

Subject Name & Code:

KINEMATICS AND THEORY OF MACHINES- BE04000171

(Disclaimer: The purpose of these AI-generated responses is just education and reference. Utilise them to grasp topics and structure, but always rewrite in your own words and double-check the content before submitting.)

Assignment – 4

Q-1: A Whitworth bolt with an angle of V-threads as 55° has a pitch of 6 mm and a mean diameter of 32 mm. the mean radius of the bearing surface where the nut is tightened is 20 mm. Determine the force required at the end of a 400 mm long spanner when the load on the bolt is 8 kN. The coefficient of friction for the nut and the bolt is 0.1 and for the nut and the bearing surface is 0.15.

Answer:

Given:

V-thread angle $2\beta = 55^\circ \rightarrow \beta = 27.5^\circ$

Pitch $p = 6$ mm, Mean diameter $d_m = 32$ mm $\rightarrow r_m = 16$ mm

Bearing mean radius $R = 20$ mm, Load $W = 8$ kN = 8000 N

Spanner length $L = 400$ mm = 0.4 m

Coefficient of friction: thread $\mu = 0.1$, collar $\mu_c = 0.15$

To Find: Force at spanner end P

Formula:

Torque to overcome thread friction:

$$T_1 = W \times r_m \times \frac{\mu \sec \beta + \tan \alpha}{1 - \mu \sec \beta \tan \alpha}$$

where $\tan \alpha = \frac{p}{\pi d_m}$ (helix angle)

Torque to overcome collar friction:

$$T_2 = \mu_c WR$$

Total torque $T = T_1 + T_2 = P \times L$

Solution:

$$\tan \alpha = \frac{p}{\pi d_m} = \frac{6}{\pi \times 32} = \frac{6}{100.53} = 0.05968 \Rightarrow \alpha = 3.416^\circ$$

$$\sec \beta = \sec 27.5^\circ = \frac{1}{\cos 27.5^\circ} = \frac{1}{0.887} = 1.127$$

$$\begin{aligned} \mu \sec \beta &= 0.1 \times 1.127 = 0.1127 \\ T_1 &= 8000 \times 0.016 \times \frac{0.1127 + 0.05968}{1 - (0.1127 \times 0.05968)} \\ T_1 &= 128 \times \frac{0.17238}{1 - 0.00673} = 128 \times \frac{0.17238}{0.99327} = 128 \times 0.1735 = 22.21 \text{ Nm} \\ T_2 &= 0.15 \times 8000 \times 0.02 = 24 \text{ Nm} \\ T &= 22.21 + 24 = 46.21 \text{ Nm} \\ P &= \frac{T}{L} = \frac{46.21}{0.4} = 115.5 \text{ N} \end{aligned}$$

Final Answer:

$$\boxed{115.5 \text{ N}}$$

Q-2: The number of teeth of spur gear is 30 and it rotates at 200 RPM. What will be its circular pitch and the pitch line velocity if it has a module of 2 mm?

Answer:

Given:

Number of teeth $Z = 30$, Speed $N = 200$ rpm, Module $m = 2$ mm

To Find: Circular pitch p_c , Pitch line velocity v

Formula:

$$p_c = \pi m, d = mZ, v = \frac{\pi dN}{60}$$

Solution:

$$\begin{aligned} p_c &= \pi \times 2 = 6.283 \text{ mm} \\ d &= 2 \times 30 = 60 \text{ mm} = 0.06 \text{ m} \\ v &= \frac{\pi \times 0.06 \times 200}{60} = \frac{37.699}{60} = 0.628 \text{ m/s} \end{aligned}$$

Final Answer:

$$\boxed{p_c = 6.283 \text{ mm}, v = 0.628 \text{ m/s}}$$

Q-3: The following data relate to two meshing gears velocity ratio = 1/3, module = 1mm, Pressure angle 20°, center distance= 200 mm. Determine the number of teeth and the base circle radius of the gear wheel.

Answer:

Given:

Velocity ratio $G = \frac{N_1}{N_2} = \frac{1}{3}$ (or gear ratio $\frac{Z_2}{Z_1} = 3$)

Module $m = 1$ mm, Pressure angle $\phi = 20^\circ$, Center distance $C = 200$ mm

To Find: Number of teeth Z_1, Z_2 , Base circle radius of gear wheel

Formula:

$$C = \frac{m(Z_1 + Z_2)}{2}, \frac{Z_2}{Z_1} = 3, r_b = r \cos \phi$$

Solution:

$$200 = \frac{1 \times (Z_1 + Z_2)}{2} \Rightarrow Z_1 + Z_2 = 400$$

$$Z_2 = 3Z_1 \Rightarrow Z_1 + 3Z_1 = 4Z_1 = 400 \Rightarrow Z_1 = 100, Z_2 = 300$$

$$r_2 = \frac{mZ_2}{2} = \frac{1 \times 300}{2} = 150 \text{ mm}$$

$$r_{b2} = r_2 \cos 20^\circ = 150 \times 0.9397 = 140.96 \text{ mm}$$

Final Answer:

$$\boxed{Z_1 = 100, Z_2 = 300, r_{b2} = 140.96 \text{ mm}}$$

Q-4: Each of the gears in a mesh has 48 teeth and a module of 8 mm. The teeth are of a 20° involute profile. The arc of contact is 2.25 times the circular pitch. Determine the addendum.

Answer:**Given:**

$Z_1 = Z_2 = 48$, Module $m = 8$ mm, Pressure angle $\phi = 20^\circ$

Arc of contact $= 2.25 \times p_c$

To Find: Addendum a

Formula:

$$p_c = \pi m, \text{Arc of contact} = \frac{\text{Path of contact}}{\cos \phi} = 2.25 p_c$$

$$\text{Path of contact} = \sqrt{(R_a)^2 - (R_b)^2} + \sqrt{(r_a)^2 - (r_b)^2} - C \sin \phi$$

$$\text{For equal gears: } R = r = \frac{mZ}{2}, R_b = r \cos \phi, R_a = r + a$$

Solution:

$$p_c = \pi \times 8 = 25.133 \text{ mm}$$

$$\begin{aligned}\text{Path of contact} &= \text{Arc of contact} \times \cos \phi = (2.25 \times 25.133) \times 0.9397 \\ &= 56.55 \times 0.9397 = 53.14 \text{ mm}\end{aligned}$$

$$r = \frac{8 \times 48}{2} = 192 \text{ mm}, r_b = 192 \times 0.9397 = 180.42 \text{ mm}$$

$$C = 2r = 384 \text{ mm}$$

$$\text{Path of contact} = 2\sqrt{(r+a)^2 - r_b^2} - C \sin \phi$$

$$53.14 = 2\sqrt{(192+a)^2 - (180.42)^2} - 384 \times 0.342$$

$$53.14 = 2\sqrt{(192+a)^2 - 32551.4} - 131.33$$

$$184.47 = 2\sqrt{(192+a)^2 - 32551.4}$$

$$92.235 = \sqrt{(192+a)^2 - 32551.4}$$

$$8507.3 = (192+a)^2 - 32551.4$$

$$(192+a)^2 = 41058.7$$

$$192+a = 202.63 \Rightarrow a = 10.63 \text{ mm}$$

Final Answer:

$$\boxed{10.63 \text{ mm}}$$

Q-5: Two involute gears in mesh have a 20° pressure angle. The gear ratio is 3 and the number of teeth on the pinion is 24. The teeth have a module of 6 mm. The pitch line velocity is 1.5 m/s and the addendum equal to one module. Determine the angle of action of a pinion (the angle turned by the pinion when one pair of teeth is in the mesh) and the maximum velocity of sliding.

Answer:

Given:

$$\phi = 20^\circ, \text{ Gear ratio } G = 3, Z_p = 24, m = 6 \text{ mm}$$

$$\text{Pitch line velocity } v = 1.5 \text{ m/s, Addendum } a = m = 6 \text{ mm}$$

To Find: Angle of action of pinion θ_p , Maximum velocity of sliding v_s

Formula:

$$r_p = \frac{mZ_p}{2}, r_g = G \times r_p$$

$$r_{ap} = r_p + a, r_{ag} = r_g + a$$

$$r_{bp} = r_p \cos \phi, r_{bg} = r_g \cos \phi$$

$$\text{Path of contact} = \sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - C \sin \phi$$

$$\text{Angle of action } \theta_p = \frac{\text{Path of contact}}{r_{bp}} \text{ (in radians)}$$

$$v_s = \omega_p \times \text{Path of contact (max at pitch point)}$$

Solution:

$$\begin{aligned} r_p &= \frac{6 \times 24}{2} = 72 \text{ mm}, r_g = 3 \times 72 = 216 \text{ mm} \\ r_{ap} &= 72 + 6 = 78 \text{ mm}, r_{ag} = 216 + 6 = 222 \text{ mm} \\ r_{bp} &= 72 \times 0.9397 = 67.66 \text{ mm}, r_{bg} = 216 \times 0.9397 = 202.98 \text{ mm} \\ C &= r_p + r_g = 72 + 216 = 288 \text{ mm} \\ \text{Path} &= \sqrt{78^2 - 67.66^2} + \sqrt{222^2 - 202.98^2} - 288 \times 0.3420 \\ &= \sqrt{6084 - 4578} + \sqrt{49284 - 41201} - 98.5 \\ &= \sqrt{1506} + \sqrt{8083} - 98.5 = 38.81 + 89.91 - 98.5 = 30.22 \text{ mm} \\ \theta_p &= \frac{30.22}{67.66} = 0.4466 \text{ rad} = 25.6^\circ \\ \omega_p &= \frac{v}{r_p} = \frac{1.5}{0.072} = 20.83 \text{ rad/s} \\ v_{s,\max} &= \omega_p \times \text{Path} = 20.83 \times 0.03022 = 0.629 \text{ m/s} \end{aligned}$$

Final Answer:

$$\theta_p = 25.6^\circ, v_{s,\max} = 0.629 \text{ m/s}$$

Q-6: Two 20° involute spur gears have a module of 10 mm. The addendum is equal to one module. The larger gear has 40 teeth while the pinion has 20 teeth will the gear interfere with the pinion?

Answer:

Given:

$$\phi = 20^\circ, m = 10 \text{ mm, Addendum } a = m = 10 \text{ mm}$$

$$Z_g = 40, Z_p = 20$$

To Find: Whether interference occurs

Formula:

$$\text{Interference occurs if } r_{ap} > \sqrt{r_{bp}^2 + (C \sin \phi)^2} \text{ (for pinion)}$$

Or maximum addendum radius without interference:

$$r_{a,\max} = \sqrt{r_b^2 + (C \sin \phi)^2}$$

Solution:

$$\begin{aligned}
 r_p &= \frac{10 \times 20}{2} = 100 \text{ mm}, r_g = \frac{10 \times 40}{2} = 200 \text{ mm} \\
 C &= 100 + 200 = 300 \text{ mm} \\
 r_{bp} &= 100 \times \cos 20^\circ = 100 \times 0.9397 = 93.97 \text{ mm} \\
 r_{ap} &= 100 + 10 = 110 \text{ mm} \\
 \sqrt{r_{bp}^2 + (C \sin \phi)^2} &= \sqrt{(93.97)^2 + (300 \times 0.3420)^2} \\
 &= \sqrt{8830.4 + (102.6)^2} = \sqrt{8830.4 + 10526.8} = \sqrt{19357.2} = 139.13 \text{ mm}
 \end{aligned}$$

Since $r_{ap} = 110 < 139.13$, **no interference** (pinion safe).

Check gear:

$$\begin{aligned}
 r_{bg} &= 200 \times 0.9397 = 187.94 \text{ mm} \\
 \sqrt{r_{bg}^2 + (C \sin \phi)^2} &= \sqrt{35320 + 10526.8} = \sqrt{45846.8} = 214.1 \text{ mm} \\
 r_{ag} &= 200 + 10 = 210 \text{ mm} < 214.1 \Rightarrow \text{no interference}
 \end{aligned}$$

Final Answer:

No interference