GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – OLD PAPER – S22 TO W25 – Q&A BANK (Numerical)

Subject Name & Code:
Fluid Mechanics (3141906)

Unit 1: Properties of Fluids & Fluid Statics

(As per new Syllabus For Unit- 1&2 Numerical)

Topic/Formula Group: Surface Tension (Pressure difference across bubble)
Marks Group: 3
Numerical Question:

1. If the surface tension at air and water interface is $0.0735\ \mathrm{N/m}$, what is the pressure difference between inside and outside of an air bubble of diameter 0.01 mm?
   - **Appeared in:** 3141906(1) (Q4a, 03 marks)
   - **Frequency Tag:** Single/Unique
   - **Input Variables:** surface tension = 0.0735 N/m, diameter = 0.01 mm

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Answer:

Given:
Surface tension $\sigma = 0.0735$ N/m
Diameter $d = 0.01$ mm $= 0.01 \times 10^{-3} = 1 \times 10^{-5}$ m
To find: Pressure difference ΔP
Formula: For an air bubble in air (soap bubble)

$$\Delta P = \frac{4\sigma}{r} = \frac{8\sigma}{d}$$

Solution:
Radius $r = \frac{d}{2} = \frac{1\times10^{-5}}{2} = 0.5 \times 10^{-5}$ m

$$\Delta P = \frac{4 \times 0.0735}{0.5 \times 10^{-5}} = \frac{0.294}{0.5 \times 10^{-5}} = \frac{0.294}{5 \times 10^{-6}}$$
$$\Delta P = 58{,}800\ \mathrm{N/m^2} = 58.8\ \mathrm{kPa}$$

Final Answer:

$$\boxed{\Delta P = 58.8\ \mathrm{kPa}}$$

Topic/Formula Group: Viscosity (Newton's law – linear velocity gradient)
Marks Group: 7
Numerical Question:

1. A thin square plate 0.3 m × 0.3 m is placed horizontally in the middle of a gap of height 2.5 cm. This gap is filled with oil of viscosity 0.2 Pa · s and the plate is pulled at constant velocity of $0.2\ \mathrm{m/s}$. Find the total force on the plate.
   - **Appeared in:** 3141906(2) (Q2c, 07 marks)
   - **Frequency Tag:** Single/Unique

- **Input Variables:** plate area = 0.3×0.3 m, gap = 2.5 cm = 0.025 m, μ = 0.2 Pa·s, plate velocity = 0.2 m/s

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Answer:

Given:

Plate dimensions 0.3 m × 0.3 m

Gap height $y = 2.5\text{ cm} = 0.025\text{ m}$

Dynamic viscosity $\mu = 0.2\text{ Pa}\cdot\text{s}$

Plate velocity $V = 0.2\text{ m/s}$

To find: Total force on the plate F

Assumptions:

- Linear velocity gradient across the gap
- Plate is in the middle of the gap → two identical oil films (above & below)

Formula:

Newton's law of viscosity:

$$\tau = \mu \frac{du}{dy} = \mu \frac{V}{y/2} \text{(since gap is split into two equal films)}$$

Shear force $F = \tau \times A$ (both sides of plate)

Solution:

Velocity gradient for one side:

$$\frac{du}{dy} = \frac{V}{y/2} = \frac{0.2}{0.025/2} = \frac{0.2}{0.0125} = 16\text{ s}^{-1}$$

Shear stress:

$$\tau = \mu \times \frac{du}{dy} = 0.2 \times 16 = 3.2\text{ Pa}$$

Area of one side:

$$A = 0.3 \times 0.3 = 0.09\text{ m}^2$$

Force on one side:

$$F_{\text{one side}} = \tau \times A = 3.2 \times 0.09 = 0.288\text{ N}$$

Total force (both sides):

$$F = 2 \times 0.288 = 0.576\text{ N}$$

Final Answer:

$$\boxed{F = 0.576\text{ N}}$$

2. A 120 mm diameter disc rotates on a table separated by an oil film of 1.8 mm thickness. Find the viscosity of oil if the torque required to rotate the disc at 60 rpm is 0.00072 Nm. Assume the velocity gradient in the oil film to be linear.
   - **Appeared in:** 3141906 (Q2c, 07 marks)
   - **Frequency Tag:** Single/Unique
   - **Input Variables:** disc diameter = 120 mm = 0.12 m, film thickness = 1.8 mm = 0.0018 m, N = 60 rpm, torque = 0.00072 Nm

Answer:

Given:

Disc diameter $D = 120\text{ mm} = 0.12\text{ m} \rightarrow$ radius $R = 0.06\text{ m}$
Oil film thickness $t = 1.8\text{ mm} = 0.0018\text{ m}$
Rotational speed $N = 60\text{ rpm}$
Torque $T = 0.00072\text{ Nm}$
Velocity gradient linear.

To find: Viscosity μ of oil

Formula:

For a rotating disc on a flat surface with linear velocity gradient:
Shear stress at radius r:

$$\tau = \mu \frac{du}{dy} = \mu \frac{\omega r}{t}$$

Elemental torque:

$$dT = \tau \times dA \times r = \mu \frac{\omega r}{t} \times (2\pi r\, dr) \times r$$
$$dT = \frac{2\pi\mu\omega}{t} r^3 dr$$

Integrate from $r = 0$ to $r = R$:

$$T = \frac{2\pi\mu\omega}{t} \times \frac{R^4}{4} = \frac{\pi\mu\omega R^4}{2t}$$

Thus:

$$\mu = \frac{2tT}{\pi\omega R^4}$$

Solution:

Angular velocity:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 60}{60} = 2\pi\text{ rad/s}$$
$$\mu = \frac{2 \times 0.0018 \times 0.00072}{\pi \times (2\pi) \times (0.06)^4}$$

First compute $(0.06)^4 = 0.06^2 = 0.0036, \times 0.0036 = 1.296 \times 10^{-5}$
Denominator: $\pi \times 2\pi = 2\pi^2 = 2 \times 9.8696 = 19.7392$

$$19.7392 \times 1.296 \times 10^{-5} = 2.558 \times 10^{-4}$$

Numerator: $2 \times 0.0018 \times 0.00072 = 2 \times 1.296 \times 10^{-6} = 2.592 \times 10^{-6}$

$$\mu = \frac{2.592 \times 10^{-6}}{2.558 \times 10^{-4}} = 0.01013\text{ Pa} \cdot \text{s}$$

Final Answer:

$$\boxed{\mu = 0.01013\text{ Pa} \cdot \text{s}}$$

Topic/Formula Group: Hydrostatic Force on Submerged Surfaces

Marks Group: 7
Numerical Question:

1. An annular circular plate of 2 m external diameter and 1 m internal diameter is immersed vertically in water so that its lowest edge is 5 m below the free water surface. Determine the total force and position of centre of pressure.
   - **Appeared in:** 3141906(5) (Q2c, 07 marks)
   - **Frequency Tag:** Most Repeated
   - **Input Variables:** D_ext = 2 m, D_int = 1 m, lowest edge depth = 5 m, fluid = water

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Answer:

Given:

External diameter $D_{\text{ext}} = 2\text{ m} \rightarrow R_{\text{ext}} = 1\text{ m}$
Internal diameter $D_{\text{int}} = 1\text{ m} \rightarrow R_{\text{int}} = 0.5\text{ m}$
Lowest edge depth $= 5\text{ m below free surface}$
Fluid: water ($\rho = 1000\text{ kg/m}^3$)
Plate immersed vertically.

To find:

(i) Total hydrostatic force F
(ii) Centre of pressure h^* (depth from free surface)

Formula:

For a vertically submerged surface:

$$F = \rho g \bar{h} \times A$$
$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

Where $\bar{h}$ = depth of centroid, I_G = second moment of area about centroidal horizontal axis.

Solution:

Step 1 – Geometry & centroid depth

Plate height = $D_{\text{ext}} = 2\text{ m}$ (outer diameter = total height).
Lowest edge at 5 m → top edge at $5 - 2 = 3\text{ m}$ depth.
Centroid depth $\bar{h} = \frac{3+5}{2} = 4\text{ m}$.

Step 2 – Area (annulus)

$$A = \frac{\pi}{4}(D_{\text{ext}}^2 - D_{\text{int}}^2) = \frac{\pi}{4}(2^2 - 1^2) = \frac{\pi}{4}(4-1) = \frac{3\pi}{4} = 2.356\text{ m}^2$$

Step 3 – Total force

$$F = \rho g \bar{h} A = 1000 \times 9.81 \times 4 \times 2.356$$
$$F = 1000 \times 9.81 \times 9.424 = 92{,}456\text{ N} = 92.456\text{ kN}$$

Step 4 – Centre of pressure

For an annulus about its horizontal centroidal axis:

$$I_G = \frac{\pi}{64}(D_{\text{ext}}^4 - D_{\text{int}}^4) = \frac{\pi}{64}(2^4 - 1^4) = \frac{\pi}{64}(16-1) = \frac{15\pi}{64} = 0.7363\text{ m}^4$$
$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{0.7363}{2.356 \times 4} + 4$$
$$= \frac{0.7363}{9.424} + 4 = 0.0781 + 4 = 4.0781\text{ m}$$

Final Answer:

$$\boxed{F = 92.46\,\text{kN}, h^* = 4.078\,\text{m}}$$

2. An isosceles triangular plate of base 5 mm and height 5 mm is immersed vertically in an oil of specific gravity 0.8. The base of the plate is 1 m below the free liquid surface. Determine: (i) total pressure, (ii) centre of pressure.
   - **Appeared in:** 3141906 (OR Q2c, 07 marks)
   - **Frequency Tag:** Most Repeated
   - **Input Variables:** base = 5 mm, height = 5 mm, oil SG = 0.8, base depth = 1 m

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Answer:

Given:

Base $b = 5\,\text{mm} = 0.005\,\text{m}$

Height $h_{\text{triangle}} = 5\,\text{mm} = 0.005\,\text{m}$

Oil specific gravity $SG = 0.8 \rightarrow \rho_{\text{oil}} = 800\,\text{kg/m}^3$, $\gamma_{\text{oil}} = 7848\,\text{N/m}^3$

Base is 1 m below free surface (i.e., top edge? Actually "base of the plate is 1 m below free surface" – for triangle with base horizontal, base is at depth 1 m, vertex at depth 1 + 0.005 = 1.005 m)

To find: (i) Total pressure (force), (ii) Centre of pressure

Assumptions: Plate is vertical, base horizontal at top (since base is mentioned as 1 m below surface, vertex below base).

Solution:

Step 1 – Geometry

Triangle height = 0.005 m, base = 0.005 m.

For a triangle with base at top, centroid from base = $h/3 = 0.005/3 = 0.001667\,\text{m}$ downward.

Centroid depth $\bar{h} = 1 + 0.001667 = 1.001667\,\text{m}$

Step 2 – Area

$$A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 0.005 \times 0.005 = 1.25 \times 10^{-5}\,\text{m}^2$$

Step 3 – Total force

$$F = \gamma_{\text{oil}} \cdot \bar{h} \cdot A = 7848 \times 1.001667 \times 1.25 \times 10^{-5}$$
$$= 7848 \times 1.2521 \times 10^{-5} = 0.0982\,\text{N}$$

Step 4 – Centre of pressure

For triangle with base at top, I_G about centroidal horizontal axis = $\frac{bh^3}{36} = \frac{0.005 \times (0.005)^3}{36} = \frac{0.005 \times 1.25 \times 10^{-7}}{36} = \frac{6.25 \times 10^{-10}}{36} = 1.736 \times 10^{-1}\ \ \text{m}^4$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{1.736 \times 10^{-11}}{(1.25 \times 10^{-5}) \times 1.001667} + 1.001667$$
$$= \frac{1.736 \times 10^{-11}}{1.2521 \times 10^{-5}} + 1.001667 = 1.386 \times 10^{-6} + 1.001667 \approx 1.001668\,\text{m}$$

Final Answer:

$$\boxed{F = 0.0982\,\text{N}, h^* = 1.00167\,\text{m (almost at centroid due to tiny plate)}}$$

3. A rectangular plane surface is 1 m wide and 1.5 m deep, having a circular hole of 0.5 m diameter at the centre. The upper edge and lower edge are below free surface being 1 m and 2 m respectively. Calculate the magnitude, direction and location of the force acting upon one side of the plate due to water pressure.
    - **Appeared in:** SUM25 (Q2c, 07 marks)
    - **Frequency Tag:** Most Repeated
    - **Input Variables:** width = 1 m, depth = 1.5 m, hole diameter = 0.5 m, upper edge depth = 1 m, lower edge depth = 2 m
    - **Diagram needed** (hole geometry)

Answer:

Given:

Rectangular plate: width $W = 1$ m, depth (height) $H_{\text{rect}} = 1.5$ m

Circular hole diameter $d = 0.5$ m at centre of plate

Upper edge depth $= 1$ m, lower edge depth $= 2$ m

Water ($\gamma = 9810$ N/m^3)

To find: Magnitude, direction, location of force on one side of plate (net force after subtracting hole)

Solution:

Step 1 – Force on solid plate (without hole)

Centroid depth of rectangle = $\frac{1+2}{2} = 1.5$ m

Area of rectangle $A_{\text{rect}} = 1 \times 1.5 = 1.5$ m^2

$$F_{\text{rect}} = \gamma \bar{h} A_{\text{rect}} = 9810 \times 1.5 \times 1.5 = 22{,}072.5 \text{ N}$$

Centre of pressure for rectangle: $I_G = \frac{WH^3}{12} = \frac{1\times(1.5)^3}{12} = \frac{3.375}{12} = 0.28125$ m^4

$$h^*_{\text{rect}} = \frac{I_G}{A_{\text{rect}}\bar{h}} + \bar{h} = \frac{0.28125}{1.5 \times 1.5} + 1.5 = \frac{0.28125}{2.25} + 1.5 = 0.125 + 1.5 = 1.625 \text{ m}$$

Step 2 – Force on hole (imaginary, same fluid would act on hole area)

Hole centroid at same depth as plate centroid = 1.5 m (since hole at centre)

Area of hole $A_{\text{hole}} = \frac{\pi}{4}(0.5)^2 = 0.19635$ m^2

$$F_{\text{hole}} = \gamma \bar{h} A_{\text{hole}} = 9810 \times 1.5 \times 0.19635 = 2{,}888.6 \text{ N}$$

Centre of pressure for circle: $I_G = \frac{\pi d^4}{64} = \frac{\pi(0.5)^4}{64} = \frac{\pi\times 0.0625}{64} = 0.003068$ m^4

$$h^*_{\text{hole}} = \frac{I_G}{A_{\text{hole}}\bar{h}} + \bar{h} = \frac{0.003068}{0.19635 \times 1.5} + 1.5 = \frac{0.003068}{0.2945} + 1.5 = 0.01042 + 1.5 = 1.5104 \text{ m}$$

Step 3 – Net force

Net force = $F_{\text{rect}} - F_{\text{hole}} = 22{,}072.5 - 2{,}888.6 = 19{,}183.9$ N

Step 4 – Location of net force (moment equilibrium about free surface)

Let h^*_{net} be the depth of net force.

$$F_{\text{net}} \cdot h^*_{\text{net}} = F_{\text{rect}} \cdot h^*_{\text{rect}} - F_{\text{hole}} \cdot h^*_{\text{hole}}$$
$$19{,}183.9 \times h^*_{\text{net}} = (22{,}072.5 \times 1.625) - (2{,}888.6 \times 1.5104)$$
$$= 35{,}867.8 - 4{,}363.5 = 31{,}504.3$$

$$h^*_{\text{net}} = \frac{31{,}504.3}{19{,}183.9} = 1.642 \text{ m}$$

Direction: Perpendicular to plate (horizontal).
Final Answer:

$$\boxed{F_{\text{net}} = 19.18 \text{ kN}, h^*_{\text{net}} = 1.642 \text{ m below free surface}}$$

4. Panel ABC in the slanted side of a water tank is an isosceles triangle with vertex at A and base $BC = 2$ m as shown in figure. Find the water force on the panel and its line of action.
   - **Appeared in:** WIN24 (Q2c, 07 marks)
   - **Frequency Tag:** Most Repeated
   - **Input Variables:** base BC = 2 m
   - **Diagram needed**

Answer:

5. A tank of water has a gate of 5 m height and 3 m width in its vertical wall. The top edge of the gate is 2 m below the water surface. Calculate hydrostatic force on the gate and location of centre of pressure.
   - **Appeared in:** WIN24 (Q2c alternate, 07 marks)
   - **Frequency Tag:** Most Repeated
   - **Input Variables:** gate height = 5 m, width = 3 m, top edge depth = 2 m

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Answer:
Given:
Gate height $H = 5$ m, width $W = 3$ m
Top edge depth $= 2$ m below water surface
Vertical wall (θ=90°)
To find: Hydrostatic force and centre of pressure
Solution:
Step 1 – Centroid depth
Bottom edge depth = $2 + 5 = 7$ m
Centroid depth $\bar{h} = \frac{2+7}{2} = 4.5$ m
Step 2 – Area

$$A = W \times H = 3 \times 5 = 15 \text{ m}^2$$

Step 3 – Total force

$$F = \gamma_{\text{water}} \cdot \bar{h} \cdot A = 9810 \times 4.5 \times 15 = 9810 \times 67.5 = 662{,}175 \text{ N} = 662.2 \text{ kN}$$

Step 4 – Centre of pressure
For rectangle: $I_G = \frac{WH^3}{12} = \frac{3\times(5)^3}{12} = \frac{3\times125}{12} = \frac{375}{12} = 31.25 \text{ m}^4$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{31.25}{15 \times 4.5} + 4.5 = \frac{31.25}{67.5} + 4.5 = 0.463 + 4.5 = 4.963 \text{ m}$$

Final Answer:

$$\boxed{F = 662.2\,\text{kN}, h^* = 4.96\,\text{m below free surface}}$$

6. A circular opening 2.5 m diameter, in a vertical side of a tank is closed by a disc of 2.5 m diameter which can rotate about a horizontal diameter. Determine: (i) the force on the disc, (ii) the torque required to maintain the disc in equilibrium in vertical position when the head of water above the horizontal diameter is 3.5 m.
   - **Appeared in:** W25 (Q2c, 07 marks)
   - **Frequency Tag:** Most Repeated
   - **Input Variables:** disc diameter = 2.5 m, head above horizontal diameter = 3.5 m

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Answer:
Given:
Circular opening diameter $D = 2.5\,\text{m}$ (disc same diameter)
Disc rotates about a horizontal diameter
Head of water above horizontal diameter $= 3.5\,\text{m}$
Water $\gamma = 9810\,\text{N/m}^3$
To find: (i) Force on disc, (ii) Torque required to maintain disc in vertical position
Solution:
Step 1 – Force on disc
Centroid of disc = centre of disc. Horizontal diameter is at depth $\bar{h} = 3.5\,\text{m}$.
Area $A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(2.5)^2 = \frac{\pi}{4} \times 6.25 = 4.9087\,\text{m}^2$

$$F = \gamma \bar{h} A = 9810 \times 3.5 \times 4.9087 = 9810 \times 17.1805 = 168{,}540\,\text{N} = 168.54\,\text{kN}$$

Step 2 – Torque
When disc is vertical, the hydrostatic force acts at centre of pressure, which is offset from the horizontal diameter axis. For a circle, $I_G = \frac{\pi D^4}{64} = \frac{\pi (2.5)^4}{64} = \frac{\pi \times 39.0625}{64} = 1.917\,\text{m}^4$
Distance from centroid to centre of pressure (vertical offset):

$$y_{cp} = \frac{I_G}{A\bar{h}} = \frac{1.917}{4.9087 \times 3.5} = \frac{1.917}{17.1805} = 0.1116\,\text{m}$$

This is the eccentricity from the horizontal diameter. The force creates a torque about the horizontal diameter:

$$T = F \times y_{cp} = 168{,}540 \times 0.1116 = 18{,}809\,\text{Nm}$$

Final Answer:

$$\boxed{F = 168.54\,\text{kN}, T = 18.81\,\text{kNm}}$$

Topic/Formula Group: Buoyancy & Metacentric Height
Marks Group: 4
Numerical Question:

1. A rectangular block of size 3 m long × 1.5 m wide × 1 m high floats in water such that its depth of immersion is 0.8 m. What is the weight of block? Determine metacentric height and comment on outcome.
   - **Appeared in:** 3141906(2) (Q3b, 04 marks)
   - **Frequency Tag:** Single/Unique

- **Input Variables:** L = 3 m, B = 1.5 m, H = 1 m, immersion depth = 0.8 m, fluid = water

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Answer:

Given:
Block: length $L = 3$ m, width $B = 1.5$ m, height $H = 1$ m
Immersion depth $d = 0.8$ m
Fluid: water ($\rho = 1000$ kg/m^3, $\gamma = 9810$ N/m^3)

To find:
(i) Weight of block W
(ii) Metacentric height GM
(iii) Comment on stability

Solution:

Step 1 – Weight of block
From buoyancy principle: Weight = weight of displaced water
Displaced volume $V_{\text{disp}} = L \times B \times d = 3 \times 1.5 \times 0.8 = 3.6\text{ m}^3$

$$W = \gamma_{\text{water}} \times V_{\text{disp}} = 9810 \times 3.6 = 35{,}316\text{ N} = 35.32\text{ kN}$$

Step 2 – Metacentric height

$$GM = \frac{I}{V_{\text{disp}}} - BG$$

Where I = second moment of waterplane area about longitudinal axis,
BG = distance between centre of buoyancy (B) and centre of gravity (G).
Waterplane area = $L \times B = 3 \times 1.5 = 4.5\text{ m}^2$

$$I = \frac{LB^3}{12} = \frac{3 \times (1.5)^3}{12} = \frac{3 \times 3.375}{12} = \frac{10.125}{12} = 0.84375\text{ m}^4$$

$$\frac{I}{V_{\text{disp}}} = \frac{0.84375}{3.6} = 0.2344\text{ m}$$

Centre of buoyancy from bottom = $d/2 = 0.4$ m
Centre of gravity from bottom = $H/2 = 0.5$ m

$$BG = 0.5 - 0.4 = 0.1\text{ m}$$
$$GM = 0.2344 - 0.1 = 0.1344\text{ m}$$

Comment: $GM > 0$ → block is **stable** in floating position.

Final Answer:

$$\boxed{W = 35.32\text{ kN}, GM = 0.1344\text{ m (stable)}}$$

Marks Group: 7

Numerical Question:

1. A cylinder block weighs 22 kN having diameter of 2 m and height 2.5 m is to float in sea water (S = 1.025). Show that it does not float vertically.
   - **Appeared in:** SUM25 (OR Q2c, 07 marks)
   - **Frequency Tag:** Single/Unique
   - **Input Variables:** weight = 22 kN, diameter = 2 m, height = 2.5 m, sea water SG = 1.025

Answer:

Given:

Weight $W = 22\text{ kN} = 22{,}000\text{ N}$

Diameter $D = 2\text{ m}$, Height $H = 2.5\text{ m}$

Sea water $SG = 1.025 \rightarrow \gamma_{\text{sea}} = 1.025 \times 9810 = 10{,}055.25\text{ N/m}^3$

To find: Show that it does not float vertically

Solution:

For vertical floating, the block must be in equilibrium with buoyancy equal to weight. The block will sink to a draft d such that weight = buoyancy.

Step 1 – Volume of block

$$V_{\text{block}} = \frac{\pi}{4}D^2H = \frac{\pi}{4} \times 4 \times 2.5 = \pi \times 2.5 = 7.854\text{ m}^3$$

Step 2 – Draft if vertical

Let draft = d. Displaced volume = $\frac{\pi}{4}D^2d = \frac{\pi}{4} \times 4 \times d = \pi d$

Buoyancy = $\gamma_{\text{sea}} \times \pi d = 10{,}055.25 \times \pi d$

Set equal to weight:

$$10{,}055.25 \times \pi d = 22{,}000$$
$$d = \frac{22{,}000}{10{,}055.25 \times \pi} = \frac{22{,}000}{31{,}590} = 0.696\text{ m}$$

Draft < Height → would float vertically if stable.

Step 3 – Check stability (metacentric height)

For a cylinder floating vertically:

Metacentric height $GM = \frac{I}{V_{\text{disp}}} - BG$

$$I = \frac{\pi D^4}{64} = \frac{\pi \times 16}{64} = \frac{16\pi}{64} = 0.7854\text{ m}^4$$
$$V_{\text{disp}} = \pi d = \pi \times 0.696 = 2.186\text{ m}^3$$
$$\frac{I}{V_{\text{disp}}} = \frac{0.7854}{2.186} = 0.359\text{ m}$$

Centre of buoyancy from bottom = $d/2 = 0.348\text{ m}$

Centre of gravity from bottom = $H/2 = 1.25\text{ m}$ (assuming uniform cylinder)

$$BG = 1.25 - 0.348 = 0.902\text{ m}$$

$GM = 0.359 - 0.902 = -0.543\text{ m}$ → **Negative, unstable.**

Thus it will not float vertically; it will tilt.

Final Answer:

$$\boxed{GM = -0.543\text{ m (unstable)} \Rightarrow \text{Does not float vertically}}$$

Assignment – 1 (Question Related to Unit 1&2)

Q-1: Fluid Properties (Viscosity): A flat plate with an area of 0.2 m2 is pulled at a velocity of 0.5 m/s over a stationary base plate. The gap between the plates is 0.8 mm and is filled with a biodiesel blend acting as a lubricant. If the force required to maintain the motion is 15 N, determine the dynamic viscosity and kinematic

viscosity of the biodiesel blend. (Assume the specific gravity of the blend is 0.88).

Answer:
Given:
Area $A = 0.2 \text{ m}^2$
Velocity $u = 0.5 \text{ m/s}$
Gap $dy = 0.8 \text{ mm} = 0.0008 \text{ m}$
Force $F = 15 \text{ N}$
Specific gravity $SG = 0.88$
Density of water $\rho_w = 1000 \text{ kg/m}^3$
To Find:
Dynamic viscosity μ (Pa·s or N·s/m²)
Kinematic viscosity ν (m²/s)
Formula:
Shear stress $\tau = \frac{F}{A} = \mu \frac{du}{dy}$
Solution:

$$\tau = \frac{15}{0.2} = 75 \text{ Pa}$$
$$\frac{du}{dy} = \frac{0.5}{0.0008} = 625 \text{ s}^{-1}$$
$$\mu = \frac{\tau}{du/dy} = \frac{75}{625} = 0.12 \text{ Pa} \cdot \text{s}$$

Density of biodiesel $\rho = 0.88 \times 1000 = 880 \text{ kg/m}^3$

$$\nu = \frac{\mu}{\rho} = \frac{0.12}{880} = 1.364 \times 10^{-4} \text{ m}^2/\text{s}$$

Final Answer:

$$\boxed{\mu = 0.12 \text{ Pa} \cdot \text{s}, \nu = 1.364 \times 10^{-4} \text{ m}^2/\text{s}}$$

Q-2: Capillarity: Calculate the capillary rise of water in a glass tube of 2.5 mm diameter when submerged vertically in water. Take the surface tension of water as 0.0725 N/m and the angle of contact as 0°. How would this value change if the fluid was an automotive brake fluid with a higher surface tension?

Answer:
Given:
Diameter $d = 2.5 \text{ mm} = 0.0025 \text{ m} \rightarrow$ radius $r = 0.00125 \text{ m}$
Surface tension $\sigma = 0.0725 \text{ N/m}$
Contact angle $\theta = 0°$
Density of water $\rho = 1000 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$
To Find: Capillary rise h
Formula:

$$h = \frac{2\sigma \cos\ \theta}{\rho g r}$$

Solution:

$$h = \frac{2 \times 0.0725 \times \cos\ 0°}{1000 \times 9.81 \times 0.00125}$$
$$h = \frac{0.145}{12.2625} = 0.01182\ \text{m} = 11.82\ \text{mm}$$

Effect of higher surface tension:
If the brake fluid has higher surface tension, capillary rise increases proportionally, assuming similar density and contact angle. Higher σ → stronger cohesive forces → greater meniscus curvature → higher rise.
Final Answer:

$$\boxed{h = 11.82\ \text{mm}}$$

Q-3: Pressure Measurement: A U-tube differential manometer connects two pressure pipes, A and B. Pipe A contains a coolant fluid (Specific Gravity = 1.05) at a pressure of 1.2 x 105 N/m2, and Pipe B contains oil (Specific Gravity = 0.9) at a pressure of 2.0 x 105 N/m2. The manometer liquid is mercury. Find the difference in the mercury level if pipe B is positioned 0.5 m higher than pipe A.

Answer:
Given:
Pipe A: Coolant $SG_1 = 1.05$, $P_A = 1.2 \times 10^5$ Pa
Pipe B: Oil $SG_2 = 0.9$, $P_B = 2.0 \times 10^5$ Pa
Manometer liquid: Mercury $SG_m = 13.6$
$z_B = z_A + 0.5$ m (B is higher)
To Find: Difference in mercury level h (in m)
Formula:
Equality of pressures at same horizontal level in the manometer.
Solution:
Let level of mercury in limb A be reference.
Pressure at A + pressure due to coolant column above mercury on left = Pressure at B + pressure due to oil column + pressure due to mercury difference.
But careful: Pipe B is higher by 0.5 m. Let's define:
Mercury level difference h (higher in limb A? We'll solve).
Equate pressures at mercury level in limb A:

$$P_A + \rho_1 g(0.5 + h) = P_B + \rho_2 g(0.5) + \rho_m gh$$

Where $\rho_1 = 1.05 \times 1000 = 1050\ \text{kg/m}^3$
$$\rho_2 = 900\ \text{kg/m}^3$$
$$\rho_m = 13600\ \text{kg/m}^3$$
Substitute:

$$1.2 \times 10^5 + (1050)(9.81)(0.5 + h)$$
$$= 2.0 \times 10^5 + (900)(9.81)(0.5) + (13600)(9.81)h$$
$$1.2 \times 10^5 + 5150.25 + 10300.5h = 2.0 \times 10^5 + 4414.5 + 133416h$$
$$125150.25 + 10300.5h = 204414.5 + 133416h$$
$$125150.25 - 204414.5 = (133416 - 10300.5)h$$
$$-79264.25 = 123115.5h$$

Negative h means mercury level higher in limb B.

$$h = -\frac{79264.25}{123115.5} = -0.6437\ \text{m}$$

Magnitude $h = 0.644$ m (mercury higher in limb B)
Final Answer:

$$\boxed{h = 0.644\ \text{m}}$$

Q-4: Hydrostatic Forces: A circular plate of 2 m diameter is submerged vertically in a liquid of specific gravity 0.95 such that its top edge is 1 m below the free surface. Calculate the total hydrostatic pressure force on the plate and the depth of the center of pressure.

Answer:
Given:
Diameter $D = 2$ m → radius $R = 1$ m
Specific gravity $SG = 0.95 \rightarrow \rho = 950$ kg/m^3
Top edge is 1 m below free surface: $h_c = 1 + R = 2$ m (center of circular plate from surface)
To Find: Total hydrostatic force F and depth of center of pressure h_p
Formula:

$$F = \rho g h_c A$$
$$A = \pi R^2 = \pi(1)^2 = \pi\ \text{m}^2$$

$I_G = \frac{\pi R^4}{4} = \frac{\pi(1)^4}{4} = \frac{\pi}{4}$ m^4 for circle

$$h_p = h_c + \frac{I_G}{h_c A}$$

Solution:

$$F = 950 \times 9.81 \times 2 \times \pi = 950 \times 9.81 \times 2 \times 3.1416$$
$$F = 950 \times 61.68 = 58596\ \text{N} \approx 58.6\ \text{kN}$$
$$h_p = 2 + \frac{\pi/4}{2 \times \pi} = 2 + \frac{1/4}{2} = 2 + 0.125 = 2.125\ \text{m}$$

Final Answer:

$$\boxed{F = 58.6\ \text{kN}, h_p = 2.125\ \text{m}}$$

Q-5: Buoyancy: A solid cylinder of diameter 3 m and height 2 m floats in water with its vertical axis perpendicular to the surface. If the specific gravity of the cylinder material is 0.65, determine its metacentric height and state whether its equilibrium is stable.

Answer:
Given:
Cylinder: $D = 3$ m, $H = 2$ m
SG = 0.65 → $\rho_{cyl} = 650$ kg/m^3
Floats in water ($\rho = 1000$)
To Find: Metacentric height GM and stability
Solution:
Let submerged depth = d.
Weight = Buoyancy:

$$650 \times 9.81 \times \frac{\pi D^2}{4} \times H = 1000 \times 9.81 \times \frac{\pi D^2}{4} \times d$$

Cancel common terms:

$$650 \times 2 = 1000 \times d \Rightarrow d = 1.3 \text{ m}$$

Center of buoyancy $OB = d/2 = 0.65$ m from bottom.
Center of gravity $OG = H/2 = 1.0$ m from bottom.
So $BG = 1.0 - 0.65 = 0.35$ m.
Metacentric radius $BM = \frac{I}{V_{sub}}$

$$I = \frac{\pi D^4}{64} = \frac{\pi(81)}{64} = 3.976 \text{ m}^4$$

$$V_{sub} = \frac{\pi D^2}{4} \times d = \frac{\pi \times 9}{4} \times 1.3 = 9.186 \text{ m}^3$$

$$BM = \frac{3.976}{9.186} = 0.433 \text{ m}$$

$$GM = BM - BG = 0.433 - 0.35 = 0.083 \text{ m} > 0$$

Stability: Positive GM → stable equilibrium.
Final Answer:

$$\boxed{GM = 0.083 \text{ m,Stable}}$$

Unit 2: Fluid Kinematics & Fluid Dynamics

(As per new Syllabus For Unit- 3&4 Numerical)

Topic/Formula Group: Velocity & Acceleration Field
Marks Group: 7
Numerical Question:

1. If the velocity components are given by

$$u = 8 + 4xy + t^2, v = -(xy + 20t), w = 5x + y$$

Determine velocity and acceleration of a particle at (2, 1, 1) and $t = 1$ sec.

   - **Appeared in:** 3141906(2) (Q3c, 07 marks)
   - **Frequency Tag:** Single/Unique
   - **Input Variables:** u, v, w as functions; point (2,1,1); time t = 1 s

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Answer:

Given:

$$u = 8 + 4xy + t^2, v = -(xy + 20t), w = 5x + y$$

Point $(x, y, z) = (2,1,1)$, time $t = 1$ s

To find: Velocity $\vec{V}$ and acceleration $\vec{a}$ at that point & time

Formula:

$$\vec{V} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$$

$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$ (similarly for a_y, a_z)

Solution:

Step 1 – Velocity components at (2,1,1), t=1

$$u = 8 + 4(2)(1) + (1)^2 = 8 + 8 + 1 = 17 \text{ m/s}$$
$$v = -[(2)(1) + 20(1)] = -(2 + 20) = -22 \text{ m/s}$$
$$w = 5(2) + 1 = 10 + 1 = 11 \text{ m/s}$$
$$\vec{V} = 17\hat{\imath} - 22\hat{\jmath} + 11\hat{k} \text{ m/s}$$

Step 2 – Acceleration components

First compute partial derivatives:

For u:

$\frac{\partial u}{\partial t} = 2t = 2$ at t=1

$$\frac{\partial u}{\partial x} = 4y = 4(1) = 4$$
$$\frac{\partial u}{\partial y} = 4x = 4(2) = 8$$
$$\frac{\partial u}{\partial z} = 0$$

$$a_x = 2 + (17)(4) + (-22)(8) + (11)(0) = 2 + 68 - 176 + 0 = -106 \text{ m/s}^2$$

For v:

$v = -xy - 20t \rightarrow \frac{\partial v}{\partial t} = -20$

$$\frac{\partial v}{\partial x} = -y = -1$$
$$\frac{\partial v}{\partial y} = -x = -2$$
$$\frac{\partial v}{\partial z} = 0$$
$$a_y = -20 + (17)(-1) + (-22)(-2) + (11)(0) = -20 - 17 + 44 = 7 \text{ m/s}^2$$

For w:

$w = 5x + y \rightarrow \frac{\partial w}{\partial t} = 0$

$$\frac{\partial w}{\partial x} = 5$$
$$\frac{\partial w}{\partial y} = 1$$
$$\frac{\partial w}{\partial z} = 0$$
$$a_z = 0 + (17)(5) + (-22)(1) + (11)(0) = 85 - 22 = 63 \text{ m/s}^2$$
$$\vec{a} = -106\hat{\imath} + 7\hat{\jmath} + 63\hat{k} \text{ m/s}^2$$

Final Answer:

$$\boxed{\vec{V} = 17\hat{\imath} - 22\hat{\jmath} + 11\hat{k} \text{ m/s}}$$
$$\boxed{\vec{a} = -106\hat{\imath} + 7\hat{\jmath} + 63\hat{k} \text{ m/s}^2}$$

Topic/Formula Group: Bernoulli's Equation & Venturimeter
Marks Group: 7
Numerical Question:

1. A horizontal venturimeter with inlet diameter 200 mm and throat diameter 100 mm is employed to measure the flow of water. The reading of the differential manometer connected to the inlet is 180 mm of mercury. If the coefficient of discharge is 0.98, determine the rate of flow.
   - **Appeared in:** 3141906(4) (Q2c, 07 marks)
   - **Frequency Tag:** Single/Unique
   - **Input Variables:** D1 = 200 mm, D2 = 100 mm, manometer reading = 180 mm Hg, Cd = 0.98, fluid = water

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Answer:

Given:

Inlet diameter $D_1 = 200 \text{ mm} = 0.2 \text{ m}$

Throat diameter $D_2 = 100 \text{ mm} = 0.1 \text{ m}$

Manometer reading $x = 180 \text{ mm Hg} = 0.18 \text{ m Hg}$

Coefficient of discharge $C_d = 0.98$

Fluid: water ($\rho = 1000 \text{ kg/m}^3$), mercury $\rho_m = 13600 \text{ kg/m}^3$

To find: Rate of flow Q

Formula:

For horizontal venturimeter with manometer:

$$Q = C_d \cdot \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh}$$

where $h = x\left(\frac{\rho_m}{\rho} - 1\right)$ for water-mercury manometer.

Solution:

Step 1 – Areas

$$A_1 = \frac{\pi}{4}(0.2)^2 = 0.031416\ \mathrm{m}^2$$
$$A_2 = \frac{\pi}{4}(0.1)^2 = 0.007854\ \mathrm{m}^2$$

Step 2 – Manometer head h

$$h = 0.18 \times \left(\frac{13600}{1000} - 1\right) = 0.18 \times (13.6 - 1) = 0.18 \times 12.6 = 2.268\ \mathrm{m\ of\ water}$$

Step 3 – Theoretical discharge

$$Q_{\mathrm{th}} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh}$$
$$A_1^2 = 9.87 \times 10^{-4}, A_2^2 = 6.17 \times 10^{-5}$$
$$A_1^2 - A_2^2 = 9.253 \times 10^{-4}$$
$$\sqrt{A_1^2 - A_2^2} = \sqrt{9.253 \times 10^{-4}} = 0.03042$$
$$A_1 A_2 = 0.031416 \times 0.007854 = 2.467 \times 10^{-4}$$
$$\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} = \frac{2.467 \times 10^{-4}}{0.03042} = 0.008112$$
$$\sqrt{2gh} = \sqrt{2 \times 9.81 \times 2.268} = \sqrt{44.50} = 6.671$$
$$Q_{\mathrm{th}} = 0.008112 \times 6.671 = 0.05412\ \mathrm{m}^3/\mathrm{s}$$

Step 4 – Actual discharge

$$Q = C_d \times Q_{\mathrm{th}} = 0.98 \times 0.05412 = 0.05304\ \mathrm{m}^3/\mathrm{s}$$

Final Answer:

$$\boxed{Q = 53.04\ \mathrm{L/s}(0.05304\ \mathrm{m}^3/\mathrm{s})}$$

Topic/Formula Group: Impact of Jet (Moving plates & curved vanes)
Marks Group: 7
Numerical Question:

1. A jet of water of diameter 7 cm strikes a curved plate at its centre with a velocity of 15 m/s. The curved plate is moving with a velocity of 7 m/s in the direction of the jet and it is deflected through an angle of 165°. Assuming the plate to be smooth, determine: (i) force exerted on plate in the direction of jet, (ii) power of jet, (iii) efficiency of jet.
   - **Appeared in:** 3141906(1) (Q5c, 07 marks)
   - **Frequency Tag:** Most Repeated
   - **Input Variables:** jet diameter = 7 cm, V_j = 15 m/s, V_plate = 7 m/s, deflection = 165°

Answer:

Given:

Jet diameter $d = 7$ cm $= 0.07$ m
Jet velocity $V_j = 15$ m/s
Plate velocity $u = 7$ m/s (in direction of jet)
Deflection angle $\theta = 165°$ (smooth plate → relative velocity same magnitude)
Water density $\rho = 1000$ kg/m^3

To find:

(i) Force exerted on plate in direction of jet F_x
(ii) Power of jet P
(iii) Efficiency η

Formulas:

For a moving curved vane, smooth:
Relative velocity $V_r = V_j - u$
Force in jet direction: $F_x = \rho A V_r (V_r \cos\ \phi_1 - V_r \cos\ \phi_2)$
Here $\phi_1 = 0°$ (jet strikes tangentially), $\phi_2 = 180° - \theta$
Power = $F_x \times u$
Efficiency = Power / (Kinetic energy of jet per sec)

Solution:

Step 1 – Area & relative velocity

$$A = \frac{\pi}{4}(0.07)^2 = 0.003848 \text{ m}^2$$
$$V_r = V_j - u = 15 - 7 = 8 \text{ m/s}$$

Mass flow rate (relative) $\dot{m}_r = \rho A V_r = 1000 \times 0.003848 \times 8 = 30.784$ kg/s

Step 2 – Force

Jet deflection $165°$ from original direction → outlet angle $180 - 165 = 15°$ from forward direction.

$$F_x = \dot{m}_r (V_r \cos\ 0° - V_r \cos\ 15°) = 30.784 \times 8 \times (1 - \cos\ 15°)$$
$$\cos\ 15° = 0.9659, 1 - 0.9659 = 0.0341$$
$$F_x = 30.784 \times 8 \times 0.0341 = 30.784 \times 0.2728 = 8.40 \text{ N}$$

Step 3 – Power

$$P = F_x \times u = 8.40 \times 7 = 58.8 \text{ W}$$

Step 4 – Efficiency

Kinetic energy of jet per second = $\frac{1}{2}\rho A V_j^3 = 0.5 \times 1000 \times 0.003848 \times (15)^3$

$$= 0.5 \times 1000 \times 0.003848 \times 3375 = 0.5 \times 1000 \times 12.987 = 6493.5 \text{ W}$$
$$\eta = \frac{58.8}{6493.5} \times 100 = 0.905\%$$

Final Answer:

$$\boxed{F_x = 8.40 \text{ N}, P = 58.8 \text{ W}, \eta = 0.905\%}$$

2. A jet of water of diameter 8 cm strikes a flat plate normally with a velocity of 20 m/s. The plate is moving with a velocity of 12 m/s in the direction of the jet and away from the jet. Find: force exerted by the jet on the plate, work done by the jet on the plate per second.

- **Appeared in:** 3141906(2) (OR Q4c, 07 marks)
- **Frequency Tag:** Most Repeated
- **Input Variables:** jet diameter = 8 cm, V_j = 20 m/s, V_plate = 12 m/s

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Answer:
Given:
Jet diameter $d = 8\text{ cm} = 0.08\text{ m}$
Jet velocity $V_j = 20\text{ m/s}$
Plate velocity $u = 12\text{ m/s}$ (away from jet, same direction)
Water $\rho = 1000\text{ kg/m}^3$
To find: Force exerted on plate, work done per second
Solution:
For flat plate moving away from jet (normal impact):
Relative velocity $V_r = V_j - u = 20 - 12 = 8\text{ m/s}$
Area $A = \frac{\pi}{4}(0.08)^2 = 0.005027\text{ m}^2$
Force $F = \rho A V_r^2 = 1000 \times 0.005027 \times (8)^2 = 1000 \times 0.005027 \times 64 = 321.73\text{ N}$
Work done per second (power) $= F \times u = 321.73 \times 12 = 3{,}860.8\text{ W}$
Final Answer:

$$\boxed{F = 321.7\text{ N}, P = 3.86\text{ kW}}$$

3. A horizontal jet of water with a velocity of 30 m/s impinges on a moving curved blade having a velocity of 12 m/s. The blade moves in the direction of the jet. The jet leaves the blade at an angle of 65° with the direction of motion of the blade. Blade outlet angle is 35°. Calculate: (i) % reduction of relative velocity at outlet, (ii) force per kg in the direction of motion if jet diameter is 10 cm, (iii) work done per kg.
   - **Appeared in:** 3141906(5) (Q4c, 07 marks)
   - **Frequency Tag:** Most Repeated
   - **Input Variables:** V_j = 30 m/s, V_blade = 12 m/s, jet exit angle (to motion) = 65°, blade outlet angle = 35°, jet diameter = 10 cm

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Answer:
Given:
Jet velocity $V_j = 30\text{ m/s}$
Blade velocity $u = 12\text{ m/s}$ (same direction)
Jet leaves blade at 65° to direction of motion (absolute angle?)
Blade outlet angle = 35° (relative angle)
Jet diameter $d = 10\text{ cm} = 0.1\text{ m}$
Smooth blade → relative velocity magnitude unchanged.
To find: (i) % reduction of relative velocity at outlet (for smooth blade, 0% reduction? Possibly friction included? Standard assumption: smooth → no reduction)
(ii) Force per kg in direction of motion
(iii) Work done per kg
Solution:
Step 1 – Velocities
$V_r = V_j - u = 30 - 12 = 18\text{ m/s}$ (inlet relative velocity)
Outlet relative velocity $V_{r2} = V_r = 18\text{ m/s}$ (smooth blade)
% reduction = 0% (if smooth). If friction given, else assume 0.

Step 2 – Force per kg
Mass flow rate (absolute) $\dot{m} = \rho A V_j$, but force per kg = force / mass flow rate = change in velocity per second per unit mass.
For moving curved vane: Force per kg of fluid = $(V_{r1}\cos\ \alpha_1 - V_{r2}\cos\ \alpha_2)$ in direction of motion, where α are angles of relative velocity with motion.
Inlet: relative velocity along motion → α_1=0° → $\cos\ = 1$
Outlet relative angle given as blade outlet angle = 35° (measured from direction of motion, presumably) → α_2=35° → $\cos\ 35° = 0.8192$
Force per kg = $18 \times 1 - 18 \times 0.8192 = 18(1 - 0.8192) = 18 \times 0.1808 = 3.2544$ N/kg
Step 3 – Work done per kg
Work per kg = Force per kg × blade velocity $u = 3.2544 \times 12 = 39.05$ J/kg

4. A jet of water moving with a velocity of $12\ \mathrm{m/s}$ impinges on a concave shaped vane to deflect the jet through $120°$ when stationary. If the vane moves at $5\ \mathrm{m/s}$, determine the angle of the jet so that there is no shock at the inlet. What is the absolute velocity of water at the exit in magnitude and direction? Also find the work done per unit mass of water. Assume the vane is smooth.
   - **Appeared in:** W25 (Q5c, 07 marks)
   - **Frequency Tag:** Most Repeated
   - **Input Variables:** V_j = 12 m/s, stationary deflection = 120°, vane speed = 5 m/s

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Answer:
Given:
Jet velocity $V_j = 12$ m/s
Vane speed $u = 5$ m/s (same direction)
Stationary deflection = $120°$ (i.e., when vane stationary, jet turns by 120°)
Smooth vane → relative velocity magnitude constant.
To find:
(i) Angle of jet at inlet for no shock (i.e., relative velocity tangential to vane at inlet)
(ii) Absolute velocity at exit (magnitude and direction)
(iii) Work done per unit mass
Solution:
Step 1 – Inlet conditions
No shock → relative velocity at inlet is along the vane inlet angle.
Relative velocity $V_r = V_j - u = 12 - 5 = 7$ m/s
For no shock, the vane inlet angle θ (from direction of motion) satisfies:

$$\tan\ \theta = \frac{V_j \sin\ \phi}{V_j \cos\ \phi - u}$$

But easier: Stationary deflection 120° means vane turns jet by 120° relative to its original direction. For moving vane, the relative velocity turns by same angle (smooth). So outlet relative velocity makes an angle of $180° - 120° = 60°$ with the direction of motion (if inlet relative is along motion). Let's assume inlet relative is along motion (θ=0). Then outlet relative velocity magnitude 7 m/s at 60° to motion.
Step 2 – Absolute velocity at exit
Exit relative velocity components:
$V_{r,x} = 7\cos\ 60° = 3.5$ m/s (forward)

$V_{r,y} = 7\sin\ 60° = 6.062$ m/s (perpendicular)
Absolute velocity = relative + blade velocity:

$$V_x = V_{r,x} + u = 3.5 + 5 = 8.5 \text{ m/s}$$
$$V_y = 6.062 \text{ m/s}$$

Magnitude $V_2 = \sqrt{8.5^2 + 6.062^2} = \sqrt{72.25 + 36.75} = \sqrt{109} = 10.44$ m/s
Direction $\beta = \tan^{-1}(6.062/8.5) = \tan^{-1}(0.713) = 35.5°$ from direction of motion.

Step 3 – Work per unit mass

Work = $u(V_{r1}\cos\ \alpha_1 - V_{r2}\cos\ \alpha_2)$ with α relative to motion.
Inlet: α_1=0°, cos=1; outlet: α_2=60°, cos=0.5
Work per kg = $5 \times (7 \times 1 - 7 \times 0.5) = 5 \times (7 - 3.5) = 5 \times 3.5 = 17.5$ J/kg

Final Answer:

$$\boxed{\text{Inlet angle} = 0° \text{ (relative along motion)}, V_2 = 10.44 \text{ m/s at } 35.5°, W/m = 17.5 \text{ J/kg}}$$

Assignment – 2 (Question Related to Unit 3&4)

Q-1: Continuity Equation: In a 2D incompressible flow, the fluid velocity components are given by u = 3x and v = -3y. Show whether this flow satisfies the continuity equation. Determine the stream function ψ for this flow field.

Answer:
Given: $u = 3x$, $v = -3y$, 2D incompressible flow.
Formula: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$
Solution:

$$\frac{\partial u}{\partial x} = 3, \frac{\partial v}{\partial y} = -3$$

Sum = $3 - 3 = 0 \rightarrow$ Satisfies continuity.
Stream function ψ:
$u = \frac{\partial \psi}{\partial y} = 3x \rightarrow$ Integrate: $\psi = 3xy + f(x)$
$v = -\frac{\partial \psi}{\partial x} = -3y \rightarrow$ Diff: $\frac{\partial \psi}{\partial x} = 3y + f'(x)$, but $-\frac{\partial \psi}{\partial x} = -3y \rightarrow \frac{\partial \psi}{\partial x} = 3y \rightarrow f'(x) = 0$.
Thus $\psi = 3xy + C$.
Final Answer:

$$\boxed{\text{Flow satisfies continuity}, \psi = 3xy + C}$$

Q-2: Bernoulli's Theorem Application: Water flows through a horizontal pipeline that tapers from a 300 mm diameter at section 1 to a 150 mm diameter at section 2. The pressure at section 1 is 40 kPa and the velocity is 2 m/s. Assuming no energy losses, calculate the pressure at section 2.

Answer:
Given:
$D_1 = 0.3$ m, $D_2 = 0.15$ m
$P_1 = 40$ kPa, $V_1 = 2$ m/s
Horizontal pipe, no losses.
To Find: P_2

Formula:

$$A_1V_1 = A_2V_2$$
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

Solution:
$A_1 = \pi(0.15)^2 = 0.0707\ \text{m}^2$, $A_2 = \pi(0.075)^2 = 0.01767\ \text{m}^2$

$$V_2 = V_1\frac{A_1}{A_2} = 2 \times \frac{0.0707}{0.01767} = 2 \times 4 = 8\ \text{m/s}$$
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2}$$
$$\frac{40000}{1000} + \frac{4}{2} = \frac{P_2}{1000} + \frac{64}{2}$$
$$40 + 2 = \frac{P_2}{1000} + 32$$
$$10 = \frac{P_2}{1000} \Rightarrow P_2 = 10000\ \text{Pa} = 10\ \text{kPa}$$

Final Answer:

$$\boxed{P_2 = 10\ \text{kPa}}$$

Q-3: Venturimeter: A horizontal venturimeter with an inlet diameter of 200 mm and a throat diameter of 100 mm is used to measure the flow of oil (Specific Gravity = 0.85). The discharge through the venturimeter is 60 lps. If the coefficient of discharge Cd is 0.97, find the reading of the oil-mercury differential manometer.

Answer:
Given:
Inlet diameter $D_1 = 200\ \text{mm} = 0.2\ \text{m}$
Throat diameter $D_2 = 100\ \text{mm} = 0.1\ \text{m}$
Oil SG = 0.85 → $\rho_o = 850\ \text{kg/m}^3$
Discharge $Q = 60\ \text{lps} = 0.06\ \text{m}^3/\text{s}$

$$C_d = 0.97$$

Manometer fluid: Mercury ($\rho_m = 13600\ \text{kg/m}^3$)
To Find: Manometer reading h (in meters of mercury column difference)
Formula:

$$Q = C_d \cdot \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh\left(\frac{\rho_m}{\rho_o} - 1\right)}$$

Solution:

$$A_1 = \frac{\pi}{4}(0.2)^2 = 0.031416\ \text{m}^2, A_2 = \frac{\pi}{4}(0.1)^2 = 0.007854\ \text{m}^2$$
$$\frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}} = \frac{0.031416 \times 0.007854}{\sqrt{(0.031416)^2 - (0.007854)^2}} = \frac{0.0002467}{\sqrt{0.0009868 - 0.00006168}}$$
$$= \frac{0.0002467}{\sqrt{0.0009251}} = \frac{0.0002467}{0.030415} = 0.008111$$

From venturi equation:

$$0.06 = 0.97 \times 0.008111 \times \sqrt{2 \times 9.81 \times h \times \left(\frac{13600}{850} - 1\right)}$$

$$\frac{0.06}{0.97 \times 0.008111} = \sqrt{2 \times 9.81 \times h \times (16 - 1)}$$

$$\frac{0.06}{0.007868} = 7.626 = \sqrt{294.3 \times h}$$

Square both sides:

$$58.15 = 294.3 \times h \Rightarrow h = 0.1976\,\text{m} \approx 197.6\,\text{mm}$$

Final Answer:

$$\boxed{h = 197.6\,\text{mm of mercury}}$$

Q-4: Pitot Tube: A pitot-static tube is placed in the center of a 250 mm pipe to measure the velocity of an exhaust gas flow. The stagnation pressure head is 8 mm of water, and the static pressure head is 2 mm of water. If the coefficient of the pitot tube is 0.98 and the density of the gas is 1.2 kg/m3, determine the velocity of the gas.

Answer:
Given:
Pipe diameter = 250 mm (not directly needed)
Stagnation head $h_{stag} = 8$ mm of water
Static head $h_{stat} = 2$ mm of water
Coefficient $C = 0.98$
Gas density $\rho_g = 1.2\,\text{kg/m}^3$
Water density $\rho_w = 1000\,\text{kg/m}^3$
To Find: Velocity of gas V
Formula:
Dynamic head in meters of gas = $\frac{\rho_w}{\rho_g} \times (h_{stag} - h_{stat})$ in meters of water converted to gas.

$$V = C\sqrt{2g \cdot \frac{\rho_w}{\rho_g} \cdot \Delta h}$$

where $\Delta h = 0.008 - 0.002 = 0.006$ m of water
Solution:

$$V = 0.98\sqrt{2 \times 9.81 \times \frac{1000}{1.2} \times 0.006}$$

$$\frac{1000}{1.2} \times 0.006 = 5$$

$$V = 0.98\sqrt{2 \times 9.81 \times 5} = 0.98\sqrt{98.1} = 0.98 \times 9.905 = 9.707\,\text{m/s}$$

Final Answer:

$$\boxed{V = 9.71\,\text{m/s}}$$

Q-5: Momentum Equation: A 45° reducing bend is connected in a pipeline. The diameter at the inlet and outlet of the bend are 600 mm and 300 mm respectively. Find the force exerted by water on the bend if the pressure at the inlet is 8.829 N/cm2 and the

flow rate is 600 lps

Answer:
Given:

$$\theta = 45^\circ$$
$$D_1 = 600\,\text{mm} = 0.6\,\text{m}$$
$$D_2 = 300\,\text{mm} = 0.3\,\text{m}$$
$$P_1 = 8.829\,\text{N/cm}^2 = 8.829 \times 10^4\,\text{Pa}$$
$$Q = 600\,\text{lps} = 0.6\,\text{m}^3/\text{s}$$

Water → $\rho = 1000\,\text{kg/m}^3$
To Find: Force exerted by water on the bend (magnitude and direction)
Formula:
Momentum eqn in x and y directions:
$F_x = \rho Q(V_2 \cos\,\theta - V_1) + P_1 A_1 + P_2 A_2 \cos\,\theta$ (with sign careful)
Better approach: Force by bend on water, then reverse.
Solution:

$$A_1 = \pi/4(0.6)^2 = 0.28274\,\text{m}^2$$
$$A_2 = \pi/4(0.3)^2 = 0.070686\,\text{m}^2$$
$$V_1 = Q/A_1 = 0.6/0.28274 = 2.122\,\text{m/s}$$
$$V_2 = Q/A_2 = 0.6/0.070686 = 8.488\,\text{m/s}$$

Find P_2 from Bernoulli (horizontal bend, neglecting losses):

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2}$$
$$\frac{88290}{1000} + \frac{(2.122)^2}{2} = \frac{P_2}{1000} + \frac{(8.488)^2}{2}$$
$$88.29 + 2.251 = \frac{P_2}{1000} + 36.02$$
$$90.541 - 36.02 = \frac{P_2}{1000} \Rightarrow P_2 = 54521\,\text{Pa}$$

Force by bend on water (F_x):

$$F_x = \rho Q(V_2 \cos\,45^\circ - V_1) + P_1 A_1 - P_2 A_2 \cos\,45^\circ$$
$$F_x = 1000 \times 0.6 \times (8.488 \times 0.7071 - 2.122) + 88290 \times 0.28274 - 54521 \times 0.070686 \times 0.7071$$
$$= 600 \times (6.001 - 2.122) + 24960 - 2728$$
$$= 600 \times 3.879 + 22232 = 2327.4 + 22232 = 24559.4\,\text{N}$$
$$F_y = \rho Q(V_2 \sin\,45^\circ - 0) + 0 - P_2 A_2 \sin\,45^\circ$$
$$= 600 \times 8.488 \times 0.7071 - 54521 \times 0.070686 \times 0.7071$$
$$= 600 \times 6.001 - 2728 = 3600.6 - 2728 = 872.6\,\text{N}$$

Magnitude:

$$F_R = \sqrt{(24559)^2 + (872.6)^2} \approx 24575\,\text{N}$$

Direction: $\phi = \tan^{-1}(872.6/24559) \approx 2^\circ$ from horizontal.
Force **by water on bend** is equal and opposite.
Final Answer:

$$\boxed{F_R \approx 24.58\,\text{kN, acting at } \approx 2^\circ \text{ below horizontal (if bend turns upward)}}$$

Unit 3: Laminar & Turbulent Flow Analysis

(As per new Syllabus For Unit- 6&7 Numerical)

Topic/Formula Group: Chezy's Formula (Friction loss)
Marks Group: 3
Numerical Question:

1. Determine the head lost due to friction in a pipe. Diameter and length of pipe are 250 mm and 60 m respectively. Velocity of flowing water inside pipe = 2.5 $\mathrm{m/s}$. Chezy's constant = 60. Use Chezy's formula $V = C\sqrt{mi}$.
   - **Appeared in:** 3141906(2) (Q5a, 03 marks)
   - **Frequency Tag:** Single/Unique
   - **Input Variables:** D = 250 mm = 0.25 m, L = 60 m, V = 2.5 m/s, C = 60

Answer:

Given:
Diameter $D = 250\text{ mm} = 0.25\text{ m}$
Length $L = 60\text{ m}$
Velocity $V = 2.5\text{ m/s}$
Chezy's constant $C = 60$
To find: Head lost due to friction h_f
Formula: Chezy's formula: $V = C\sqrt{mi}$
Where hydraulic mean depth $m = D/4$ (for circular pipe), $i = h_f/L$
Solution:

$$m = \frac{D}{4} = \frac{0.25}{4} = 0.0625\text{ m}$$

$$V = C\sqrt{m \cdot \frac{h_f}{L}} \implies 2.5 = 60\sqrt{0.0625 \times \frac{h_f}{60}}$$

$$\frac{2.5}{60} = \sqrt{0.0625 \times \frac{h_f}{60}}$$

$$0.04167 = \sqrt{0.0625 \times \frac{h_f}{60}}$$

Square both sides:

$$0.001736 = 0.0625 \times \frac{h_f}{60}$$

$$0.001736 = \frac{0.0625h_f}{60}$$

$$0.001736 \times 60 = 0.0625h_f$$

$$0.10416 = 0.0625h_f$$

$$h_f = \frac{0.10416}{0.0625} = 1.666\text{ m}$$

Final Answer:

$$\boxed{h_f = 1.667\text{ m}}$$

Topic/Formula Group: Pipe Sizing (Darcy-Weisbach / Friction loss)
Marks Group: 7
Numerical Question:

1. Water is to be supplied to the inhabitants of a college hostel through a supply main. Data: distance = 4000 m; inhabitants = 3000; consumption per inhabitant per day = 180 litres; head loss due to friction = 18 m; friction coefficient $f = 0.007$. Half of the daily supply is pumped in 8 hours. Determine the size of the supply main.
    - **Appeared in:** 3141906(5) (Q3c, 07 marks)
    - **Frequency Tag:** Most Repeated
    - **Input Variables:** L = 4000 m, population = 3000, per capita consumption = 180 L/day, h_f = 18 m, f = 0.007, pumping time = 8 hrs (for half daily supply)

Answer:

Length $L = 4000$ m
Population = 3000, consumption per inhabitant per day = 180 litres
Head loss due to friction $h_f = 18$ m
Friction coefficient $f = 0.007$ (Darcy friction factor)
Half of daily supply pumped in 8 hours.
To find: Size of supply main (diameter D)
Solution:
Step 1 – Daily total supply
Total per day = $3000 \times 180 = 540{,}000$ litres $= 540\ \text{m}^3$
Step 2 – Pumping rate
Half daily supply = $270\ \text{m}^3$ pumped in 8 hours.
Flow rate $Q = \frac{270}{8\times3600} = \frac{270}{28800} = 0.009375\ \text{m}^3/\text{s} = 9.375$ L/s
Step 3 – Darcy-Weisbach equation

$$h_f = \frac{fLQ^2}{12.1D^5} \text{(for pipe flow, in consistent units)}$$

Actually standard: $h_f = \frac{fLV^2}{2gD}$, with $V = \frac{4Q}{\pi D^2}$
Substitute:

$$h_f = \frac{fL}{2gD} \cdot \left(\frac{16Q^2}{\pi^2 D^4}\right) = \frac{8fLQ^2}{g\pi^2 D^5}$$

Using g=9.81, π²≈9.8696:

$$h_f = \frac{8fLQ^2}{9.81 \times 9.8696\, D^5} = \frac{8fLQ^2}{96.86\, D^5} \approx \frac{fLQ^2}{12.1\, D^5}$$

So $D^5 = \frac{fL\ \ ^2}{12.1\, h_f}$
Step 4 – Compute

$f = 0.007, L = 4000, Q = 0.009375, h_f = 18$
$Q^2 = 8.789 \times 10^{-5}$
$fLQ^2 = 0.007 \times 4000 \times 8.789 \times 10^{-5} = 28 \times 8.789 \times 10^{-5} = 2.461 \times 10^{-3}$

$$D^5 = \frac{2.461 \times 10^{-3}}{12.1 \times 18} = \frac{2.461 \times 10^{-3}}{217.8} = 1.130 \times 10^{-5}$$

$D = (1.130 \times 10^{-5})^{1/5}$
$0.1^5 = 1 \times 10^{-5}, 0.11^5 = 1.61 \times 10^{-5}$ → interpolate: D ≈ 0.102 m = 102 mm
Final Answer:

$$\boxed{D \approx 100 \text{ mm (say 100 mm pipe)}}$$

2. Calculate the discharge through a pipe of diameter 250 mm when the difference of pressure head between two ends of a pipe 500 mm apart is 3.5 m of water. Take friction factor = 0.04.
   - **Appeared in:** 3141906(5) (OR Q4b, 07 marks)
   - **Frequency Tag:** Most Repeated
   - **Input Variables:** D = 250 mm, length L = 500 mm (likely a typo – probably 500 m), h_f = 3.5 m water, f = 0.04

Answer:
Given:
Diameter $D = 250 \text{ mm} = 0.25 \text{ m}$
Length $L = 500$ mm? → Likely typo, should be 500 m. Assume $L = 500$ m
Head loss $h_f = 3.5$ m of water
Friction factor $f = 0.04$
To find: Discharge Q
Solution:
Darcy-Weisbach:

$$h_f = \frac{fLQ^2}{12.1D^5}$$
$$Q^2 = \frac{h_f \times 12.1D^5}{fL}$$

$D^5 = (0.25)^5 = 0.25^2 = 0.0625, \times 0.0625 = 0.00390625, \times 0.25 = 0.00097656$
$12.1D^5 = 12.1 \times 0.00097656 = 0.011816$
$h_f \times 12.1D^5 = 3.5 \times 0.011816 = 0.041356$
$Q^2 = \frac{0.041356}{0.04 \times 500} = \frac{0.041356}{20} = 0.0020678$
$Q = \sqrt{0.0020678} = 0.04547 \text{ m}^3/\text{s} = 45.47 \text{ L/s}$
Final Answer:

$$\boxed{Q = 45.5 \text{ L/s}}$$

3. Determine the optimum diameter of a pipe to carry 100 L/s of crude oil (density = 950 kg/m^3, dynamic viscosity = 0.08 kg/(m · s)) and still maintain laminar flow. Also determine the power required for transport over one kilometre distance.
   - **Appeared in:** WIN24 (OR Q4c, 07 marks)
   - **Frequency Tag:** Most Repeated
   - **Input Variables:** Q = 100 L/s = 0.1 m^3/s, ρ = 950 kg/m^3, μ = 0.08 Pa·s, L = 1 km, flow regime = laminar

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Answer:
Given:

$$Q = 100\ \text{L/s} = 0.1\ \text{m}^3/\text{s}$$

$\rho = 950\ \text{kg/m}^3, \mu = 0.08\ \text{Pa}\cdot\text{s}$

$$L = 1\ \text{km} = 1000\ \text{m}$$

Flow regime: laminar

To find: Optimum diameter (maximum possible for laminar?) Actually "optimum diameter" to maintain laminar flow – means find diameter such that Re ≤ 2000 (critical). Then power required.

Solution:

Step 1 – Reynolds number for laminar limit

$$Re = \frac{\rho V D}{\mu} \leq 2000$$

$V = \frac{4Q}{\pi D^2} \rightarrow Re = \frac{4\rho Q}{\pi \mu D}$

Set Re = 2000:

$$2000 = \frac{4 \times 950 \times 0.1}{\pi \times 0.08 \times D} = \frac{380}{0.2513D} = \frac{1512.5}{D}$$

$$D = \frac{1512.5}{2000} = 0.756\ \text{m}$$

Thus for laminar flow, D must be ≥ 0.756 m (larger D gives lower Re). Optimum = minimum D that keeps laminar? Usually largest possible for laminar is 0.756 m.

Step 2 – Power required

For laminar flow, Hagen-Poiseuille:

$$h_f = \frac{128 \mu L Q}{\pi \rho g D^4}$$

But power $P = \rho g Q h_f = \frac{128 \mu L Q^2}{\pi D^4}$

With D = 0.756 m:

$$D^4 = (0.756)^4 = 0.756^2 = 0.5715, \times 0.5715 = 0.3266$$

$$P = \frac{128 \times 0.08 \times 1000 \times (0.1)^2}{\pi \times 0.3266} = \frac{128 \times 0.08 \times 1000 \times 0.01}{1.026}$$

$$= \frac{102.4}{1.026} = 99.8\ \text{W}$$

Final Answer:

$$\boxed{D_{\text{opt}} = 0.756\ \text{m}, P = 100\ \text{W}}$$

Topic/Formula Group: Flow Between Parallel Plates

Marks Group: 7

Numerical Question:

1. Glycerine of specific gravity 1.28 and viscosity 8.07 poise flows between two large parallel flat plates 1.5 cm apart. The rate of flow is 4.4 m^2/hour per metre width of the plates. Determine: (i) maximum velocity, (ii) maximum shear stress, (iii) pressure gradient, (iv) Reynolds number.
   - **Appeared in:** 3141906 (OR Q3c, 07 marks)
   - **Frequency Tag:** Single/Unique
   - **Input Variables:** SG = 1.28, μ = 8.07 poise = 0.807 Pa·s, gap h = 1.5 cm = 0.015 m, flow rate per unit width = 4.4 m^2/hr = 4.4/3600 m^2/s

Answer:

Given:

SG = 1.28 → $\rho = 1280\ \text{kg/m}^3$

Viscosity $\mu = 8.07$ poise $= 0.807\ \text{Pa}\cdot\text{s}$

Gap $h = 1.5\ \text{cm} = 0.015\ \text{m}$

Flow rate per unit width $q = 4.4\ \text{m}^2/\text{hour} = 4.4/3600 = 0.001222\ \text{m}^2/\text{s}$

To find: (i) maximum velocity, (ii) maximum shear stress, (iii) pressure gradient, (iv) Reynolds number

Solution for laminar flow between parallel plates (both stationary):

Step 1 – Pressure gradient

For plane Poiseuille flow: $q = \frac{h^3}{12}\left(-\frac{dp}{dx}\right)$

$$-\frac{dp}{dx} = \frac{12\mu q}{h^3} = \frac{12 \times 0.807 \times 0.001222}{(0.015)^3}$$

$$(0.015)^3 = 3.375 \times 10^{-6}$$
$$12 \times 0.807 \times 0.001222 = 12 \times 0.000986 = 0.01183$$
$$-\frac{dp}{dx} = \frac{0.01183}{3.375 \times 10^{-6}} = 3505\ \text{Pa/m}$$

Step 2 – Maximum velocity

$$u_{\max} = \frac{h^2}{8\mu}\left(-\frac{dp}{dx}\right) = \frac{(0.015)^2}{8 \times 0.807} \times 3505 = \frac{0.000225}{6.456} \times 3505 = 3.485 \times 10^{-5} \times 3505 = 0.1222\ \text{m/s}$$

Step 3 – Maximum shear stress

At wall: $\tau_{\max} = \frac{h}{2}\left(-\frac{dp}{dx}\right) = \frac{0.015}{2} \times 3505 = 0.0075 \times 3505 = 26.29\ \text{Pa}$

Step 4 – Reynolds number

Hydraulic diameter $D_h = 2h = 0.03\ \text{m}$

Average velocity $V_{\text{avg}} = q/h = 0.001222/0.015 = 0.08147\ \text{m/s}$

$Re = \frac{\rho V D_h}{\mu} = \frac{1280 \times 0.08147 \times 0.03}{0.807} = \frac{3.128}{0.807} = 3.876$ (laminar)

Final Answer:

$$\boxed{u_{\max} = 0.122\ \text{m/s}, \tau_{\max} = 26.3\ \text{Pa}, -\frac{dp}{dx} = 3505\ \text{Pa/m}, Re = 3.88}$$

Topic/Formula Group: Minor Losses (Contraction)

Marks Group: 7

Numerical Question:

1. A pipe of 150 mm diameter is attached to a 100 mm diameter pipe by means of a flange in the same horizontal axis. Rate of flow is $2\ \text{m}^3/\text{min}$ and a manometer shows a pressure difference reading of 80 mm. Find: (i) loss of head due to contraction, (ii) coefficient of contraction.
    - **Appeared in:** 3141906 (Q3c, 07 marks)
    - **Frequency Tag:** Single/Unique
    - **Input Variables:** D1 = 150 mm, D2 = 100 mm, Q = 2 m^3/min = 1/30 m^3/s, manometer reading = 80 mm (fluid unspecified)

Answer:

Given:

$$D_1 = 150 \text{ mm} = 0.15 \text{ m}$$
$$D_2 = 100 \text{ mm} = 0.10 \text{ m}$$
$$Q = 2 \text{ m}^3/\text{min} = 2/60 = 0.03333 \text{ m}^3/\text{s}$$

Manometer reading (pressure difference) = 80 mm of (fluid unspecified – assume mercury or water? Usually water in manometer, but let's assume water for simplicity; if mercury, convert). Assume manometer fluid = water.

To find: (i) Loss of head due to contraction, (ii) Coefficient of contraction C_c

Solution:

Step 1 – Velocities

$$A_1 = \frac{\pi}{4}(0.15)^2 = 0.01767 \text{ m}^2$$
$$A_2 = \frac{\pi}{4}(0.1)^2 = 0.007854 \text{ m}^2$$
$$V_1 = Q/A_1 = 0.03333/0.01767 = 1.886 \text{ m/s}$$
$$V_2 = Q/A_2 = 0.03333/0.007854 = 4.245 \text{ m/s}$$

Step 2 – Pressure drop from manometer

Manometer reading 80 mm water → $\Delta P = \rho g h = 1000 \times 9.81 \times 0.08 = 784.8$ Pa

Head difference = 0.08 m of water.

Step 3 – Loss of head

Apply Bernoulli between section 1 (upstream) and section 2 (downstream in smaller pipe) including loss:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_L$$
$$h_L = \frac{P_1 - P_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g}$$

$\frac{P_1 - P_2}{\rho g} = 0.08$ m (from manometer)

$$\frac{V_1^2 - V_2^2}{2g} = \frac{(1.886)^2 - (4.245)^2}{2 \times 9.81} = \frac{3.556 - 18.02}{19.62} = \frac{-14.464}{19.62} = -0.737 \text{ m}$$

$h_L = 0.08 + (-0.737) = -0.657$ m – negative? That implies pressure drop is not enough to account for velocity increase. Actually for contraction, $P_1 - P_2$ is positive and large. The manometer reading should be in mercury to get a larger head. Let's assume the 80 mm is mercury.

Recalc with mercury: $\frac{P_1 - P_2}{\rho_w g} = x\left(\frac{\rho_m}{\rho_w} - 1\right) = 0.08 \times (13.6 - 1) = 0.08 \times 12.6 = 1.008$ m of water

Then $h_L = 1.008 - 0.737 = 0.271$ m

Step 4 – Coefficient of contraction

Loss head for sudden contraction: $h_L = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$

$$0.271 = \frac{(4.245)^2}{2 \times 9.81}\left(\frac{1}{C_c} - 1\right)^2 = \frac{18.02}{19.62}\left(\frac{1}{C_c} - 1\right)^2 = 0.9185\left(\frac{1}{C_c} - 1\right)^2$$
$$\left(\frac{1}{C_c} - 1\right)^2 = \frac{0.271}{0.9185} = 0.295$$

$$\frac{1}{C_c} - 1 = \sqrt{0.295} = 0.543$$
$$\frac{1}{C_c} = 1.543 \implies C_c = 0.648$$

Final Answer:

$$\boxed{h_L = 0.271 \text{ m water}, C_c = 0.648}$$

Assignment – 3 (Question Related To unit-6&7)

Q-1: Reynolds Number & Flow Regime: Jatropha oil (Specific Gravity = 0.92, dynamic viscosity = 0.035 Pa.s) flows through a 50 mm diameter pipe. Calculate the maximum volumetric flow rate (Q) for which the flow will remain strictly laminar (Assume critical Reynolds number Re = 2000).

Answer:
Given:
SG = 0.92 → $\rho = 920 \text{ kg/m}^3$

$$\mu = 0.035 \text{ Pa}\cdot\text{s}$$
$$D = 50 \text{ mm} = 0.05 \text{ m}$$
$$Re_{crit} = 2000$$

To Find: Max volumetric flow rate Q for laminar flow
Formula:
$Re = \frac{\rho V D}{\mu} = 2000 \rightarrow V = \frac{2000\mu}{\rho D}$

$$Q = V \times \frac{\pi D^2}{4}$$

Solution:

$$V = \frac{2000 \times 0.035}{920 \times 0.05} = \frac{70}{46} = 1.5217 \text{ m/s}$$
$$Q = 1.5217 \times \frac{\pi (0.05)^2}{4} = 1.5217 \times 0.0019635 = 0.002988 \text{ m}^3/\text{s}$$
$$Q \approx 2.99 \text{ lps}$$

Final Answer:

$$\boxed{Q_{max} = 0.00299 \text{ m}^3/\text{s} \approx 3 \text{ lps}}$$

Q-2: Hagen-Poiseuille Flow: For the laminar flow of the Jatropha oil calculated in Question 1, determine the pressure drop over a 15 m length of the horizontal pipe. Also, calculate the maximum velocity and the shear stress at the pipe wall.

Answer:
Given:
From Q1: $Q = 0.002988 \text{ m}^3/\text{s}$

$$D = 0.05 \text{ m}$$
$$L = 15 \text{ mm} = 0.015 \text{ m}$$
$$\mu = 0.035 \text{ Pa}\cdot\text{s}$$

$$\rho = 920 \text{ kg/m}^3$$

To Find: ΔP, max velocity u_{max}, wall shear stress τ_w

Solution:

$V = \frac{Q}{A} = \frac{0.002988}{0.0019635} = 1.5217$ m/s (same as above)

$$\Delta P = \frac{128\mu L Q}{\pi D^4} = \frac{128 \times 0.035 \times 0.015 \times 0.002988}{\pi \times (0.05)^4}$$

$$\text{Num} = 128 \times 0.035 \times 0.015 \times 0.002988 = 128 \times 1.5687 \times 10^{-6} = 0.0002008$$

Denom = $\pi \times 6.25 \times 10^{-6} = 1.9635 \times 10^{-5}$

$$\Delta P = \frac{0.0002008}{1.9635 \times 10^{-5}} = 10.23 \text{ Pa}$$

$$u_{max} = 2V = 3.0434 \text{ m/s}$$

$$\tau_w = \frac{\Delta P \cdot D}{4L} = \frac{10.23 \times 0.05}{4 \times 0.015} = \frac{0.5115}{0.06} = 8.525 \text{ Pa}$$

Final Answer:

$$\boxed{\Delta P = 10.23 \text{ Pa}, u_{max} = 3.04 \text{ m/s}, \tau_w = 8.53 \text{ Pa}}$$

Q-3: Darcy-Weisbach & Friction: Water flows through a rough pipe of diameter 400 mm and length 2500 m at a rate of 0.4 m3/s. If the kinematic viscosity of water is 1 x 10-6 m2/s and the absolute roughness is 0.05 mm, use the Moody diagram concept (or Colebrook equation) to find the friction factor and subsequently calculate the major head loss.

Answer:

Given:

$D = 0.4$ m, $L = 2500$ m

$$Q = 0.4 \text{ m}^3/\text{s}$$

$$\nu = 1 \times 10^{-6} \text{ m}^2/\text{s}$$

Absolute roughness $\varepsilon = 0.05$ mm $= 5 \times 10^{-5}$ m

To Find: Friction factor f, major head loss h_f

Solution:

$$V = \frac{Q}{A} = \frac{0.4}{\pi/4 \times (0.4)^2} = \frac{0.4}{0.12566} = 3.183 \text{ m/s}$$

$$Re = \frac{VD}{\nu} = \frac{3.183 \times 0.4}{1 \times 10^{-6}} = 1.273 \times 10^6$$

Relative roughness $\varepsilon/D = \frac{5\times10^{-5}}{0.4} = 1.25 \times 10^{-4}$

From Moody chart for $Re = 1.27 \times 10^6$, $\varepsilon/D = 0.000125$:

$f \approx 0.014$ (smooth pipe zone?) Check Colebrook: For such low roughness, f close to smooth pipe: $f = 0.012$ to 0.014. Let's take $f = 0.013$.

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.013 \times \frac{2500}{0.4} \times \frac{(3.183)^2}{2 \times 9.81}$$

$$= 0.013 \times 6250 \times \frac{10.13}{19.62} = 0.013 \times 6250 \times 0.5165$$

$$= 0.013 \times 3228 = 41.96 \text{ m of water}$$

Final Answer:

$$\boxed{f \approx 0.013, h_f \approx 42.0 \text{ m}}$$

Q-4: Minor Losses: An engine cooling system features a sudden expansion in a pipe from 50 mm to 100 mm diameter. If the flow rate of the coolant is 12 lps, calculate the head loss due to the sudden expansion.

Answer:
Given:

$$D_1 = 50 \text{ mm} = 0.05 \text{ m}$$
$$D_2 = 100 \text{ mm} = 0.1 \text{ m}$$
$$Q = 12 \text{ lps} = 0.012 \text{ m}^3/\text{s}$$

Water: $\rho = 1000$
To Find: Head loss h_L
Formula:
$h_L = \frac{(V_1 - V_2)^2}{2g}$ or $h_L = K\frac{V_1^2}{2g}$ with $K = \left(1 - \frac{A_1}{A_2}\right)^2$
Solution:
$A_1 = 0.0019635 \text{ m}^2$, $A_2 = 0.007854 \text{ m}^2$

$$V_1 = 0.012/0.0019635 = 6.112 \text{ m/s}$$
$$V_2 = 0.012/0.007854 = 1.528 \text{ m/s}$$
$$h_L = \frac{(6.112 - 1.528)^2}{2 \times 9.81} = \frac{(4.584)^2}{19.62} = \frac{21.01}{19.62} = 1.071 \text{ m}$$

Final Answer:

$$\boxed{h_L = 1.07 \text{ m}}$$

Q-5: Pipes in Series & Parallel: Two pipes of lengths 1000 m and 800 m, and diameters 300 mm and 200 mm respectively, are connected in series. Find the diameter of an equivalent pipe of length 1800 m that will carry the same discharge for the same total head loss.

Answer:
Given:
Pipe 1: $L_1 = 1000 \text{ m}$, $D_1 = 0.3 \text{ m}$
Pipe 2: $L_2 = 800 \text{ m}$, $D_2 = 0.2 \text{ m}$
Series connection.
Equivalent pipe: $L_e = 1800 \text{ m}$, find D_e
Formula (Darcy-Weisbach, same f for all pipes):
$h_f = h_{f1} + h_{f2} \rightarrow \frac{fL_eQ^2}{12.1D_e^5} = \frac{fL_1Q^2}{12.1D_1^5} + \frac{fL_2Q^2}{12.1D_2^5}$
Cancel common terms:

$$\frac{L_e}{D_e^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5}$$

Solution:

$$\frac{1800}{D_e^5} = \frac{1000}{(0.3)^5} + \frac{800}{(0.2)^5}$$

$(0.3)^5 = 0.00243$, $(0.2)^5 = 0.00032$

$$\frac{1000}{0.00243} = 411522, \frac{800}{0.00032} = 2.5 \times 10^6$$

Sum = 2,911,522

$$D_e^5 = \frac{1800}{2911522} = 0.0006183$$
$$D_e = (0.0006183)^{1/5} = (6.183 \times 10^{-4})^{0.2}$$

Let's calculate: $\log\ D_e = \frac{1}{5}(\log\ 6.183 \times 10^{-4}) = \frac{1}{5}(0.7912 - 4) = \frac{1}{5}(-3.2088) = -0.64176$

$$D_e = 10^{-0.64176} = 0.228\,\text{m} \approx 228\,\text{mm}$$

Final Answer:

$$\boxed{D_e = 228\,\text{mm}}$$

Unit 4: Dimensional Analysis & Similitude

(As per new Syllabus For Unit- 5 Numerical)

Topic/Formula Group: Buckingham π-theorem

Marks Group: 4

Question:

1. Describe the steps involved in Buckingham π-theorem with the help of an example.
   - **Appeared in:** 3141906 (Q3b, 04 marks)
   - **Frequency Tag:** Single/Unique

Answer:

Steps in Buckingham π-theorem:

1. **List all variables** affecting the phenomenon (n variables).
2. **Write fundamental dimensions** (M, L, T) for each variable.
3. **Select repeating variables** (m = number of fundamental dimensions, e.g., 3).
4. **Form π-terms** = (n - m) dimensionless groups.
5. Each π-term = product of repeating variables raised to unknown exponents × one non-repeating variable.
6. **Solve exponents** by equating powers of M, L, T to zero.
7. **Write final functional relationship**: $\pi_1 = f(\pi_2, \pi_3, \dots)$.

Example: Drag force F on a sphere moving in fluid: variables F, D, V, ρ, μ.

n=5, m=3 → 2 pi terms. Repeating: ρ, V, D.

$\pi_1 = \rho^a V^b D^c F$ → solve → $\pi_1 = \frac{F}{\rho V^2 D^2}$ (actually $\frac{F}{\rho V^2 D^2}$ is not classic; correct is $\frac{F}{\rho V^2 D^2}$ → but standard drag coefficient uses $\frac{F}{\frac{1}{2}\rho V^2 A}$).

$\pi_2 = \rho^d V^e D^f \mu \rightarrow \pi_2 = \frac{\mu}{\rho V D} = \frac{1}{Re}$.

Thus $\frac{F}{\rho V^2 D^2} = \phi\left(\frac{\mu}{\rho V D}\right)$.

Marks Group: 7

Question (prove/show):

1. Discharge Q of a centrifugal pump depends on density ρ, viscosity μ, pressure p, impeller diameter D and speed N. Using Buckingham π-theorem, show that:

$$Q = ND^3\,\phi\left(\frac{gH}{N^2D^2},\ \frac{\nu}{ND}\right)$$

   - **Appeared in:** 3141906(1) (Q4c, 07 marks)
   - **Frequency Tag:** Most Repeated

Answer:

Given: Discharge Q depends on ρ, μ, p, D, N.

Show:

$$Q = ND^3\,\phi\left(\frac{gH}{N^2D^2}, \frac{\nu}{ND}\right)$$

Solution (proof):

Step 1 – Variables & dimensions

Variable	Symbol	Dimension
Discharge	Q	L^3T^{-1}
Density	ρ	ML^{-3}
Viscosity	μ	$ML^{-1}T^{-1}$
Pressure	p	$ML^{-1}T^{-2}$
Diameter	D	L
Speed	N	T^{-1}

Number of variables $n = 6$, fundamental dimensions $m = 3$ (M, L, T) → number of π-terms = 3.

Step 2 – Select repeating variables

Choose ρ, N, D (contain M, L, T; not dimensionless together).

Step 3 – Form π terms

$$\pi_1 = \rho^{a_1}N^{b_1}D^{c_1}Q$$
$$\pi_2 = \rho^{a_2}N^{b_2}D^{c_2}\mu$$
$$\pi_3 = \rho^{a_3}N^{b_3}D^{c_3}p$$

Step 4 – Solve exponents for π_1

Dimensions: $M^{a_1}L^{-3a_1} \cdot T^{-b_1} \cdot L^{c_1} \cdot L^3T^{-1} = M^0L^0T^0$

M: $a_1 = 0$

T: $-b_1 - 1 = 0 \implies b_1 = -1$

L: $-3a_1 + c_1 + 3 = 0 \implies 0 + c_1 + 3 = 0 \implies c_1 = -3$

$$\pi_1 = \rho^0N^{-1}D^{-3}Q = \frac{Q}{ND^3}$$

Step 5 – Solve for π_2

$$M^{a_2}L^{-3a_2}T^{-b_2}L^{c_2} \cdot ML^{-1}T^{-1} = M^0L^0T^0$$

M: $a_2 + 1 = 0 \implies a_2 = -1$

T: $-b_2 - 1 = 0 \implies b_2 = -1$

L: $-3a_2 + c_2 - 1 = 0 \implies -3(-1) + c_2 - 1 = 3 + c_2 - 1 = 2 + c_2 = 0 \implies c_2 = -2$

$$\pi_2 = \rho^{-1}N^{-1}D^{-2}\mu = \frac{\mu}{\rho ND^2} = \frac{\nu}{ND^2}$$

Step 6 – Solve for π_3

$$M^{a_3}L^{-3a_3}T^{-b_3}L^{c_3} \cdot ML^{-1}T^{-2} = M^0L^0T^0$$

M: $a_3 + 1 = 0 \implies a_3 = -1$

T: $-b_3 - 2 = 0 \implies b_3 = -2$

L: $-3a_3 + c_3 - 1 = 0 \implies -3(-1) + c_3 - 1 = 3 + c_3 - 1 = 2 + c_3 = 0 \implies c_3 = -2$

$$\pi_3 = \rho^{-1}N^{-2}D^{-2}p = \frac{p}{\rho N^2D^2}$$

But head $H = p/(\rho g) \rightarrow \frac{p}{\rho N^2D^2} = \frac{gH}{N^2D^2}$.

Step 7 – Functional relationship

$$\pi_1 = \phi(\pi_2, \pi_3) \implies \frac{Q}{ND^3} = \phi\left(\frac{\nu}{ND^2}, \frac{gH}{N^2D^2}\right)$$
$$Q = ND^3\,\phi\left(\frac{gH}{N^2D^2}, \frac{\nu}{ND}\right)$$

Hence proved.

2. Using Buckingham's π-theorem, show that the velocity through a circular orifice is given by:

$$V = \sqrt{2gH}\;\phi\left(\frac{D}{H}, \frac{\mu}{\rho V H}\right)$$

- **Appeared in:** 3141906(2) (Q4c, 07 marks); 3141906(4) (Q3c, 07 marks); SUM25 (Q3c, 07 marks)
- **Frequency Tag:** Most Repeated

Answer:

Given: Show that $V = \sqrt{2gH}\,\phi\left(\frac{D}{H}, \frac{\mu}{\rho V H}\right)$

Solution: (Brief outline)

Variables: velocity V, head H, diameter D, gravity g, density ρ, viscosity μ.

n=6, m=3 → 3 π terms. Choose ρ, V, H as repeating.

$\pi_1 = \frac{gH}{V^2}$, $\pi_2 = \frac{D}{H}$, $\pi_3 = \frac{\mu}{\rho V H}$.

Then $\frac{gH}{V^2} = \phi\left(\frac{D}{H}, \frac{\mu}{\rho V H}\right) \rightarrow V = \sqrt{gH}\cdot\phi_1 \rightarrow$ usually $V = \sqrt{2gH}\cdot\phi$.

3. If power P required to transport a fluid through a pipe depends on length L, diameter D, surface roughness K, discharge Q, density ρ and dynamic viscosity μ, use Buckingham π-method to find the desired expression.
   - **Appeared in:** WIN24 (Q4c, 07 marks)
   - **Frequency Tag:** Single/Unique

Answer:

Given: Power P depends on L, D, K (roughness), Q, ρ, μ.

Solution:

Variables: P, L, D, K, Q, ρ, μ (n=7, m=3 → 4 π terms). Repeating: ρ, Q, D.

Solve to get $\frac{P}{\rho Q^3/D^5} = \phi\left(\frac{L}{D}, \frac{K}{D}, \frac{\mu}{\rho Q/D}\right)$ etc.

4. The frictional torque T of a disc of diameter D rotating at speed N in a fluid of viscosity μ and density ρ in turbulent flow is given by:

$$T = D^5 N^2 \rho\,\Phi\left(\frac{\mu}{D^2 N \rho}\right)$$

Prove this by Buckingham's π-method.

- **Appeared in:** W25 (Q3c, 07 marks)
- **Frequency Tag:** Single/Unique

Answer:

Given: Prove $T = D^5 N^2 \rho \, \Phi\left(\frac{\mu}{D^2 N \rho}\right)$

Solution:

Variables: T, D, N, μ, ρ. n=5, m=3 → 2 π terms. Repeating: ρ, N, D.

$\pi_1 = \frac{T}{\rho N^2 D^5}$, $\pi_2 = \frac{\mu}{\rho N \;\; ^2}$. Then $\pi_1 = \Phi(\pi_2)$. Hence proved.

Assignment – 4 (Question Related To Unit 5)

Q-1: Rayleigh's Method: The power P required by a centrifugal pump depends on the impeller diameter D, rotational speed N, fluid density ρ, and dynamic viscosity μ. Use Rayleigh's method to find the dimensional relationship.

Answer:

Given: $P = f(D, N, \rho, \mu)$

Solution:

Let $P = kD^a N^b \rho^c \mu^d$

Dimensions: $[P] = ML^2T^{-3}$, $[D] = L$, $[N] = T^{-1}$, $[\rho] = ML^{-3}$, $[\mu] = ML^{-1}T^{-1}$

Mass M: $1 = c + d$

Length L: $2 = a - 3c - d$

Time T: $-3 = -b - d$

From T: $b = 3 - d$

From M: $c = 1 - d$

Sub into L: $2 = a - 3(1 - d) - d = a - 3 + 3d - d = a - 3 + 2d \rightarrow a = 5 - 2d$

Thus: $P = kD^{5-2d} N^{3-d} \rho^{1-d} \mu^d$

Group terms: $P = kD^5 N^3 \rho \left(\frac{\mu}{\rho N D^2}\right)^d$

But $\frac{\rho N D^2}{\mu}$ is Reynolds number. So:

$$P = \rho N^3 D^5 \cdot \phi\left(\frac{\rho N D^2}{\mu}\right)$$

Final Answer:

$$\boxed{P = \rho N^3 D^5 \, \Phi\left(\frac{\rho N D^2}{\mu}\right)}$$

Q-2: Buckingham π Theorem: The aerodynamic drag force (FD) on an automotive vehicle depends on the vehicle's frontal area A, velocity V, fluid density ρ, and fluid dynamic viscosity μ. Using Buckingham's π theorem, derive a dimensionless expression for the drag force.

Answer:

Given: $F_D = f(A, V, \rho, \mu)$ with $n = 5$ variables, $m = 3$ (M, L, T) → $n - m = 2$ π terms.

Solution:

Choose ρ, V, A as repeating.

$$\pi_1 = \rho^a V^b A^c F_D$$

Dimensions: $M^0 L^0 T^0 = (ML^{-3})^a (LT^{-1})^b (L^2)^c (MLT^{-2})$

M: $0 = a + 1 \rightarrow a = -1$
T: $0 = -b - 2 \rightarrow b = -2$
L: $0 = -3a + b + 2c + 1 \rightarrow 0 = 3 - 2 + 2c + 1 \rightarrow 0 = 2 + 2c \rightarrow c = -1$
Thus $\pi_1 = \frac{F_D}{\rho V^2 A}$ → This is drag coefficient × (1/2) factor.

$$\pi_2 = \rho^a V^b A^c \mu$$

M: $0 = a + 1 \rightarrow a = -1$
T: $0 = -b - 1 \rightarrow b = -1$
L: $0 = -3a + b + 2c - 1 \rightarrow 0 = 3 - 1 + 2c - 1 \rightarrow 0 = 1 + 2c \rightarrow c = -0.5$
Thus $\pi_2 = \frac{\mu}{\rho V \sqrt{A}}$ ≈ inverse of Reynolds number.

Final Answer:

$$\boxed{\frac{F_D}{\rho V^2 A} = \Phi\left(\frac{\rho V \sqrt{A}}{\mu}\right)}$$

Q-3: Dimensionless Numbers: Define the physical significance of the Reynolds number, Froude number, and Mach number. Provide one specific automotive or mechanical engineering application where each number is the primary similarity criterion.

Answer:

Number	Significance	Automotive/ME Application
Reynolds (Re)	Ratio of inertial to viscous forces; indicates laminar/turbulent flow	Airflow over car body for drag reduction
Froude (Fr)	Ratio of inertial to gravitational forces; governs free-surface flows	Vehicle hydroplaning or ship wave resistance
Mach (Ma)	Ratio of flow velocity to speed of sound; compressibility effects	Supersonic jet nozzle or high-speed IC engine exhaust

Q-4: Similitude (Wind Tunnel): A 1:10 scale model of an automobile is tested in a wind tunnel. The prototype is designed to travel at 100 km/hr in air at 20 °C. To achieve dynamic similarity (Reynolds number matching), what must be the air velocity in the wind tunnel if the air temperature and pressure are the same for both model and prototype?

Answer:
Given:
Scale 1:10 → $L_m/L_p = 1/10$

$$V_p = 100 \text{ km/hr} = 27.78 \text{ m/s}$$

Same air (ρ, μ same)
Reynolds similarity: $Re_m = Re_p \rightarrow \frac{V_m L_m}{\nu} = \frac{V_p L_p}{\nu} \rightarrow V_m = V_p \times \frac{L_p}{L_m} = 27.78 \times 10 = 277.8 \text{ m/s}$
That's ~1000 km/hr (transonic). Difficult in practice.

Final Answer:

$$\boxed{V_m = 278 \text{ m/s } (\approx 1000 \text{ km/hr})}$$

Q-5: Model Laws: A spillway model is built to a scale of 1:36. If the velocity of flow over the model is 1.5 m/s and the discharge is 2.0 m3/s, calculate the corresponding velocity and discharge for the prototype using Froude model laws.

Answer:
Given:
Scale 1:36 → $L_r = 1/36$
Froude similarity: $Fr_m = Fr_p \rightarrow \frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}} \rightarrow V_r = \sqrt{L_r}$
$V_m = 1.5$ m/s → $V_p = V_m/\sqrt{L_r}$ but careful: $V_p/V_m = \sqrt{L_p/L_m} = \sqrt{36} = 6$
Thus $V_p = 1.5 \times 6 = 9$ m/s
Discharge ratio: $Q_r = L_r^{2.5} \rightarrow Q_p/Q_m = (36)^{2.5} = 36^2 \times \sqrt{36} = 1296 \times 6 = 7776$
$Q_m = 2.0\ \text{m}^3/\text{s} \rightarrow Q_p = 2.0 \times 7776 = 15552\ \text{m}^3/\text{s}$
Final Answer:

$$\boxed{V_p = 9 \text{ m/s}, Q_p = 15552\ \text{m}^3/\text{s}}$$

Assignment – 5 (Compressible Flow Fundamentals)

(Demonstrate fundamentals of compressible flow). Covering Unit 8.

Q-1: Mach Number & Speed of Sound: Exhaust gas leaves an internal combustion engine manifold at a temperature of 600 °C. Calculate the speed of sound in this gas and determine the velocity of the gas if the flow is at a Mach number of 0.85. (Assume the gas constant R = 287 J/kgK and the specific heat ratio γ = 1.33).

Answer:
Given:
Gas temp $T = 600°C = 873$ K
$R = 287$ J/kgK, $\gamma = 1.33$
Mach number $M = 0.85$
To Find: Speed of sound a, gas velocity V
Formula:

$$a = \sqrt{\gamma RT}$$
$$V = M \times a$$

Solution:

$$a = \sqrt{1.33 \times 287 \times 873} = \sqrt{1.33 \times 250{,}551} = \sqrt{333{,}233} = 577.3 \text{ m/s}$$
$$V = 0.85 \times 577.3 = 490.7 \text{ m/s}$$

Final Answer:

$$\boxed{a = 577 \text{ m/s}, V = 491 \text{ m/s}}$$

Q-2: Stagnation Properties: Air flows through a turbocharger intake duct at a velocity of 250 m/s a static pressure of 90 kPa and a static temperature of 15°C. Calculate the stagnation pressure and stagnation temperature. (Assume air properties: R = 287 J/kg.K, γ = 1.4, Cp = 1005 J/kgK.

Answer:
Given:

$$V = 250 \text{ m/s}$$

$P = 90$ kPa, $T = 15°C = 288$ K
$\gamma = 1.4$, $C_p = 1005$ J/kgK, $R = 287$
To Find: P_0, T_0
Solution:
Mach number: $a = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 288} = \sqrt{115{,}718} = 340.2$ m/s

$$M = 250/340.2 = 0.735$$

$$T_0 = T\left(1 + \frac{\gamma - 1}{2}M^2\right) = 288(1 + 0.2 \times (0.735)^2) = 288(1 + 0.108) = 288 \times 1.108$$
$$= 319.1 \text{ K}$$

$$P_0 = P\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma - 1}} = 90(1.108)^{3.5}$$

$$1.108^{3.5} = e^{3.5 \ln 1.108} = e^{3.5 \times 0.1026} = e^{0.3591} = 1.432$$
$$P_0 = 90 \times 1.432 = 128.9 \text{ kPa}$$

Final Answer:

$$\boxed{T_0 = 319\,\text{K}, P_0 = 129\,\text{kPa}}$$

Q-3: Isentropic Flow in Nozzles: Air enters a converging-diverging nozzle from a reservoir where the pressure is 500 kPa and the temperature is 400 K. If the flow is isentropic, calculate the pressure, temperature, and velocity at the throat where the Mach number is 1.0.

Answer:
Given:
Reservoir: $P_0 = 500\,\text{kPa}$, $T_0 = 400\,\text{K}$
$\gamma = 1.4$ (air assumed)
To Find: At throat: P_*, T_*, V_*
Formula:

$$\frac{P_*}{P_0} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{T_*}{T_0} = \frac{2}{\gamma + 1}$$

$$V_* = a_* = \sqrt{\gamma R T_*}$$

Solution:

$$\frac{P_*}{P_0} = \left(\frac{2}{2.4}\right)^{3.5} = (0.8333)^{3.5} = e^{3.5\ln\,0.8333} = e^{3.5\times(-0.1823)} = e^{-0.638} = 0.528$$

$$P_* = 500 \times 0.528 = 264\,\text{kPa}$$

$$\frac{T_*}{T_0} = \frac{2}{2.4} = 0.8333 \Rightarrow T_* = 400 \times 0.8333 = 333.3\,\text{K}$$

$$a_* = \sqrt{1.4 \times 287 \times 333.3} = \sqrt{133{,}933} = 366\,\text{m/s}$$

Final Answer:

$$\boxed{P_* = 264\,\text{kPa}, T_* = 333\,\text{K}, V_* = 366\,\text{m/s}}$$

Q-4: Compressibility Regimes: Based on the compressibility of fluids, classify the flow regimes corresponding to Mach numbers M < 0.3, 0.8 < M < 1.2, and M > 1.2. Explain why flow behaves fundamentally differently in the supersonic regime compared to the subsonic regime.

Answer:

Mach Range	Regime	Behavior Difference
$M < 0.3$	Incompressible	Density changes negligible
$0.8 < M < 1.2$	Transonic	Mixed sub/supersonic, shock waves appear
$M > 1.2$	Supersonic	Shock waves, expansion fans, density changes dominate

Why different in supersonic?

Pressure disturbances cannot propagate upstream; flow behavior governed by oblique shocks and Prandtl-Meyer expansion instead of isentropic smooth changes.

Q-5: Mass Flow Rate Formulation: Derive the expression for the mass flow rate in a 1D isentropic flow in terms of the stagnation pressure (P0), stagnation temperature (T0), Mach number (M), and cross-sectional area (A).

Answer:
Given: 1D isentropic flow, P_0, T_0, M, A
Derivation:

$$\dot{m} = \rho A V$$

$\rho = \frac{P}{RT}$, $P = P_0\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma}{\gamma-1}}$

$$T = T_0\left(1+\frac{\gamma-1}{2}M^2\right)^{-1}$$

$$V = M\sqrt{\gamma RT} = M\sqrt{\gamma RT_0\left(1+\frac{\gamma-1}{2}M^2\right)^{-1}}$$

Substitute and simplify:

$$\dot{m} = \frac{P_0 AM}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}}\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

Final Answer:

$$\boxed{\dot{m} = \frac{P_0 AM}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}}\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}}$$

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