Subject Name & Code:

PHYSICS- BE01R00021

MODULE: QUANTUM MECHANICS

Q-1: Write a note on Heisenberg uncertainty principle and its significance.

Answer:

Statement:
According to Werner Heisenberg (1927), it is impossible to determine simultaneously both the position and momentum of a microscopic particle with absolute accuracy. The product of the uncertainties in position (Δx) and momentum (Δp) is at least of the order of ħ/2:

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}, \text{where } \hbar = \frac{h}{2\pi}$$

Significance:

- It is a fundamental result of quantum mechanics, arising from the wave-particle duality.
- It rejects the classical idea of deterministic trajectories for particles.
- Explains why electrons cannot exist inside the nucleus (see Q3).
- Leads to the concept of zero-point energy in potential wells.
- Forms the basis for quantum non-locality and the probabilistic interpretation of wave functions.

Q-2: Write a note on probability density and normalization of wave function.

Answer:

Probability Density (P(x,t)):
For a wave function ψ(x,t), the probability density is:

$$P(x,t) = |\psi(x,t)|^2 = \psi^*(x,t)\psi(x,t)$$

It gives the probability per unit length (in 1D) of finding the particle at position x at time t.

Normalization Condition:
Since the particle must be found somewhere in space:

$$\int_{-\infty}^{+\infty} |\, \psi(x,t)\, |^2 \ dx = 1$$

Significance:

- Normalization makes the probabilistic interpretation consistent.
- A non-normalized ψ can be scaled by a constant factor to satisfy the condition.
- For bound states, normalization is essential to compute expectation values.

Q-3: Prove that a free electron cannot exist inside a nucleus using the Heisenberg uncertainty principle.

Answer:

Given:
Nucleus radius ≈ 10^{-14} m. If an electron is inside the nucleus, its position uncertainty Δx ≤ 10^{-14} m.

Solution:
From Δx·Δp ≥ ħ/2, we get:

$$\Delta p \geq \frac{\hbar}{2\Delta x} = \frac{1.054 \times 10^{-3}}{2 \times 10^{-14}} \approx 5.27 \times 10^{-21}\ \mathrm{kg \cdot m/s}$$

If we assume Δp ≈ p (minimum momentum), then kinetic energy:

$$E = \frac{p^2}{2m} = \frac{(5.27 \times 10^{-21})^2}{2 \times 9.11 \times 10^{-3}} \approx 1.52 \times 10^{-11}\ \mathrm{J}$$

Converting to eV: $1.52 \times 10^{-11}/1.6 \times 10^{-1}$ ≈ 95 MeV

Conclusion:
This energy is much higher than typical nuclear binding energies (~8 MeV/nucleon). Hence, an electron cannot be confined inside the nucleus; if forced, it would escape immediately. This supports the existence of neutrons and protons as nuclear constituents.

Q-4: Derive Schrödinger wave equation.

Answer:

Time-dependent form (1D):

Starting from classical energy: $E = \frac{p^2}{2m} + V(x,t)$

Using de Broglie relations: $E \to i\hbar\frac{\partial}{\partial t}, p \to -i\hbar\frac{\partial}{\partial x}$

Operate on ψ(x,t):

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x,t)\psi$$

For time-independent case (V not a function of t):
Assume ψ(x,t) = φ(x) e^{-iEt/ħ}. Then:

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2} + V(x)\phi = E\phi$$

This is the **time-independent Schrödinger equation** (TISE).

Q-5:

What is Eigen value and Eigen function? Derive the expression for Eigen function and Eigen energy values for a particle in a infinite potential well of finite width.

Answer:

Definitions:

- **Eigenfunction $\varphi_n(x)$:** A solution to TISE that satisfies boundary conditions.
- **Eigenvalue E_n:** The corresponding energy.

Derivation for 1D infinite well (0 < x < L):
Potential: $V = 0$ inside, $V = \infty$ outside.
TISE: $-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2} = E\phi$

Solution: $\phi(x) = A\sin(kx) + B\cos(kx), k = \sqrt{2mE}/\hbar$

Boundary conditions: φ(0)=0 ⇒ B=0 ⇒ φ(x)=A sin(kx)
φ(L)=0 ⇒ sin(kL)=0 ⇒ kL = nπ, n=1,2,3,…

Thus $k_n = \frac{n\pi}{L}$ ⇒ **Eigenfunctions:**

$$\phi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \text{(normalized)}$$

Eigenenergies:

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2h^2}{8mL^2}$$

Q-6: Derive an expression for the energy Eigen values of a free particle.

Answer:

For free particle (V=0 everywhere), TISE:

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2} = E\phi \Rightarrow \frac{d^2\phi}{dx^2} = -k^2\phi, k = \sqrt{2mE}/\hbar$$

General solution: $\phi(x) = Ae^{ikx} + Be^{-i}$

No boundary conditions ⇒ any E ≥ 0 allowed ⇒ **continuous energy spectrum**:

$$E = \frac{\hbar^2k^2}{2m}$$

Thus, a free particle is not quantized in energy.

Q-7: Discuss the wave functions and probability density for particle in an infinite potential well for first three allowed states.

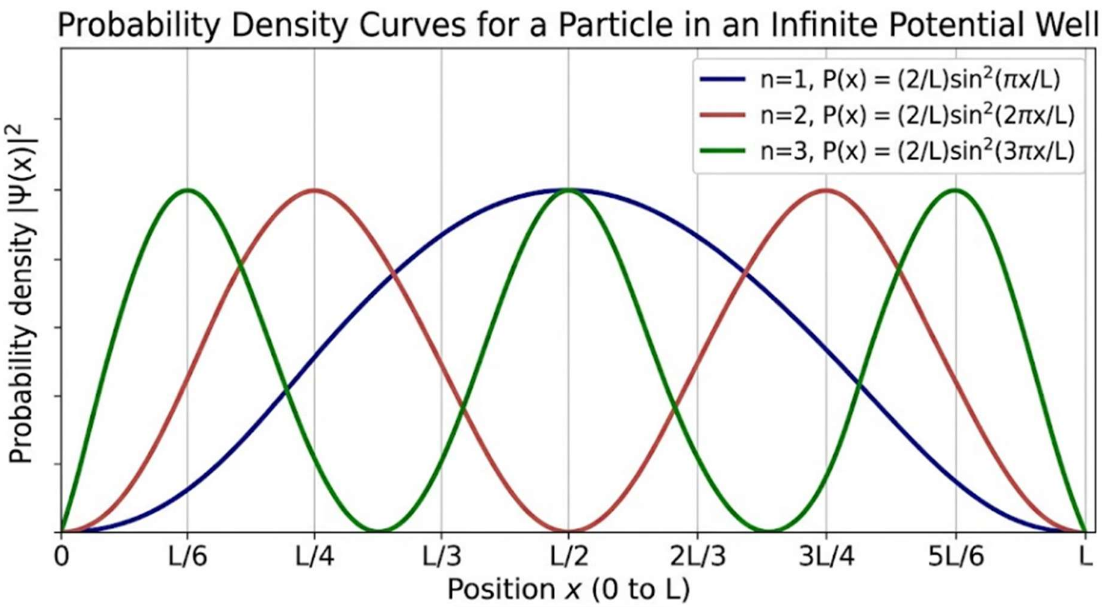
Answer:

n=1 (Ground state):
$\varphi_1(x) = \sqrt{(2/L)} \sin(\pi x/L)$
$P_1(x) = (2/L) \sin^2(\pi x/L)$ — maximum at x=L/2, zero at boundaries.

n=2 (First excited):
$\varphi_2(x) = \sqrt{(2/L)} \sin(2\pi x/L)$
$P_2(x) = (2/L) \sin^2(2\pi x/L)$ — maximum at x=L/4 and 3L/4, node at L/2.

n=3 (Second excited):
$\varphi_3(x) = \sqrt{(2/L)} \sin(3\pi x/L)$
$P_3(x) = (2/L) \sin^2(3\pi x/L)$ — maxima at L/6, L/2, 5L/6, nodes at L/3 and 2L/3.

Diagram:



Numerical:

Q-1:

Electrons moving with a velocity of 3.32×10^5 m/s if this velocity is measured with an inaccuracy of 0.53% then estimate the uncertainty in the position of an electron.

Given that: Velocity of the electron is $v = 3.32 \times 10^5$ m/s
Inaccuracy in the measurement of velocity = 0.53 %

Answer:

Given:
$v = 3.32 \times 10^5$ m/s
Inaccuracy in v = 0.53%
$m_e = 9.11 \times 10^{-31}$ kg
$\hbar = 1.054 \times 10^{-34}$ J·s

To Find: Δx

Formula: $\Delta x \cdot \Delta p \geq \hbar/2$, $\Delta p = m \cdot \Delta v$

Solution:
Δv = 0.53% of v = $(0.53/100) \times 3.32\times10^5 = 1759.6$ m/s
$\Delta p = 9.11\times10^{-31} \times 1759.6 \approx 1.603\times10^{-27}$ kg·m/s
$\Delta x \geq \hbar/(2\Delta p) = 1.054\times10^{-34} / (2 \times 1.603\times10^{-27}) \approx 3.287\times10^{-8}$ m

Final Answer:

$$\boxed{\Delta x \geq 3.29 \times 10^{-8}\ \text{m}}$$

Q-2:

In an experimental determination of displacement of an electron in 10^{-6} second is 3.6m. Calculate the uncertainty involved in the determination of position if the inherent error involved in the measurement of displacement of the electron in given time is 0.23%.

Given that: The displacement of an electron in 10^{-6} second = 3.6 m
The error in the displacement measurement = 0.23%

Answer:

Given:
Time = 10^{-6} s
Displacement = 3.6 m
Error in displacement = 0.23%
Uncertainty in position = ?

To Find: Δx

Solution:
This problem is unusual — displacement is macroscopic, but inherent error suggests uncertainty in measurement. In quantum context, if displacement is 3.6 m in 10^{-6} s, velocity ≈ 3.6×10^{6} m/s. Error in velocity ≈ same % as in displacement (since time fixed):

Δx = error in position = 0.23% of 3.6 m = (0.23/100) × 3.6 = 0.00828 m

Final Answer:

$$\boxed{8.28 \times 10^{-3}\ \text{m}}$$

Q-3:

The velocity of an electron confined in an infinite potential well is found to be 3×10^{4}m/s for the ground state. Calculate the velocity of the electron in first and second excited state.

Given that: Velocity of the electron in ground state v_{-} = 3 × 10^{4}m/s

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Answer:

Given:
v_1 (ground state) = 3×10^{4} m/s
Infinite well, $v \propto \sqrt{E_n}$, $E_n \propto n^2 \Rightarrow v_n \propto n$

To Find: v_2 (first excited), v_3 (second excited)

Formula: $v_n = n\cdot v_1$

Solution:
$v_2 = 2 \times 3\times10^{4} = 6\times10^{4}$ m/s
$v_3 = 3 \times 3\times10^{4} = 9\times10^{4}$ m/s

Final Answer:

$$\boxed{v_2 = 6 \times 10^{4}\ \text{m/s}, v_3 = 9 \times 10^{4}\ \text{m/s}}$$

Q-4:

An electron is bound in one dimensional potential well of width 0.18nm. Find the energy value in eV of the second excited state. Given that: Width of the potential well a = 0.18 nm = 0.18×10^{-9} m

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Answer:

Given:
$L = 0.18$ nm $= 0.18\times10^{-9}$ m
n = 3 (second excited state: n=1 ground, n=2 first excited, n=3 second excited)
$m_e = 9.11\times10^{-31}$ kg
$h = 6.626\times10^{-34}$ J·s
1 eV = 1.6×10^{-19} J

To Find: E_3 in eV

Formula: $E_n = \frac{n^2h^2}{8mL^2}$

Solution:
$E_3 = 9 \times (6.626\times10^{-34})^2 / [8 \times 9.11\times10^{-31} \times (0.18\times10^{-9})^2]$
$= 9 \times 4.39\times10^{-67} / [8 \times 9.11\times10^{-31} \times 3.24\times10^{-20}]$
Numerator: 3.951×10^{-66}
Denominator: $8 \times 9.11\times10^{-31} \times 3.24\times10^{-20} = 8 \times 2.95164\times10^{-50} = 2.3613\times10^{-49}$
E_3 (J) $= 3.951\times10^{-66} / 2.3613\times10^{-49} \approx 1.673\times10^{-17}$ J
E_3 (eV) $= 1.673\times10^{-17} / 1.6\times10^{-19} \approx 104.6$ eV

Final Answer:

$$\boxed{E_3 \approx 104.6 \text{ eV}}$$

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