Subject Name & Code:

FLUID MECHANICS - BE04000161

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Assignment – 4

Q-1: Rayleigh's Method: The power P required by a centrifugal pump depends on the impeller diameter D, rotational speed N, fluid density ρ, and dynamic viscosity μ. Use Rayleigh's method to find the dimensional relationship.

Answer:

Given: $P = f(D, N, \rho, \mu)$

Solution:
Let $P = kD^a N^b \rho^c \mu^d$
Dimensions: $[P] = ML^2T^{-3}$, $[D] = L$, $[N] = T^{-1}$, $[\rho] = ML^{-3}$, $[\mu] = ML^{-1}T^{-1}$

Mass M: $1 = c + d$
Length L: $2 = a - 3c - d$
Time T: $-3 = -b - d$

From T: $b = 3 - d$
From M: $c = 1 - d$
Sub into L: $2 = a - 3(1 - d) - d = a - 3 + 3d - d = a - 3 + 2d \rightarrow a = 5 - 2d$

Thus: $P = kD^{5-2d}N^{3-d}\rho^{1-d}\mu^d$

Group terms: $P = kD^5N^3\rho\left(\frac{\mu}{\rho ND^2}\right)^d$

But $\frac{\rho ND^2}{\mu}$ is Reynolds number. So:

$$P = \rho N^3 D^5 \cdot \phi\left(\frac{\rho ND^2}{\mu}\right)$$

Final Answer:

$$\boxed{P = \rho N^3 D^5\, \Phi\left(\frac{\rho ND^2}{\mu}\right)}$$

Q-2: Buckingham π Theorem: The aerodynamic drag force (FD) on an automotive vehicle depends on the vehicle's frontal area A, velocity V, fluid density ρ, and fluid dynamic viscosity μ. Using Buckingham's π theorem, derive a dimensionless expression for the drag force.

Answer:

Given: $F_D = f(A, V, \rho, \mu)$ with $n = 5$ variables, $m = 3$ (M, L, T) → $n - m = 2$ π terms.

Solution:
Choose ρ, V, A as repeating.

$$\pi_1 = \rho^a V^b A^c F_D$$

Dimensions: $M^0 L^0 T^0 = (ML^{-3})^a (LT^{-1})^b (L^2)^c (MLT^{-2})$
M: $0 = a + 1 \rightarrow a = -1$
T: $0 = -b - 2 \rightarrow b = -2$
L: $0 = -3a + b + 2c + 1 \rightarrow 0 = 3 - 2 + 2c + 1 \rightarrow 0 = 2 + 2c \rightarrow c = -1$
Thus $\pi_1 = \frac{F_D}{\rho V^2 A}$ → This is drag coefficient × (1/2) factor.

$$\pi_2 = \rho^a V^b A^c \mu$$

M: $0 = a + 1 \rightarrow a = -1$
T: $0 = -b - 1 \rightarrow b = -1$
L: $0 = -3a + b + 2c - 1 \rightarrow 0 = 3 - 1 + 2c - 1 \rightarrow 0 = 1 + 2c \rightarrow c = -0.5$
Thus $\pi_2 = \frac{\mu}{\rho V \sqrt{A}}$ ≈ inverse of Reynolds number.

Final Answer:

$$\boxed{\frac{F_D}{\rho V^2 A} = \Phi\left(\frac{\rho V \sqrt{A}}{\mu}\right)}$$

Q-3: Dimensionless Numbers: Define the physical significance of the Reynolds number, Froude number, and Mach number. Provide one specific automotive or mechanical engineering application where each number is the primary similarity criterion.

Answer:

Number	Significance	Automotive/ME Application
Reynolds (Re)	Ratio of inertial to viscous forces; indicates laminar/turbulent flow	Airflow over car body for drag reduction
Froude (Fr)	Ratio of inertial to gravitational forces; governs free-surface flows	Vehicle hydroplaning or ship wave resistance
Mach (Ma)	Ratio of flow velocity to speed of sound; compressibility effects	Supersonic jet nozzle or high-speed IC engine exhaust

Q-4: Similitude (Wind Tunnel): A 1:10 scale model of an automobile is tested in a wind tunnel. The prototype is designed to travel at 100 km/hr in air at 20 °C. To achieve dynamic similarity (Reynolds number matching), what must be the air velocity in the wind tunnel if the air temperature and pressure are the same for both model and prototype?

Answer:

Given:
Scale 1:10 → $L_m/L_p = 1/10$

$$V_p = 100 \text{ km/hr} = 27.78 \text{ m/s}$$

Same air (ρ, μ same)
Reynolds similarity: $Re_m = Re_p \rightarrow \frac{V_m L_m}{\nu} = \frac{V_p L_p}{\nu} \rightarrow V_m = V_p \times \frac{L_p}{L_m} = 27.78 \times 10 = 277.8 \text{ m/s}$

That's ~1000 km/hr (transonic). Difficult in practice.

Final Answer:

$$\boxed{V_m = 278 \text{ m/s } (\approx 1000 \text{ km/hr})}$$

Q-5: Model Laws: A spillway model is built to a scale of 1:36. If the velocity of flow over the model is 1.5 m/s and the discharge is 2.0 m3/s, calculate the corresponding velocity and discharge for the prototype using Froude model laws.

Answer:

Given:
Scale 1:36 → $L_r = 1/36$
Froude similarity: $Fr_m = Fr_p \rightarrow \frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}} \rightarrow V_r = \sqrt{L_r}$
$V_m = 1.5 \text{ m/s} \rightarrow V_p = V_m/\sqrt{L_r}$ but careful: $V_p/V_m = \sqrt{L_p/L_m} = \sqrt{36} = 6$
Thus $V_p = 1.5 \times 6 = 9 \text{ m/s}$

Discharge ratio: $Q_r = L_r^{2.5} \rightarrow Q_p/Q_m = (36)^{2.5} = 36^2 \times \sqrt{36} = 1296 \times 6 = 7776$
$Q_m = 2.0 \text{ m}^3/\text{s} \rightarrow Q_p = 2.0 \times 7776 = 15552 \text{ m}^3/\text{s}$

Final Answer:

$$\boxed{V_p = 9 \text{ m/s}, Q_p = 15552 \text{ m}^3/\text{s}}$$