Subject Name & Code:

FLUID MECHANICS - BE04000161

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Assignment – 2

Q-1: Continuity Equation: In a 2D incompressible flow, the fluid velocity components are given by u = 3x and v = -3y. Show whether this flow satisfies the continuity equation. Determine the stream function ψ for this flow field.

Answer:

Given: $u = 3x$, $v = -3y$, 2D incompressible flow.

Formula: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Solution:

$$\frac{\partial u}{\partial x} = 3, \frac{\partial v}{\partial y} = -3$$

Sum = $3 - 3 = 0$ → Satisfies continuity.

Stream function ψ:
$u = \frac{\partial \psi}{\partial y} = 3x$ → Integrate: $\psi = 3xy + f(x)$
$v = -\frac{\partial \psi}{\partial x} = -3y$ → Diff: $\frac{\partial \psi}{\partial x} = 3y + f'(x)$, but $-\frac{\partial \psi}{\partial x} = -3y \rightarrow \frac{\partial \psi}{\partial x} = 3y \rightarrow f'(x) = 0$.
Thus $\psi = 3xy + C$.

Final Answer:

$$\boxed{\text{Flow satisfies continuity}, \psi = 3xy + C}$$

Q-2: Bernoulli's Theorem Application: Water flows through a horizontal pipeline that tapers from a 300 mm diameter at section 1 to a 150 mm diameter at section 2. The pressure at section 1 is 40 kPa and the velocity is 2 m/s. Assuming no energy losses, calculate the pressure at section 2.

Answer:

Given:
$D_1 = 0.3$ m, $D_2 = 0.15$ m
$P_1 = 40$ kPa, $V_1 = 2$ m/s
Horizontal pipe, no losses.

To Find: P_2

Formula:

$$A_1V_1 = A_2V_2$$
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

Solution:
$A_1 = \pi(0.15)^2 = 0.0707 \text{ m}^2$, $A_2 = \pi(0.075)^2 = 0.01767 \text{ m}^2$

$$V_2 = V_1\frac{A_1}{A_2} = 2 \times \frac{0.0707}{0.01767} = 2 \times 4 = 8 \text{ m/s}$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2}$$
$$\frac{40000}{1000} + \frac{4}{2} = \frac{P_2}{1000} + \frac{64}{2}$$
$$40 + 2 = \frac{P_2}{1000} + 32$$
$$10 = \frac{P_2}{1000} \Rightarrow P_2 = 10000 \text{ Pa} = 10 \text{ kPa}$$

Final Answer:

$$\boxed{P_2 = 10 \text{ kPa}}$$

Q-3: Venturimeter: A horizontal venturimeter with an inlet diameter of 200 mm and a throat diameter of 100 mm is used to measure the flow of oil (Specific Gravity = 0.85). The discharge through the venturimeter is 60 lps. If the coefficient of discharge Cd is 0.97, find the reading of the oil-mercury differential manometer.

Answer:

Given:
Inlet diameter $D_1 = 200 \text{ mm} = 0.2$ m
Throat diameter $D_2 = 100 \text{ mm} = 0.1$ m
Oil SG = 0.85 → $\rho_o = 850 \text{ kg/m}^3$
Discharge $Q = 60 \text{ lps} = 0.06 \text{ m}^3\text{/s}$

$$C_d = 0.97$$

Manometer fluid: Mercury ($\rho_m = 13600 \text{ kg/m}^3$)

To Find: Manometer reading h (in meters of mercury column difference)

Formula:

$$Q = C_d \cdot \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh\left(\frac{\rho_m}{\rho_o} - 1\right)}$$

Solution:

$$A_1 = \frac{\pi}{4}(0.2)^2 = 0.031416\ \text{m}^2, A_2 = \frac{\pi}{4}(0.1)^2 = 0.007854\ \text{m}^2$$

$$\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} = \frac{0.031416 \times 0.007854}{\sqrt{(0.031416)^2 - (0.007854)^2}} = \frac{0.0002467}{\sqrt{0.0009868 - 0.00006168}}$$

$$= \frac{0.0002467}{\sqrt{0.0009251}} = \frac{0.0002467}{0.030415} = 0.008111$$

From venturi equation:

$$0.06 = 0.97 \times 0.008111 \times \sqrt{2 \times 9.81 \times h \times \left(\frac{13600}{850} - 1\right)}$$

$$\frac{0.06}{0.97 \times 0.008111} = \sqrt{2 \times 9.81 \times h \times (16 - 1)}$$

$$\frac{0.06}{0.007868} = 7.626 = \sqrt{294.3 \times h}$$

Square both sides:

$$58.15 = 294.3 \times h \Rightarrow h = 0.1976\ \text{m} \approx 197.6\ \text{mm}$$

Final Answer:

$$\boxed{h = 197.6\ \text{mm of mercury}}$$

Q-4: Pitot Tube: A pitot-static tube is placed in the center of a 250 mm pipe to measure the velocity of an exhaust gas flow. The stagnation pressure head is 8 mm of water, and the static pressure head is 2 mm of water. If the coefficient of the pitot tube is 0.98 and the density of the gas is 1.2 kg/m3, determine the velocity of the gas.

Answer:

Given:

Pipe diameter = 250 mm (not directly needed)
Stagnation head $h_{stag} = 8$ mm of water
Static head $h_{stat} = 2$ mm of water
Coefficient $C = 0.98$

Gas density $\rho_g = 1.2\ \text{kg/m}^3$
Water density $\rho_w = 1000\ \text{kg/m}^3$

To Find: Velocity of gas V

Formula:
Dynamic head in meters of gas = $\frac{\rho_w}{\rho_g} \times (h_{stag} - h_{stat})$ in meters of water converted to gas.

$$V = C\sqrt{2g \cdot \frac{\rho_w}{\rho_g} \cdot \Delta h}$$

where $\Delta h = 0.008 - 0.002 = 0.006$ m of water

Solution:

$$V = 0.98\sqrt{2 \times 9.81 \times \frac{1000}{1.2} \times 0.006}$$
$$\frac{1000}{1.2} \times 0.006 = 5$$
$$V = 0.98\sqrt{2 \times 9.81 \times 5} = 0.98\sqrt{98.1} = 0.98 \times 9.905 = 9.707\ \text{m/s}$$

Final Answer:

$$\boxed{V = 9.71\ \text{m/s}}$$

Q-5: Momentum Equation: A 45° reducing bend is connected in a pipeline. The diameter at the inlet and outlet of the bend are 600 mm and 300 mm respectively. Find the force exerted by water on the bend if the pressure at the inlet is 8.829 N/cm2 and the flow rate is 600 lps

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Answer:

Given:

$$\theta = 45^\circ$$
$$D_1 = 600\ \text{mm} = 0.6\ \text{m}$$
$$D_2 = 300\ \text{mm} = 0.3\ \text{m}$$
$$P_1 = 8.829\ \text{N/cm}^2 = 8.829 \times 10^4\ \text{Pa}$$
$$Q = 600\ \text{lps} = 0.6\ \text{m}^3/\text{s}$$

Water → $\rho = 1000\ \text{kg/m}^3$

To Find: Force exerted by water on the bend (magnitude and direction)

Formula:
Momentum eqn in x and y directions:

$F_x = \rho Q(V_2\cos\ \theta - V_1) + P_1A_1 + P_2A_2\cos\ \theta$ (with sign careful)
Better approach: Force by bend on water, then reverse.

Solution:

$$A_1 = \pi/4(0.6)^2 = 0.28274\ \text{m}^2$$
$$A_2 = \pi/4(0.3)^2 = 0.070686\ \text{m}^2$$
$$V_1 = Q/A_1 = 0.6/0.28274 = 2.122\ \text{m/s}$$
$$V_2 = Q/A_2 = 0.6/0.070686 = 8.488\ \text{m/s}$$

Find P_2 from Bernoulli (horizontal bend, neglecting losses):

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2}$$
$$\frac{88290}{1000} + \frac{(2.122)^2}{2} = \frac{P_2}{1000} + \frac{(8.488)^2}{2}$$
$$88.29 + 2.251 = \frac{P_2}{1000} + 36.02$$
$$90.541 - 36.02 = \frac{P_2}{1000} \Rightarrow P_2 = 54521\ \text{Pa}$$

Force by bend on water (F_x):

$$F_x = \rho Q(V_2\cos\ 45^\circ - V_1) + P_1A_1 - P_2A_2\cos\ 45^\circ$$
$$F_x = 1000 \times 0.6 \times (8.488 \times 0.7071 - 2.122) + 88290 \times 0.28274$$
$$- 54521 \times 0.070686 \times 0.7071$$
$$= 600 \times (6.001 - 2.122) + 24960 - 2728$$
$$= 600 \times 3.879 + 22232 = 2327.4 + 22232 = 24559.4\ \text{N}$$
$$F_y = \rho Q(V_2\sin\ 45^\circ - 0) + 0 - P_2A_2\sin\ 45^\circ$$
$$= 600 \times 8.488 \times 0.7071 - 54521 \times 0.070686 \times 0.7071$$
$$= 600 \times 6.001 - 2728 = 3600.6 - 2728 = 872.6\ \text{N}$$

Magnitude:

$$F_R = \sqrt{(24559)^2 + (872.6)^2} \approx 24575\ \text{N}$$

Direction: $\phi = \tan^{-1}(872.6/24559) \approx 2^\circ$ from horizontal.

Force **by water on bend** is equal and opposite.

Final Answer:

$$\boxed{F_R \approx 24.58\ \text{kN, acting at } \approx 2^\circ \text{ below horizontal (if bend turns upward)}}$$

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