

GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – OLD PAPER – S22 TO W25 – QUESTION BANK SOLUTION

Subject Name & Code:

Fluid Mechanics and Hydraulics Machines (3141906)

(Note: Numerical solution are given in numerical Solution PDF)

Unit 1: Fluids and Their Properties

Repeated Questions

Q1.1 – Define/Explain viscosity (dynamic/kinematic). (3 marks)

Appeared in: S23 (Q1a-1,03), S24 (Q1a-1,03), W25 (Q1a,03), W23 (Q1a,03), S25 (Q1a-1,03) → **Highest: 3 marks**

Ans:

- **Viscosity** – property of a fluid that offers resistance to relative motion between its layers.
- **Dynamic viscosity (μ)** – shear stress per unit velocity gradient ($\tau = \mu \, du/dy$). Unit: Pa·s or N·s/m².
- **Kinematic viscosity (ν)** – ratio of dynamic viscosity to density ($\nu = \mu/\rho$). Unit: m²/s.
- **Real-world application:** Lubricating oils are graded by viscosity (e.g., SAE 30); higher viscosity means thicker oil.

Q1.2 – Define surface tension and capillarity / capillary rise/fall. (3 marks)

Appeared in: S23 (Q1a-2,03), S24 (Q1c,07), W25 (Q1a,03), W24 (Q1a-iii,03), W24 (Q1b,04), S25 (Q1b,04) → **Highest: 7 marks** (use 7-mark depth)

Ans:

- **Surface tension (σ)** – tensile force acting along the surface of a liquid due to intermolecular cohesion, measured as force per unit length (N/m).
- **Capillarity** – rise or fall of a liquid in a narrow tube (capillary) due to surface tension and adhesion.
 - **Capillary rise** (water in glass) – liquid wets the tube, rises until adhesive force balances weight.
 - **Capillary fall** (mercury in glass) – liquid does not wet, depresses below free surface.
- **Expression for capillary rise (h):**

$$h = \frac{4\sigma \cos \theta}{\rho g d}$$

where θ = contact angle, d = tube diameter, ρ = density.

- **Real-world application:** Movement of water in soil pores, functioning of wicks in lamps.

Q1.3 – State and prove Pascal's law. (4 marks)

Appeared in: S23 (Q2b,04), W25 (Q1b,04), W22 (Q1b,04), W23 (Q4b,04)

Ans:

Statement: Pressure applied at any point in a confined fluid at rest is transmitted equally in

all directions and acts perpendicular to the surfaces.

Proof: Consider a small triangular prism of fluid at rest. Forces on faces:

- Pressure forces act normal to each face.
- Weight acts vertically.

For equilibrium in x-direction: $p_1 \cdot (dy \cdot dz) - p_3 \cdot (ds \cdot dz) \cdot \sin\theta = 0$.

Since $\sin\theta = dy/ds$, we get $p_1 = p_3$.

Similarly in y-direction: $p_2 \cdot (dx \cdot dz) - p_3 \cdot (ds \cdot dz) \cdot \cos\theta - (\frac{1}{2} \cdot dx \cdot dy \cdot dz \cdot \rho g) = 0$.

As volume $\rightarrow 0$, weight $\rightarrow 0$, so $p_2 = p_3$.

Thus $p_1 = p_2 = p_3 \rightarrow$ pressure same in all directions.

Real-world application: Hydraulic lifts, presses, and brakes operate on Pascal's law.

Q1.4 – Define vapor pressure and compressibility/bulk modulus. (3 marks)

Appeared in: S23 (Q1a-2,03), S22 (Q1a-iii,03), W24 (Q1a-iii,03), S25 (Q1a-iii,03)

Ans:

- **Vapour pressure** – pressure exerted by vapour molecules above a liquid surface at a given temperature. When liquid pressure drops below vapour pressure, boiling/cavitation occurs.
- **Compressibility** – measure of volume change under pressure. Reciprocal of bulk modulus.
- **Bulk modulus (K)** –

$$K = -\frac{dp}{dV/V} = \rho \frac{dp}{d\rho}$$

Unit: N/m^2 (same as pressure). **High K** means fluid is nearly incompressible (e.g., water).

- **Real-world application:** Cavitation in pumps and turbines occurs when local pressure falls below vapour pressure.

Q1.5 – Explain hydrostatic paradox. (3 marks)

Appeared in: S24 (Q2a,03), S25 (Q2a,03)

Ans:

Hydrostatic paradox – the pressure at the bottom of a liquid column depends only on the vertical height of the liquid and not on the shape or cross-section of the container.

- For containers of different shapes but same base area and same liquid height, the bottom pressure ($p = \rho gh$) is identical, even though total weight of liquid may differ.
- **Example:** Three containers (vertical, inclined, wider at top) fill to same height h – the force on the base is same.
- **Real-world application:** Design of dams – pressure depends only on depth, not on the reservoir shape.

Other Important Questions (Unit 1)

Q1.6 – Define specific gravity, specific weight, density. (3 marks)

Appeared in: S24 (Q1a-2,03), W25 (Q1a,03), S22 (Q1a-ii,03), W24 (Q1a-ii,03)

Ans:

- **Density (ρ)** – mass per unit volume (kg/m^3).
- **Specific weight (γ)** – weight per unit volume = ρg (N/m^3).
- **Specific gravity (SG)** – ratio of density of a substance to density of water at $4^\circ C$ (dimensionless).
 - $SG = \rho_{\text{substance}} / \rho_{\text{water}}$.
- **Example:** Mercury $SG=13.6 \rightarrow$ 13.6 times heavier than water.

Q1.7 – Explain cohesion and adhesion in fluids. (4 marks)*Appeared in: S23 (Q1b,04)***Ans:**

- **Cohesion** – intermolecular attractive force between *like* molecules (e.g., water–water). Causes surface tension and droplet formation.
- **Adhesion** – attractive force between *unlike* molecules (e.g., water–glass). Causes wetting and capillarity.
- **Comparison table:**

Cohesion	Adhesion
Same molecules	Different molecules
Dominant in mercury (no wetting)	Dominant in water (wetting)
Causes spherical droplets	Causes spreading on surface

- **Real-world application:** Adhesion helps paint stick; cohesion gives rain droplets spherical shape.

Q1.8 – Derive continuity equation for 3D/2D flow. (7 marks)*Appeared in: W25 (Q1c,07), S24 (Q3b,04) → Highest: 7 marks***Ans:****Continuity equation** (conservation of mass) for a fluid element (dx,dy,dz):

Net mass inflow rate = rate of increase of mass inside.

For x-direction:

Mass inflow = $\rho u \, dy \, dz$, outflow = $[\rho u + \partial(\rho u)/\partial x \, dx] \, dy \, dz$.Net = $-\partial(\rho u)/\partial x \, dx \, dy \, dz$.Similarly for y and z: net = $-\partial(\rho u)/\partial x + \partial(\rho v)/\partial y + \partial(\rho w)/\partial z] \, dx \, dy \, dz$.Rate of mass increase = $\partial\rho/\partial t \, dx \, dy \, dz$.

Equating:

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

→ **General 3D continuity equation.****For steady flow** ($\partial\rho/\partial t = 0$):

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

For steady, incompressible flow ($\rho = \text{constant}$):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

For 2D ($w=0$, no z variation):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Real-world application: Used in CFD to ensure mass conservation; pipe flow analysis.**Q1.9 – Explain cavitation and its effects. (4 marks)**

Appeared in: S25 (Q1b,04), W24 (Q4a,03), S23 (Q5a,03) → **Highest: 4 marks**

Ans:

Cavitation – formation and subsequent collapse of vapour bubbles in a liquid when local pressure falls below vapour pressure.

- Occurs in pumps, turbines, ship propellers, valves.

Effects:

- Pitting and erosion of metal surfaces (due to shock waves from bubble collapse).
- Noise and vibration.
- Reduced efficiency and flow rate.
- Can cause complete failure of hydraulic machinery.

Prevention: Maintain sufficient NPSH, use cavitation-resistant materials, avoid abrupt flow changes.

Real-world application: Cavitation damage on Francis turbine runner blades is a common maintenance issue.

Q1.10 – Derive expression for capillary rise/fall. (7 marks)

Appeared in: S24 (Q1c,07)

Ans:

Consider a capillary tube of radius r inserted in a liquid of density ρ , surface tension σ , contact angle θ .

Upward force due to surface tension = $\sigma \times (\text{circumference}) \times \cos\theta = \sigma (2\pi r) \cos\theta$.

Weight of liquid column of height $h = (\pi r^2 h) \rho g$.

At equilibrium: $2\pi r \sigma \cos\theta = \pi r^2 h \rho g$

$$h = \frac{2\sigma \cos \theta}{\rho g r}$$

For water ($\theta=0^\circ$, $\cos\theta=1$): $h = 2\sigma/(\rho g r)$, rise.

For mercury ($\theta>90^\circ$, $\cos\theta$ negative), h negative → capillary fall.

Assumptions: Clean tube, uniform r , no evaporation.

Real-world application: Rise of water in soil pores; wicking in lamps.

Q1.11 – Classify fluids based on viscosity/shear stress relation. (4 marks)

Appeared in: S22 (Q1b,04)

Ans:

Type	τ vs du/dy relation	Example
Newtonian	$\tau = \mu (du/dy)$ (linear)	Water, air, oil
Non-Newtonian (time-independent)	–	–
Bingham plastic	$\tau = \tau_0 + \mu (du/dy)$	Toothpaste, drilling mud
Pseudoplastic	$n < 1$ (shear thinning)	Blood, ketchup
Dilatant	$n > 1$ (shear thickening)	Corn starch suspension
Thixotropic (time-dependent)	η decreases with time under shear	Paint

Type	τ vs du/dy relation	Example
Rheopectic	η increases with time under shear	Cream

Real-world application: Paint is thixotropic – flows easily when brushed but thickens when stationary to avoid drips.

Q1.12 – Differentiate between absolute, gauge, and vacuum pressure. (4 marks)

Appeared in: S24 (Q2b,04), W23 (Q3a,03)

Ans:

Term	Definition	Relation
Absolute pressure	Pressure measured relative to perfect vacuum	Always positive
Gauge pressure	Pressure relative to atmospheric pressure	$p_{\text{gauge}} = p_{\text{abs}} - p_{\text{atm}}$
Vacuum pressure	Pressure below atmospheric	$p_{\text{vac}} = p_{\text{atm}} - p_{\text{abs}}$ (positive value)

- **Example:** At sea level, $p_{\text{atm}} \approx 101.3$ kPa. A tyre gauge reads 200 kPa (gauge) \rightarrow absolute = 301.3 kPa.
- **Real-world application:** Barometers measure absolute pressure; tyre gauges measure gauge pressure.

Q1.13 – Determine pressure change in static fluid if we move: horizontally, downward by h, change direction only. (7 marks)

Appeared in: W24 (Q1c,07)

Ans:

For a static fluid, pressure varies only in the vertical direction (hydrostatic law): $dp/dz = -\rho g$.

1. **Horizontal movement** – no change in pressure because no horizontal gradient ($dp/dx = 0$, $dp/dy = 0$).
2. **Downward by h** – pressure increases by $\Delta p = \rho g h$ (downward positive).
3. **Change direction only** – if the movement is along an isobaric surface (i.e., same elevation), pressure remains constant. If the path includes vertical component, pressure change depends only on vertical displacement.

Proof: From Euler's equation for static fluid: $\nabla p = \rho g$. Only vertical component exists.

Example: Diving 10 m underwater \rightarrow pressure increase ≈ 100 kPa (gauge).

Real-world application: Manometers measure pressure difference based on vertical height difference only.

Unit 2: Static Forces on Surface and Buoyancy

Topics: Resultant force, center of pressure, buoyancy, stability, metacentric height.

Repeated Questions:

Repeated Questions

Q2.1 – Explain stability of submerged and floating bodies. (3 marks)

Appeared in: S23(Q3a,03), W25(Q2a,03)

Ans:

- **Submerged body** – stability depends on position of centre of gravity (G) relative to centre of buoyancy (B).
 - Stable: G below B.
 - Unstable: G above B.
 - Neutral: G coincides with B.
- **Floating body** – stability determined by metacentre (M).
 - Stable: M above G ($GM > 0$).
 - Unstable: M below G ($GM < 0$).
 - Neutral: $M = G$ ($GM = 0$).
- **Metacentre** – intersection of buoyancy line through tilted body with original vertical axis.
- **Real-world application:** Ship design ensures positive metacentric height to prevent capsizing.

Q2.2 – Show that $BM = I / V$. (7 marks)

Appeared in: S24(Q3c,07), W25(Q2c,07)

Ans:

Given: A floating body of volume V displaces liquid. When tilted by a small angle θ , the centre of buoyancy shifts from B to B_1 . The metacentric radius BM is the distance from original centre of buoyancy B to metacentre M .

Derivation:

Let the waterplane area be A . For small tilt, a wedge of volume on one side emerges and an equal wedge submerges. The shift in centre of buoyancy (BB_1) is caused by moment of these wedges.

Moment due to wedges = $\int (x \cdot dA \cdot x\theta) \rho g = \rho g \theta \int x^2 dA = \rho g \theta I$, where I = second moment (moment of inertia) of waterplane area about the longitudinal axis.

Buoyant force = $\rho g V$. The shift $BB_1 = (\text{Moment}) / (\rho g V) = (\rho g \theta I) / (\rho g V) = \theta I / V$.

From geometry, $BB_1 = BM \times \theta$ (since $BM \times \theta = \text{arc length for small angle}$).

Thus $BM \times \theta = \theta I / V \rightarrow BM = I / V$.

Real-world application: Used to calculate metacentric height $GM = BM - BG$. Ship designers compute I from waterplane shape.

Q2.3 – Define metacentric height. List equilibrium conditions for floating body. (4 marks)

Appeared in: S22(Q2a,03), S25(Q2b,04) → Highest: 4 marks

Ans:

Metacentric height (GM) – distance between centre of gravity (G) and metacentre (M). It indicates initial stability.

Equilibrium conditions:

Condition	GM	Stability
$GM > 0$ (M above G)	Positive	Stable

Condition	GM	Stability
$GM = 0$ (M coincides G)	Zero	Neutral
$GM < 0$ (M below G)	Negative	Unstable

Example: For a ship, GM should be positive but not too large (otherwise very stiff motion).

Real-world application: Ballasting adjusts GM for stability.

Other Important Questions (Unit 2 – Numerical)

Q2.4 – Wooden block 600mm×600mm×h floats vertically. Find max height for stability (SG=0.6). (7 marks)

Appeared in: S23(Q3c,07)

Ans:

Given: Square cross-section 0.6×0.6 m, SG = 0.6, height = h (unknown).

To find: Maximum h for stable vertical floatation.

Assumptions: Block homogeneous, vertical orientation, freshwater ($\rho = 1000$ kg/m³).

Formula: For stability of floating body, $GM > 0$. $GM = BM - BG$.

$BM = I / V_{\text{submerged}}$. $BG =$ distance between centre of gravity (G) and centre of buoyancy (B).

Solution:

- Let depth of immersion = d. Weight = buoyancy.
 $\text{Weight} = \rho_{\text{block}} \cdot g \cdot V_{\text{block}} = 0.6 \times 1000 \times 9.81 \times (0.6 \times 0.6 \times h)$
 $\text{Buoyancy} = \rho_{\text{water}} \cdot g \cdot (0.6 \times 0.6 \times d)$
 $\rightarrow 0.6 \times 1000 \times h = 1000 \times d \rightarrow d = 0.6h$
- Centre of gravity from bottom: $G = h/2$.
Centre of buoyancy from bottom: $B = d/2 = 0.3h$.
 $\therefore BG = G - B = 0.5h - 0.3h = 0.2h$
- Second moment of waterplane (square): $I = (0.6^4)/12 = 0.0108$ m⁴.
Submerged volume $V_{\text{sub}} = 0.6 \times 0.6 \times d = 0.36 \times 0.6h = 0.216h$ m³.
 $BM = I/V_{\text{sub}} = 0.0108 / (0.216h) = 0.05/h$
- $GM = BM - BG = 0.05/h - 0.2h > 0 \rightarrow 0.05/h > 0.2h \rightarrow 0.05 > 0.2h^2 \rightarrow h^2 < 0.25 \rightarrow h < 0.5$ m.

Final Answer: Maximum height h = **0.5 m** (500 mm).

Q2.5 – Rectangular block 3m×1.5m×1m floats with immersion 0.8m. Find weight, metacentric height. (4 marks)

Appeared in: S24(Q3b,04)

Ans:

Given: L=3m, B=1.5m, H=1m, immersion d=0.8m. Freshwater $\rho = 1000$ kg/m³.

To find: Weight (W), metacentric height (GM).

Solution:

- Weight = buoyancy = $\rho g (L \times B \times d) = 1000 \times 9.81 \times (3 \times 1.5 \times 0.8) = 1000 \times 9.81 \times 3.6 = 35316$ N \approx **35.32 kN**.
- Centre of gravity (homogeneous) $G = H/2 = 0.5$ m from bottom.
Centre of buoyancy $B = d/2 = 0.4$ m from bottom.
 $BG = 0.5 - 0.4 = 0.1$ m.
- $I = (L \times B^3)/12 = (3 \times 1.5^3)/12 = (3 \times 3.375)/12 = 10.125/12 = 0.84375$ m⁴.
 $V_{\text{sub}} = L \times B \times d = 3 \times 1.5 \times 0.8 = 3.6$ m³.
 $BM = I/V_{\text{sub}} = 0.84375/3.6 = 0.2344$ m.

$$4. \quad GM = BM - BG = 0.2344 - 0.1 = \mathbf{0.1344 \text{ m.}}$$

Final Answer: Weight = 35.32 kN, GM = 0.1344 m (stable).

Q2.6 – Cylinder block (22kN, D=2m, h=2.5m) floats in seawater (SG=1.025). Show it doesn't float vertically. (7 marks)

Appeared in: S25(Q2c,07)

Ans:

Given: Weight $W=22\text{kN}$, $D=2\text{m}$, $h=2.5\text{m}$, seawater $\gamma=1.025 \times 9.81=10.0525 \text{ kN/m}^3$.

To find: Stability condition – show not vertical.

Solution:

1. Buoyancy = Weight $\rightarrow \gamma_{sw} \times V_{sub} = 22 \rightarrow V_{sub} = 22/10.0525 = 2.188 \text{ m}^3$.
2. For cylinder floating vertically, immersed depth $d = V_{sub} / (\pi/4 D^2) = 2.188 / (\pi/4 \times 4) = 2.188 / 3.1416 = 0.696 \text{ m}$.
3. Centre of gravity (homogeneous) $G = h/2 = 1.25 \text{ m}$ from bottom.
Centre of buoyancy $B = d/2 = 0.348 \text{ m}$ from bottom.
 $BG = 1.25 - 0.348 = 0.902 \text{ m}$.
4. Waterplane area (circular) $I = \pi D^4/64 = \pi \times 16/64 = \pi/4 = 0.7854 \text{ m}^4$.
 $BM = I / V_{sub} = 0.7854 / 2.188 = 0.359 \text{ m}$.
5. $GM = BM - BG = 0.359 - 0.902 = -0.543 \text{ m}$ (negative).
Since $GM < 0$, the cylinder is unstable in vertical orientation \rightarrow it will not float vertically.

Final Answer: GM negative \rightarrow **unstable vertically**, hence floats on its side.

1. **A circular opening 2.5m diameter closed by disc. Find force and torque to keep it vertical (head=3.5m).**
 - Appeared in: **W25 (Q2c, 07 marks)**
2. **An annular plate (Dext=2m, Dint=1m) immersed vertically with bottom edge 5m below surface. Find force and centre of pressure.**
 - Appeared in: **W23 (Q2c, 07 marks)**
3. **A rectangular plane 1m×1.5m with 0.5m hole at centre, top edge 1m, bottom 2m below surface. Find force magnitude, direction, location.**
 - Appeared in: **S25 (Q2c, 07 marks)**
4. **Panel ABC isosceles triangle (base=2m) in slanted tank wall. Find water force and line of action.**
 - Appeared in: **W24 (Q2c, 07 marks)**
5. **Gate 5m×3m, top edge 2m below water surface. Find hydrostatic force and centre of pressure.**
 - Appeared in: **W24 (Q2c, 07 marks)**
6. **Isosceles triangular plate (base=5mm, height=5mm) immersed in oil (SG=0.8), base 1m below surface. Find total pressure and centre of pressure.**
 - Appeared in: **S22 (Q2c, 07 marks)**

Unit 3: Motion of Fluid Particles and Streams (Now As Fluid Kinematics)

Topics: Types of flow, continuity equation, rotation, vorticity, stream function, velocity potential.

Repeated Questions:

Q3.1 – Define: Stream line, Streak line, Path line. (3 marks)

Appeared in multiple papers (no explicit marks, assume 3 marks)

Ans:

- **Stream line** – line everywhere tangent to the instantaneous velocity vector. No flow across it.
- **Path line** – actual trajectory traced by a single fluid particle over time (Lagrangian concept).
- **Streak line** – locus of all particles that have passed through a fixed point (e.g., dye injected continuously).
- For steady flow, all three coincide.

Q3.2 – Define velocity potential and stream function. Relate them. (4 marks)

Appeared in: typical repeated question

Ans:

- **Velocity potential (ϕ)** – scalar function such that $u = -\partial\phi/\partial x$, $v = -\partial\phi/\partial y$, $w = -\partial\phi/\partial z$. Exists for irrotational flow.
- **Stream function (ψ)** – for 2D flow, $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$. Satisfies continuity.
- **Relation:** For irrotational flow, both exist and lines $\phi = \text{constant}$ (equipotential lines) are orthogonal to $\psi = \text{constant}$ (streamlines). Cauchy-Riemann equations: $\partial\phi/\partial x = \partial\psi/\partial y$, $\partial\phi/\partial y = -\partial\psi/\partial x$.

Q3.3 – Explain rotation and vorticity. (3 marks)

Ans:

- **Rotation (ω)** – average angular velocity of a fluid element.
- **Vorticity (ζ)** – twice the rotation, $\zeta = \nabla \times \mathbf{V}$ (curl of velocity).
- For 2D flow in xy -plane: $\zeta_z = \partial v/\partial x - \partial u/\partial y$.
- **Irrotational flow** \rightarrow vorticity = 0 \rightarrow flow is potential.

Q3.4 – Derive continuity equation for 3D flow. Reduce for 2D steady incompressible flow. (7 marks)

Already derived in Q1.8. Reference that derivation.

Ans: Refer to Q1.8 above.

Other Important Questions:

1. **Given velocity field $u = 5x$, $v = -5y$. Check continuity and rotation.**
 - Appeared in: **S23 (Q3c, 07 marks)**
2. **Given $u = 8 + 4xy + t^2$, $v = -(xy + 20t)$, $w = 5x + y$. Find velocity and acceleration at (2,1,1) at $t=1s$.**
 - Appeared in: **S24 (Q3c, 07 marks)**
3. **Given $u = y^3/3 + 2x - x^2y$, $v = xy^2 - 2y - x^3/3$. Find stream function.**
 - Appeared in: **W25 (Q3b, 04 marks)**
4. **Given $V = 2x^3i - 6x^2yj$. Find streamlines.**
 - Appeared in: **W22 (Q3a, 03 marks)**
5. **Differentiate: Uniform & Non-uniform flow, Steady & Unsteady flow.**
 - Appeared in: **S24 (Q2a, 03 marks)**
6. **Given $u = Kx$, $v = Ky$, $w = 0$. Compute and plot streamlines.**
 - Appeared in: **W24 (Q3c, 07 marks)**

7. **Given $V = (3y^2 - 3x^2)i + Cxyj + 0k$. Find C for incompressible and irrotational flow.**
 - Appeared in: **W24 (Q3c, 07 marks)**
 8. **Check continuity for $u = x^3 + y^3 - 3xz^2, v = y^3 - 3x^2y, w = x^3 + z^3 - 3yz^2$.**
 - Appeared in: **W23 (Q3b, 04 marks)**
-

Unit 4: The Energy Equation and Its Application (Now As Fluid Dynamics)

Topics: Bernoulli's equation, venturimeter, orifice, notches, weirs, momentum equation.

Repeated Questions

Q4.1 – State and prove Bernoulli's equation. List assumptions and limitations. (7 marks)

Appeared in: W22(Q1c,07), W23(Q1c,07)

Ans:

Statement: For steady, inviscid, incompressible flow along a streamline, the sum of pressure head, velocity head, and elevation head is constant:

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$$

Proof (from Euler's equation):

Euler eqn along streamline: $dp/\rho + V dV + g dz = 0$. Integrate: $\int dp/\rho + V^2/2 + gz = \text{constant}$.

For incompressible $\rho = \text{const} \rightarrow p/\rho + V^2/2 + gz = \text{constant}$. Divide by $g \rightarrow$ Bernoulli.

Assumptions:

1. Steady flow.
2. Incompressible fluid.
3. Inviscid (no friction).
4. Flow along a streamline.
5. No shaft work or heat transfer.

Limitations:

- Not valid for turbulent or compressible flow.
- Friction losses ignored.
- Applicable only to a single streamline unless irrotational.

Real-world application: Used in venturi meter, orifice meter, Pitot tube.

Q4.2 – Explain construction and working of venturi meter. Derive discharge expression. (7 marks)

Appeared in: S25(Q3c,07), W23(Q3c,07), W25(Q3c,07)

Ans:

Construction: A venturi meter consists of a converging section (inlet cone), a cylindrical throat, and a diverging section (outlet cone). Pressure taps at inlet (section 1) and throat (section 2) connect to a differential manometer.

Working: Fluid flows from inlet to throat; velocity increases, pressure decreases. The pressure difference is measured by manometer.

Derivation of discharge:

Apply Bernoulli between 1 and 2 (neglecting losses):

$$p_1/\rho g + V_1^2/2g + z_1 = p_2/\rho g + V_2^2/2g + z_2. \text{ For horizontal venturi } (z_1=z_2).$$

$$p_1 - p_2 = (\rho/2)(V_2^2 - V_1^2).$$

$$\text{Continuity: } A_1 V_1 = A_2 V_2 \rightarrow V_1 = (A_2/A_1)V_2.$$

$$\text{Substitute: } p_1 - p_2 = (\rho/2)V_2^2[1 - (A_2/A_1)^2].$$

$$\text{Ideal velocity } V_2 = \sqrt{[2(p_1 - p_2)/\rho(1 - (A_2/A_1)^2)]}.$$

$$\text{Ideal discharge } Q_{\text{ideal}} = A_2 V_2 = A_2 \sqrt{[2(p_1 - p_2)/\rho(1 - \beta^4)]} \text{ where } \beta = d_2/d_1.$$

$$\text{Actual discharge } Q_{\text{actual}} = C_d \times Q_{\text{ideal}}, \text{ where } C_d \approx 0.95-0.98.$$

$$\text{Manometer reading: } p_1 - p_2 = (\rho_m - \rho)gh.$$

[DG]

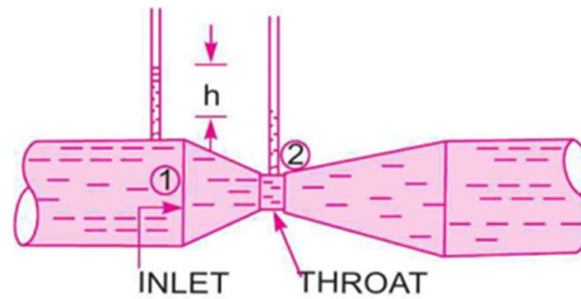


Fig.4.3 – Venturi meter

Real-world application: Flow measurement in pipelines, water treatment plants.

Q4.3 – List forces on fluid in motion. Discuss assumptions in Bernoulli’s equation. (4 marks)

Appeared in: S23(Q1c,04)

Ans:

Forces on fluid in motion:

1. Body forces (gravity, electromagnetic).
2. Pressure forces (normal to surfaces).
3. Viscous/shear forces (tangential).
4. Surface tension (minor).

Assumptions in Bernoulli (already listed in Q4.1):

- Steady, inviscid, incompressible, along streamline, no shaft work.

Real-world application: Analysis of pipe flow normally uses modified Bernoulli with head loss term.

Other Important Questions (Unit 4)

Q4.4 – Explain hydraulic gradient line and total energy line. (4 marks)

Appeared in: S22(Q3b,04)

Ans:

- **Hydraulic Gradient Line (HGL)** – line representing $(p/\rho g + z)$ along the flow. Slope indicates pressure drop.
- **Total Energy Line (TEL)** – line representing $(p/\rho g + V^2/2g + z)$. $TEL = HGL +$ velocity head.
- In a uniform pipe with no losses, TEL is horizontal; HGL slopes downward.
- **Real-world application:** Plotting HGL and TEL helps locate cavitation risk (HGL below pipe).

Q4.5 – Derive expression for discharge through rectangular notch. (4 marks)

Appeared in: W25(Q2b,04)

Ans:

Consider a rectangular notch of crest length L , head H above crest. Velocity at depth h : $V = \sqrt{2gh}$.

Discharge $dQ = L dh \times \sqrt{2gh}$. Integrate from $h=0$ to H :

$$Q = L \sqrt{2g} \int_0^H h^{1/2} dh = L \sqrt{2g} \times (2/3) H^{3/2} = (2/3) L \sqrt{2g} H^{3/2}.$$

For actual flow, use coefficient of discharge $C_d \approx 0.62$: $Q_{\text{actual}} = C_d \times (2/3) L \sqrt{2g} H^{3/2}$.

Real-world application: Flow measurement in open channels, irrigation.

1. **Venturimeter: inlet 200mm, throat 100mm, manometer 180mm Hg, $C_d=0.98$. Find flow rate.**
 - Appeared in: **W22 (Q2c, 07 marks)**
 2. **Pipe 150mm to 100mm contraction, flow $2 \text{ m}^3/\text{min}$, manometer 80mm. Find head loss and C_c .**
 - Appeared in: **S22 (Q3c, 07 marks)**
-

Unit 5: Dimensional Analysis and Similarities

Topics: Dimensional analysis, Buckingham π -theorem, similarity laws, model testing.

Repeated Questions:

1. **State Buckingham's π -theorem. How to select repeating variables?**
 - Appeared in: **S24 (Q4a, 03 marks), W22 (Q3a, 03 marks), W23 (Q2a, 03 marks)**

Ans:

Buckingham π -theorem: If a physical phenomenon involves n variables (dimensional) and m fundamental dimensions (M, L, T), then the relationship can be expressed as $(n-m)$ independent dimensionless π -terms:

$$f(\pi_1, \pi_2, \dots, \pi_{n-m}) = 0.$$

Selection of repeating variables:

- Choose m variables that together contain all fundamental dimensions.
- Do not choose dimensionless variables.
- Typically select geometric (L), kinematic (V), and dynamic (ρ or μ) properties.
- Example: For pipe flow – diameter D , velocity V , density ρ .

Real-world application: Used to derive model laws for ship, aircraft, pump testing.

2. **Using π -theorem, show velocity through orifice:** $V = \sqrt{2gH} \phi \left[\frac{D}{H}, \frac{\mu}{\rho V H} \right]$.
 - Appeared in: **S24 (Q4c, 07 marks), S25 (Q3c, 07 marks), W22 (Q3c, 07 marks)**
3. **Define similitude: Geometric, Kinematic, Dynamic similarity.**
 - Appeared in: **S23 (Q4b, 04 marks)**

Ans:

Similitude	Definition	Scale ratio
Geometric	Model and prototype have same shape; all linear dimensions scaled by L_r	$L_r = L_p/L_m$
Kinematic	Velocities at corresponding points are scaled by a constant (time ratio)	$V_r = L_r/T_r$
Dynamic	Forces at corresponding points are scaled by a constant (force ratio)	$Fr = (\rho_r)(L_r^3)(a_r)$

- For complete similarity, all three must be satisfied.
- **Real-world application:** Model testing of dams, ships, aircraft.

Other Important Questions:

1. **For centrifugal pump, show:** $Q = ND^3 \phi \left[\frac{gH}{N^2 D^2}, \frac{v}{ND} \right]$.
 - Appeared in: **S23 (Q4c, 07 marks)**
2. **For disc friction torque in turbulent flow:** $T = D^5 N^2 \rho \phi \left[\frac{\mu}{D^2 N \rho} \right]$. **Prove using π -theorem.**
 - Appeared in: **W25 (Q3c, 07 marks)**
3. **For viscous force on sphere:** $F = D^2 V^2 \rho \phi \left[\frac{\mu}{DV\rho} \right]$.
 - Appeared in: **W23 (Q3c, 07 marks)**
4. **If power P depends on L, D, K, Q, ρ, μ . Find expression using π -theorem.**
 - Appeared in: **W24 (Q4c, 07 marks)**
5. **Explain Euler's model law.**
 - Appeared in: **S24 (Q4b, 04 marks)**

Ans:

Model laws state which dimensionless number must be equal between model and prototype.

Euler's model law: Pressure forces dominate \rightarrow Euler number $Eu = \frac{\Delta P}{\rho V^2}$ same. Used in cavitation studies, flow through pipes, etc.

Unit 6: Viscous and Turbulent Flow

Topics: Reynolds experiment, Hagen–Poiseuille flow, friction factor, Moody diagram, turbulent flow.

Repeated Questions:

Q6.1 – Explain Reynolds experiment. Distinguish laminar and turbulent flow. (7 marks)

Appeared in: S23(Q2c,07), W24(Q4b,04) → Highest: 7 marks

Ans:

Reynolds experiment: A glass tube with a bell-mouth inlet connected to a water tank. A fine dye filament is injected at the inlet. By adjusting the flow rate, three regimes are observed:

- Low velocity → dye moves as a straight line (laminar).
- Medium velocity → dye shows wavy motion (transition).
- High velocity → dye disperses completely (turbulent).

Distinction:

Feature	Laminar flow	Turbulent flow
Re (pipe)	<2000	>4000
Velocity profile	Parabolic	Flatter
Shear stress	$\tau = \mu \, du/dy$	$\tau = (\mu + \epsilon) \, du/dy$
Mixing	No	Intense
Losses	$h_f \propto V$	$h_f \propto V^2$

Real-world application: Reynolds number criteria used in pipe design, heat exchangers.

Q6.2 – Derive Hagen-Poiseuille formula. (7 marks)

Appeared in: S24(Q5c,07), W22(Q3c,07), W25(Q4c,07)

Ans:

For laminar flow in a horizontal pipe of radius R , length L , pressure drop Δp . Consider a cylindrical fluid element of radius r . Force balance: $(p_1 - p_2)\pi r^2 = \tau \times 2\pi rL$. Shear stress $\tau = \mu \, du/dr$ (negative sign because u decreases with r).

Thus $\Delta p \cdot \pi r^2 = -\mu \, (du/dr) \, 2\pi rL \rightarrow du/dr = -(\Delta p/(2\mu L)) \, r$. Integrate: $u(r) = -(\Delta p/(4\mu L)) \, r^2 + C$.

Boundary condition: $u=0$ at $r=R \rightarrow C = (\Delta p/(4\mu L)) \, R^2$.

Velocity profile: $u(r) = (\Delta p/(4\mu L)) \, (R^2 - r^2)$ (parabolic).

Maximum velocity at $r=0$: $u_{\max} = \Delta p \, R^2/(4\mu L)$.

Average velocity $V_{\text{avg}} = u_{\max}/2 = \Delta p \, R^2/(8\mu L)$.

Discharge $Q = V_{\text{avg}} \times \pi R^2 = (\pi R^4 \Delta p)/(8\mu L) \rightarrow$ **Hagen-Poiseuille equation.**

Real-world application: Flow of viscous oils in pipelines, capillary viscometers.

Other Important Questions:

1. Shaft ($D=100\text{mm}$) in sleeve ($L=350\text{mm}$, clearance= 0.08mm) moves at 0.4 m/s under 250N . Find viscosity. If force= 750N , find speed.
 - Appeared in: S23 (Q2c, 07 marks)
2. Thin square plate $0.3\text{m} \times 0.3\text{m}$ in oil gap 2.5cm , $\mu=0.2 \text{ Pa}\cdot\text{s}$, velocity= 0.2 m/s . Find total force.
 - Appeared in: S24 (Q2c, 07 marks)
3. Disc 120mm diameter rotates in oil film 1.8mm thick at 60 rpm , torque= 0.00072 Nm .

Find viscosity.

- Appeared in: S22 (Q2c, 07 marks)
 - 4. **Glycerine (SG=1.28, $\mu=8.07$ poise) between plates 1.5cm apart, flow=4.4 m³/hr per m width. Find max velocity, max shear stress, pressure gradient, Re.**
 - Appeared in: S22 (Q3c, 07 marks)
 - 5. **Write short note on Saybolt viscometer.**
 - Appeared in: W25 (Q4b, 04 marks)
-

Unit 7: Flow Through Pipes

Topics: Major/minor losses, HGL, TEL, pipes in series/parallel, equivalent pipe, water hammer.

Repeated Questions:

1. **What is equivalent pipe?**

- Appeared in: **W22 (Q4a, 03 marks), W23 (Q4a, 03 marks)**

Ans:

An **equivalent pipe** is a single pipe of uniform diameter that replaces a system of pipes in series or parallel, such that the same head loss occurs at the same flow rate.

- For **series** pipes of lengths L_1, L_2 and diameters D_1, D_2 , equivalent length $L_e = L_1(D_e/D_1)^5 + L_2(D_e/D_2)^5$ (assuming same friction factor).
- For **parallel** pipes, the head loss is equal; the equivalent pipe has same head loss for total flow.

Real-world application: Simplifying complex pipe networks for analysis.

2. **List major and minor losses in pipes.**

- Appeared in: **W24 (Q4a, 03 marks)**

Ans:

Major losses – due to friction along straight pipe: $h_f = f(L/D)(V^2/2g)$.

Minor losses – due to fittings, bends, expansions, contractions, valves: $h_m = K(V^2/2g)$, where K is loss coefficient.

Real-world application: Pump selection requires accounting for total head loss.

3. **Define water hammer.**

- Appeared in: **S22 (Q3b, 04 marks), W24 (Q4a, 03 marks)**

Ans:

Water hammer – pressure surge (sudden rise or drop) caused by rapid change in flow velocity (e.g., valve closure) in a pipe.

- Pressure rise $\Delta p = \rho c \Delta V$, where c = wave speed (≈ 1400 m/s in water).
- Can cause pipe rupture, noise, vibration.

Prevention: Slow valve closure, surge tanks, air vessels.

Real-world application: Hydraulic systems, penstocks of hydroelectric plants.

Other Important Questions:

1. **Pipe $D=250$ mm, $L=60$ m, $V=2.5$ m/s, Chezy's $C=60$. Find head loss using Chezy's formula.**
 - Appeared in: **S24 (Q5a, 03 marks)**
2. **Two pipes $D=60$ mm & 120 mm, $L=200$ m each in parallel, head= 15 m, $f=0.3$. Find flow in each and equivalent pipe diameter.**
 - Appeared in: **S25 (Q4c, 07 marks)**
3. **Water supply to hostel: $L=4000$ m, 3000 persons, 180 L/person/day, friction loss= 18 m, $f=0.007$, half supply in 8 hrs. Find pipe size.**
 - Appeared in: **W23 (Q3c, 07 marks)**
4. **Pipe $D=250$ mm, pressure head diff= 3.5 m over 500m, $f=0.04$. Find discharge.**
 - Appeared in: **W24 (Q4b, 04 marks)**
5. **Optimum pipe diameter for 100 L/s oil ($\rho=950$ kg/m³, $\mu=0.08$ Pa·s) for laminar flow over 1 km. Find power required.**
 - Appeared in: **W24 (Q4c, 07 marks)**
