}$	$\mathbf{y - 3x}$

Step 3: Apply given conditions.

- Arm A rotates at -100 rpm.

$$y = -100$$

- Gear B is fixed.

$$y - 0.6x = 0 \implies -100 - 0.6x = 0 \implies -0.6x = 100 \implies x = -\frac{100}{0.6} = -\frac{1000}{6} = -166.67$$

Step 4: Calculate speed of gear C.

- $N_C = y - 3x = (-100) - 3(-166.67) = -100 + 500$
- $N_C = 400$ rpm.

- **Step 5: Determine direction.**
  The result is **positive**. Since we took anticlockwise as positive, and the arm was given a negative (clockwise) value, a positive result for C means it rotates in the **opposite direction** to the arm. Therefore, the direction is **Anticlockwise**.
- **Final Answer:**

$$\boxed{N_C = 400 \text{ rpm (Anticlockwise)}}$$

Belts and Chains

1. Define belt drive and list different types of belt drives.
(2-3 Marks)

- **Belt Drive:** A belt drive is a flexible power transmission element used to transfer power between two parallel shafts. It consists of a **flexible belt** (leather, rubber, etc.) passing over two **pulleys** mounted on the shafts. Power is transmitted by the **friction** between the belt and the pulleys.
- **Types of Belt Drives:**
    1. **Based on Belt Cross-section:**
        - **Flat Belt Drive:** Rectangular cross-section.
        - **V-Belt Drive:** Trapezoidal cross-section.
        - **Circular Belt (Rope) Drive:** Circular cross-section.
    2. **Based on Pulley Arrangement:**
        - **Open Belt Drive:** Shafts rotate in the **same direction**.
        - **Crossed Belt Drive:** Shafts rotate in the **opposite direction**.

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2. State different materials used for belts and their applications.
(2-3 Marks)

Material	Characteristics	Applications
Leather	Strong, flexible, high coefficient of friction. Requires maintenance.	Factory line shafts, agricultural machinery.
Rubber (Fabric / Cotton Duck)	Good friction, operates in moist conditions, less expensive.	General industrial power transmission, conveyors.
Balata (Gum)	Resistant to moisture and acids. Not for high temperatures >40°C.	Sawmills, chemical plants, mining.
Nylon / Polyester (Synthetic)	High strength, lightweight, durable, resistant to chemicals and temperature.	High-speed machinery, automotive engines (timing belts), textile machinery.

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3. Define slip and creep in belt drive.
(2-3 Marks)

- **Slip:** Slip is the **relative motion between the belt and the pulley** caused by insufficient friction to grip the pulley. It occurs over the entire arc of contact and results in a loss of power and a reduction in the velocity ratio. Slip is a **gross phenomenon** that can be prevented by proper tensioning.
- **Creep:** Creep is the **relative motion between the belt and the pulley** caused by the **uneven stretching** of the belt as it travels from the tight side (more stretch) to the slack side (less stretch). This differential strain causes the belt to slightly "creep" back on the pulley surface. Creep is a **local phenomenon** that is inevitable in a friction drive and causes a minor loss in velocity ratio.

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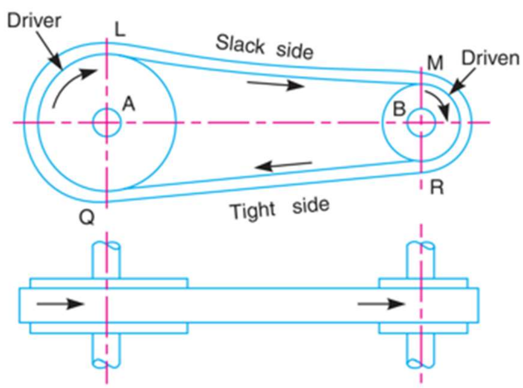
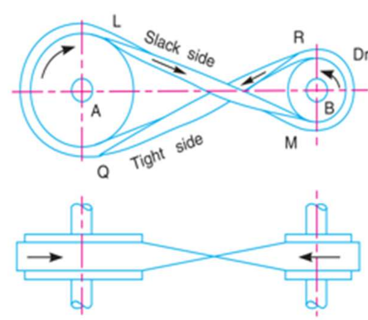
4. Compare open belt drive and crossed belt drive with a neat sketch.
(5 Marks)

Feature	**Open Belt Drive**	**Crossed Belt Drive**
Arrangement	Belts are arranged **without crossing**. The two shafts are parallel and rotate in the **same direction**.	Belts are arranged with a **cross (figure-eight)** . The two parallel shafts rotate in the **opposite direction**.
Angle of Lap	Smaller angle of lap, especially on the smaller pulley. This can lead to slip.	**Larger angle of lap** on both pulleys due to the crossing, which increases the power transmission capacity.
Wear	Belt experiences less wear as it is not twisted.	Belt experiences **more wear** due to twisting and rubbing against itself at the crossing point.
Applications	Most common drive, used in machine tools, fans, etc.	Used in specific applications like rope drives, textile machinery, and where opposite rotation is needed.

Diagram:





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5. Derive the expression for the velocity ratio of belt drive considering slip.
(5 Marks)

- **Derivation:**
  - **Step 1: Without Slip**
    Let d_1, d_2 = diameters of driver and driven pulleys.
    N_1, N_2 = speeds of driver and driven pulleys in rpm.
    If there is no slip, the linear speed of the belt is the same as the pulley surface speeds.

$$\text{Velocity of belt on driver} = \frac{\pi d_1 N_1}{60}$$

$$\text{Velocity of belt on driven} = \frac{\pi d_2 N_2}{60}$$

Equating them (assuming no slip and no thickness):

$$\frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60} \implies \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

This is the theoretical velocity ratio.

- **Step 2: Considering Slip**
  Slip is a loss of velocity due to insufficient friction. It is usually expressed as a percentage.
  Let s_1 = slip % between driver and belt.
  Let s_2 = slip % between belt and driven.
  Total slip $s = s_1 + s_2$? Not exactly additive. It's better to consider the effect step by step.
  - The velocity of the belt (v_b) is less than the speed of the driver pulley due to slip s_1.
  $$v_b = \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right)$$
  - The velocity of the driven pulley (v_2) is less than the speed of the belt due to slip s_2.
  $$\frac{\pi d_2 N_2}{60} = v_b\left(1 - \frac{s_2}{100}\right) = \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$
  - If the total slip S is considered as the net loss, we can use an approximation. The product of the two slip factors can be simplified if s_1 and s_2 are small. The total effective slip S is approximately $s_1 + s_2$. Therefore, the combined factor becomes $\left(1 - \frac{S}{100}\right)$.
- **Step 3: Final Expression**
  Cancelling $\frac{\pi}{60}$ from both sides:
  $$d_2 N_2 = d_1 N_1\left(1 - \frac{S}{100}\right)$$
  $$\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S}{100}\right)}$$

This is the velocity ratio considering the total percentage slip S.

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6. Derive the ratio of belt tensions $T_1/T_2 = e^{\mu\theta}$.
(7 Marks)
This derivation analyzes the forces on a small element of the belt as it is about to slip on the pulley.

- **Assumptions:**
  1. The belt is perfectly flexible and has no mass.
  2. The material obeys Hooke's law.
  3. The effect of centrifugal tension is neglected (or considered separately).
  4. The coefficient of friction μ is constant over the arc of contact.
- **Derivation:**

Step 1: Consider a small element of the belt.
Let a small element of the belt PQ subtend an angle $\delta\theta$ at the center of the pulley. The belt is on the point of slipping on the pulley. Let the tension at end P be T and at end Q be $T + \delta T$. The normal reaction from the pulley on the element is R, and the frictional force $F = \mu R$ acts

tangentially to resist motion.

Step 2: Resolve forces radially (equilibrium in the radial direction).

The radial components of the tensions provide the normal reaction.

- Component of T radially inward: $T\sin\left(\frac{\delta\theta}{2}\right)$
- Component of $T + \delta T$ radially outward? No, both are pulling the element toward the center. The correct radial equilibrium is:

$$R = T\sin\left(\frac{\delta\theta}{2}\right) + (T + \delta T)\sin\left(\frac{\delta\theta}{2}\right)$$

For very small $\delta\theta$, $\sin(\delta\theta/2) \approx \delta\theta/2$. Neglecting the product of small quantities ($\delta T \cdot \delta\theta$), we get:

$$R = 2T\left(\frac{\delta\theta}{2}\right) = T\delta\theta \qquad (1)$$

Step 3: Resolve forces tangentially (equilibrium in the tangential direction).

The forces in the tangential direction are the difference in tension and the frictional force. Since the belt is about to slip, friction is at its maximum μR. The frictional force acts to prevent the increase in tension.

$$(T + \delta T)\cos\left(\frac{\delta\theta}{2}\right) - T\cos\left(\frac{\delta\theta}{2}\right) = \mu R$$

For very small $\delta\theta$, $\cos(\delta\theta/2) \approx 1$. Therefore:

$$\delta T = \mu R \qquad (2)$$

Step 4: Combine equations.

Substitute the value of R from equation (1) into equation (2):

$$\delta T = \mu(T\delta\theta)$$
$$\frac{\delta T}{T} = \mu\,\delta\theta$$

Step 5: Integrate over the entire angle of lap.

As $\delta\theta \to 0$, we integrate over the total angle of lap θ (in radians). The tension increases from T_2 (slack side) to T_1 (tight side).

$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^\theta \mu\, d\theta$$
$$[\ln T]_{T_2}^{T_1} = \mu\theta$$
$$\ln T_1 - \ln T_2 = \mu\theta$$
$$\ln\left(\frac{T_1}{T_2}\right) = \mu\theta$$

- **Final Expression:**

$$\boxed{\frac{T_1}{T_2} = e^{\mu\theta}}$$

This is the ratio of belt tensions, considering only the friction between belt and pulley.

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7. Derive the expression for centrifugal tension in a belt.
(5 Marks)

- **Derivation:**

Step 1: Concept.
As the belt runs over the pulley, centrifugal force acts on the belt, throwing it outward. This creates an additional tension in the belt called centrifugal tension (T_c).
Step 2: Consider a small element of the belt.
Consider a small element of the belt AB of length $\delta l = r\delta\theta$, where r is the radius of the pulley and $\delta\theta$ is the small angle subtended.
Let m = mass of the belt per unit length (kg/m).
Mass of element = $m \cdot \delta l = mr\delta\theta$.
The belt is moving with a linear velocity v (m/s).
Step 3: Calculate the centrifugal force.
The centrifugal force F_c acting on the element is given by:

$$F_c = (\text{mass}) \times \frac{v^2}{r} = (mr\delta\theta) \cdot \frac{v^2}{r} = mv^2\delta\theta$$

This force acts radially outward.
Step 4: Relate centrifugal force to centrifugal tension.
This outward centrifugal force is balanced by the inward components of the centrifugal tension T_c acting at both ends of the element. The centrifugal tension T_c is constant throughout the belt (unlike T_1 and T_2, which are for power transmission).
For very small $\delta\theta$, the inward component is $2T_c \sin(\delta\theta/2) \approx 2T_c(\delta\theta/2) = T_c\delta\theta$.
Equating the outward centrifugal force to the inward tension component:

$$T_c\delta\theta = mv^2\delta\theta$$

- **Final Expression:**

$$\boxed{T_c = mv^2}$$

Conclusion: Centrifugal tension depends only on the mass per unit length of the belt and the square of its velocity. It is independent of the radius of the pulley. The maximum tension in the belt (T_{max}) is the sum of the tight side tension (T_1) and the centrifugal tension (T_c), i.e., $T_{max} = T_1 + T_c$.

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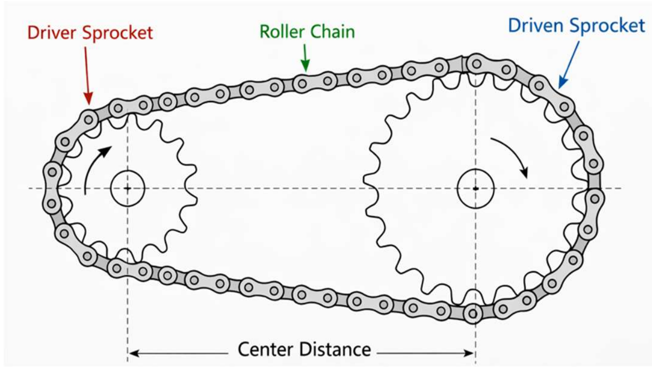
8. Explain chain drive with neat sketch and list its advantages over belt drive. (5 Marks)

- **Chain Drive:** A chain drive is a form of flexible power transmission where a **chain** (a series of interconnected links) meshes with **toothed sprockets** on the driving and driven shafts. It combines the features of a belt drive (flexible) and a gear drive (positive engagement).
- **Diagram Description:**
  - A small driver sprocket (with teeth) connected to a larger driven sprocket.
  - A roller chain wraps around both sprockets, with its rollers engaging in the sprocket teeth.
  - The centers of the sprockets are at a fixed distance (center distance).
  - The direction of rotation is the same for both sprockets (like an open belt drive).

Diagram:



- **Advantages over Belt Drive:**
    1. **No Slip, Constant Velocity Ratio:** Due to the positive engagement between chain and sprocket, there is no slip or creep, ensuring a positive and accurate velocity ratio.
    2. **High Power Transmission:** Can transmit more power than belts of comparable width because the load is distributed over many teeth.
    3. **Compact:** Requires less width than a belt drive for the same power transmission.
    4. **High Efficiency:** Efficiency is very high, often up to 98-99%.
    5. **Ability to Operate in Adverse Conditions:** Can operate in wet, oily, and dusty environments where belt drives would fail.
    6. **No Initial Tension Required:** Unlike belts, chains do not require high initial tension to operate, reducing bearing loads.

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9. Explain different types of chains used in chain drives.
(5 Marks)

Based on their construction and application, chains are classified into three main types:

1. **Hoisting and Hauling Chains:**
    - **Construction:** These are simple chains with oval or circular links. They may be welded or have studs to maintain shape.
    - **Types:**
        - **Coil Chain:** Links are formed from oval sections.
        - **Stud-Link Chain:** Has a stud across the middle of the link to prevent kinking and increase strength.
    - **Application:** Used in cranes, elevators, and mooring of ships for low-speed heavy lifting.
2. **Conveyor Chains:**
    - **Construction:** Similar to hoisting chains but with attachments or special link plates designed to carry or push materials. They are usually made of malleable cast iron.
    - **Types:**
        - **Detachable Chain:** Links can be easily detached and reattached.
        - **Closed-End Pintle Chain:** Links have a closed end and are connected by pins.
    - **Application:** Used for moving materials in conveyor systems at moderate speeds, such as in agricultural machinery (e.g., hay balers) and assembly lines.
3. **Power Transmission (Driving) Chains:**
    - **Construction:** Precisely engineered to run on toothed sprockets at high speeds. They are designed for efficient power transmission.
    - **Types:**

- **Block Chain (Bush Roller Chain):** A simple, inexpensive type with blocks and pins.
- **Roller Chain (Bush Roller Chain):** The most common type. It consists of inner links (with rollers and bushes), outer links (with pins), and pins. The rollers reduce friction with the sprocket teeth.
- **Silent Chain (Inverted Tooth Chain):** Has toothed link plates that engage with the sprocket teeth without rollers, providing very smooth and quiet operation at high speeds.

- **Application:** Widely used in motorcycles, bicycles, industrial machinery, camshaft drives, and conveyor systems.

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10. Numerical Problem (Belt Drive Power Transmission) (5 Marks)

- **Given:**
  - Pulley diameter $d = 600$ mm $= 0.6$ m
  - Speed $N = 200$ rpm
  - Coefficient of friction $\mu = 0.25$
  - Angle of lap $\theta = 160^\circ = 160 \times \frac{\pi}{180} = \frac{8\pi}{9}$ rad ≈ 2.7925 rad
  - Maximum tension $T_{max} = 2500$ N. (Assume this is the tight side tension T_1, and centrifugal tension is negligible unless stated otherwise).
- **To Find:** Power transmitted (P).
- **Formula:**
  1. Velocity of belt: $v = \frac{\pi d N}{60}$
  2. Tension ratio: $\frac{T_1}{T_2} = e^{\mu\theta}$
  3. Power: $P = (T_1 - T_2) \times v$
- **Solution:**

Step 1: Calculate belt velocity.

$$v = \frac{\pi \times 0.6 \times 200}{60} = \frac{\pi \times 120}{60} = 2\pi \approx 6.283 \text{ m/s}$$

Step 2: Calculate tension ratio.

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.25 \times 2.7925} = e^{0.698125}$$

Using $e^{0.698} \approx 2.009$ (or from calculator $e^{0.698125} \approx 2.01$).
So, $\frac{T_1}{T_2} \approx 2.01$.

Step 3: Find T_2.
Given $T_1 = 2500$ N.

$$T_2 = \frac{T_1}{2.01} = \frac{2500}{2.01} \approx 1243.78 \text{ N}$$

Step 4: Calculate power transmitted.

$$P = (T_1 - T_2) \times v = (2500 - 1243.78) \times 6.283$$
$$P = (1256.22) \times 6.283 \approx 7891.5 \text{ W}$$

- **Final Answer:**

$$\boxed{P \approx 7.89 \text{ kW}}$$

11. Numerical Problem (Chain Drive)
(5 Marks)

- **Given:**
  - Driver speed $N_1 = 360$ rpm
  - Driven speed $N_2 = 120$ rpm
  - Teeth on driver $T_1 = 10$
  - Pitch radius of driven sprocket $r_2 = 250$ mm
  - Center distance $C = 400$ mm
- **To Find:**
  1. Number of teeth on driven sprocket (T_2).
  2. Pitch (p) of the chain.
  3. Length of the chain (L).
- **Formula:**
  1. Velocity ratio: $\frac{N_2}{N_1} = \frac{T_1}{T_2}$ (for chain drives with no slip).
  2. Pitch circle diameter (PCD) relation: $\sin\left(\frac{180^\circ}{T}\right) = \frac{p}{D}$, or $D = \frac{p}{\sin(180^\circ/T)}$.
     Therefore, pitch radius $r = \frac{p}{2\sin(180^\circ/T)}$.
  3. Length of chain (for an open drive, approximate formula): $L = 2C + \frac{D_2+D_1}{2} + \frac{(D_2-D_1)^2}{4C}$, where D_1, D_2 are PCDs.
- **Solution:**

Part 1: Find T_2.

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \implies \frac{120}{360} = \frac{10}{T_2}$$
$$\frac{1}{3} = \frac{10}{T_2} \implies T_2 = 30$$

Answer 1: $\boxed{T_2 = 30 \text{ teeth}}$

Part 2: Find the pitch p.

We know the pitch radius for the driven sprocket $r_2 = 250$ mm.
The formula for pitch radius is:

$$r_2 = \frac{p}{2\sin\left(\frac{180^\circ}{T_2}\right)}$$
$$250 = \frac{p}{2\sin\left(\frac{180^\circ}{30}\right)} = \frac{p}{2\sin(6^\circ)}$$
$$\sin(6^\circ) \approx 0.1045$$
$$250 = \frac{p}{2 \times 0.1045} = \frac{p}{0.209}$$
$$p = 250 \times 0.209 = 52.25 \text{ mm}$$

Answer 2: $\boxed{p \approx 52.25 \text{ mm}}$

Part 3: Find the length of the chain L.

First, find the PCD of both sprockets (D_1, D_2).

- $D_2 = 2 \times r_2 = 500$ mm.
- For driver sprocket: $D_1 = \frac{p}{\sin(180^\circ/T_1)} = \frac{52.25}{\sin(18^\circ)}$.

$$\sin(18°) \approx 0.3090$$

$D_1 = \frac{52.25}{0.3090} \approx 169.09$ mm.

Now, use the length formula. It is usually expressed in terms of pitches, but we can calculate the length in mm first. The formula $L = 2C + \frac{\pi(D_2+D_1)}{2} + \frac{(D_2-D_1)^2}{4C}$ is an approximation.

$$L = 2(400) + \frac{\pi(500 + 169.09)}{2} + \frac{(500-169.09)^2}{4 \times 400}$$

$$L = 800 + \frac{\pi \times 669.09}{2} + \frac{(330.91)^2}{1600}$$

$$L = 800 + \frac{2102.2}{2} + \frac{109501.6}{1600}$$

Wait, $\pi \times 669.09 \approx 2101.9$. Dividing by 2 gives 1050.95.

$$L = 800 + 1050.95 + \frac{109501.6}{1600}$$

$$L = 800 + 1050.95 + 68.44$$

$$L \approx 1919.39 \text{ mm}$$

- **Final Answer 3:**

$$\boxed{L \approx 1919.4 \text{ mm}}$$

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