

GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – OLD PAPER – S22 TO W25 – Q&A BANK (Numerical)

Subject Name & Code:
Fluid Mechanics (3141906)

Unit 1: Properties of Fluids & Fluid Statics

(As per new Syllabus For Unit- 1&2 Numerical)

Topic/Formula Group: Surface Tension (Pressure difference across bubble)

Marks Group: 3

Numerical Question:

1. If the surface tension at air and water interface is 0.0735 N/m, what is the pressure difference between inside and outside of an air bubble of diameter 0.01 mm?
 - **Appeared in:** 3141906(1) (Q4a, 03 marks)
 - **Frequency Tag:** Single/Unique
 - **Input Variables:** surface tension = 0.0735 N/m, diameter = 0.01 mm

Answer:

Given:

Surface tension $\sigma = 0.0735$ N/m

Diameter $d = 0.01$ mm $= 0.01 \times 10^{-3} = 1 \times 10^{-5}$ m

To find: Pressure difference ΔP

Formula: For an air bubble in air (soap bubble)

$$\Delta P = \frac{4\sigma}{r} = \frac{8\sigma}{d}$$

Solution:

$$\text{Radius } r = \frac{d}{2} = \frac{1 \times 10^{-5}}{2} = 0.5 \times 10^{-5} \text{ m}$$

$$\Delta P = \frac{4 \times 0.0735}{0.5 \times 10^{-5}} = \frac{0.294}{0.5 \times 10^{-5}} = \frac{0.294}{5 \times 10^{-6}}$$

$$\Delta P = 58,800 \text{ N/m}^2 = 58.8 \text{ kPa}$$

Final Answer:

$$\boxed{\Delta P = 58.8 \text{ kPa}}$$

Topic/Formula Group: Viscosity (Newton's law – linear velocity gradient)

Marks Group: 7

Numerical Question:

1. A thin square plate $0.3 \text{ m} \times 0.3 \text{ m}$ is placed horizontally in the middle of a gap of height 2.5 cm. This gap is filled with oil of viscosity $0.2 \text{ Pa} \cdot \text{s}$ and the plate is pulled at constant velocity of 0.2 m/s . Find the total force on the plate.
 - **Appeared in:** 3141906(2) (Q2c, 07 marks)
 - **Frequency Tag:** Single/Unique

- **Input Variables:** plate area = 0.3×0.3 m, gap = 2.5 cm = 0.025 m, $\mu = 0.2$ Pa·s, plate velocity = 0.2 m/s

Answer:

Given:

Plate dimensions $0.3 \text{ m} \times 0.3 \text{ m}$

Gap height $y = 2.5 \text{ cm} = 0.025 \text{ m}$

Dynamic viscosity $\mu = 0.2 \text{ Pa} \cdot \text{s}$

Plate velocity $V = 0.2 \text{ m/s}$

To find: Total force on the plate F

Assumptions:

- Linear velocity gradient across the gap
- Plate is in the middle of the gap $\rightarrow$ two identical oil films (above & below)

Formula:

Newton's law of viscosity:

$$\tau = \mu \frac{du}{dy} = \mu \frac{V}{y/2} \text{ (since gap is split into two equal films)}$$

Shear force $F = \tau \times A$ (both sides of plate)

Solution:

Velocity gradient for one side:

$$\frac{du}{dy} = \frac{V}{y/2} = \frac{0.2}{0.025/2} = \frac{0.2}{0.0125} = 16 \text{ s}^{-1}$$

Shear stress:

$$\tau = \mu \times \frac{du}{dy} = 0.2 \times 16 = 3.2 \text{ Pa}$$

Area of one side:

$$A = 0.3 \times 0.3 = 0.09 \text{ m}^2$$

Force on one side:

$$F_{\text{one side}} = \tau \times A = 3.2 \times 0.09 = 0.288 \text{ N}$$

Total force (both sides):

$$F = 2 \times 0.288 = 0.576 \text{ N}$$

Final Answer:

$$\boxed{F = 0.576 \text{ N}}$$

2. A 120 mm diameter disc rotates on a table separated by an oil film of 1.8 mm thickness. Find the viscosity of oil if the torque required to rotate the disc at 60 rpm is 0.00072 Nm. Assume the velocity gradient in the oil film to be linear.

- **Appeared in:** 3141906 (Q2c, 07 marks)
- **Frequency Tag:** Single/Unique
- **Input Variables:** disc diameter = 120 mm = 0.12 m, film thickness = 1.8 mm = 0.0018 m, N = 60 rpm, torque = 0.00072 Nm

Answer:

Given:

Disc diameter $D = 120 \text{ mm} = 0.12 \text{ m} \rightarrow \text{radius } R = 0.06 \text{ m}$

Oil film thickness $t = 1.8 \text{ mm} = 0.0018 \text{ m}$

Rotational speed $N = 60 \text{ rpm}$

Torque $T = 0.00072 \text{ Nm}$

Velocity gradient linear.

To find: Viscosity μ of oil

Formula:

For a rotating disc on a flat surface with linear velocity gradient:

Shear stress at radius r :

$$\tau = \mu \frac{du}{dy} = \mu \frac{\omega r}{t}$$

Elemental torque:

$$dT = \tau \times dA \times r = \mu \frac{\omega r}{t} \times (2\pi r dr) \times r$$

$$dT = \frac{2\pi\mu\omega}{t} r^3 dr$$

Integrate from $r = 0$ to $r = R$:

$$T = \frac{2\pi\mu\omega}{t} \times \frac{R^4}{4} = \frac{\pi\mu\omega R^4}{2t}$$

Thus:

$$\mu = \frac{2tT}{\pi\omega R^4}$$

Solution:

Angular velocity:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 60}{60} = 2\pi \text{ rad/s}$$

$$\mu = \frac{2 \times 0.0018 \times 0.00072}{\pi \times (2\pi) \times (0.06)^4}$$

First compute $(0.06)^4 = 0.06^2 = 0.0036, \times 0.0036 = 1.296 \times 10^{-5}$

Denominator: $\pi \times 2\pi = 2\pi^2 = 2 \times 9.8696 = 19.7392$

$$19.7392 \times 1.296 \times 10^{-5} = 2.558 \times 10^{-4}$$

Numerator: $2 \times 0.0018 \times 0.00072 = 2 \times 1.296 \times 10^{-6} = 2.592 \times 10^{-6}$

$$\mu = \frac{2.592 \times 10^{-6}}{2.558 \times 10^{-4}} = 0.01013 \text{ Pa} \cdot \text{s}$$

Final Answer:

$$\boxed{\mu = 0.01013 \text{ Pa} \cdot \text{s}}$$

Topic/Formula Group: Hydrostatic Force on Submerged Surfaces

Marks Group: 7**Numerical Question:**

1. An annular circular plate of 2 m external diameter and 1 m internal diameter is immersed vertically in water so that its lowest edge is 5 m below the free water surface. Determine the total force and position of centre of pressure.
 - **Appeared in:** 3141906(5) (Q2c, 07 marks)
 - **Frequency Tag:** Most Repeated
 - **Input Variables:** $D_{\text{ext}} = 2$ m, $D_{\text{int}} = 1$ m, lowest edge depth = 5 m, fluid = water

Answer:**Given:**

External diameter $D_{\text{ext}} = 2$ m $\rightarrow R_{\text{ext}} = 1$ m

Internal diameter $D_{\text{int}} = 1$ m $\rightarrow R_{\text{int}} = 0.5$ m

Lowest edge depth = 5 m below free surface

Fluid: water ($\rho = 1000$ kg/m³)

Plate immersed vertically.

To find:

- (i) Total hydrostatic force F
- (ii) Centre of pressure h^* (depth from free surface)

Formula:

For a vertically submerged surface:

$$F = \rho g \bar{h} \times A$$

$$h^* = \frac{I_G}{A \bar{h}} + \bar{h}$$

Where $\bar{h}$ = depth of centroid, I_G = second moment of area about centroidal horizontal axis.

Solution:**Step 1 – Geometry & centroid depth**

Plate height = $D_{\text{ext}} = 2$ m (outer diameter = total height).

Lowest edge at 5 m $\rightarrow$ top edge at $5 - 2 = 3$ m depth.

Centroid depth $\bar{h} = \frac{3+5}{2} = 4$ m.

Step 2 – Area (annulus)

$$A = \frac{\pi}{4} (D_{\text{ext}}^2 - D_{\text{int}}^2) = \frac{\pi}{4} (2^2 - 1^2) = \frac{\pi}{4} (4 - 1) = \frac{3\pi}{4} = 2.356 \text{ m}^2$$

Step 3 – Total force

$$F = \rho g \bar{h} A = 1000 \times 9.81 \times 4 \times 2.356$$

$$F = 1000 \times 9.81 \times 9.424 = 92,456 \text{ N} = 92.456 \text{ kN}$$

Step 4 – Centre of pressure

For an annulus about its horizontal centroidal axis:

$$I_G = \frac{\pi}{64} (D_{\text{ext}}^4 - D_{\text{int}}^4) = \frac{\pi}{64} (2^4 - 1^4) = \frac{\pi}{64} (16 - 1) = \frac{15\pi}{64} = 0.7363 \text{ m}^4$$

$$h^* = \frac{I_G}{A \bar{h}} + \bar{h} = \frac{0.7363}{2.356 \times 4} + 4$$

$$= \frac{0.7363}{9.424} + 4 = 0.0781 + 4 = 4.0781 \text{ m}$$

Final Answer:

$$F = 92.46 \text{ kN}, h^* = 4.078 \text{ m}$$

2. An isosceles triangular plate of base 5 mm and height 5 mm is immersed vertically in an oil of specific gravity 0.8. The base of the plate is 1 m below the free liquid surface. Determine: (i) total pressure, (ii) centre of pressure.
- **Appeared in:** 3141906 (OR Q2c, 07 marks)
 - **Frequency Tag:** Most Repeated
 - **Input Variables:** base = 5 mm, height = 5 mm, oil SG = 0.8, base depth = 1 m

Answer:

Given:

Base $b = 5 \text{ mm} = 0.005 \text{ m}$

Height $h_{\text{triangle}} = 5 \text{ mm} = 0.005 \text{ m}$

Oil specific gravity $SG = 0.8 \rightarrow \rho_{\text{oil}} = 800 \text{ kg/m}^3, \gamma_{\text{oil}} = 7848 \text{ N/m}^3$

Base is 1 m below free surface (i.e., top edge? Actually “base of the plate is 1 m below free surface” – for triangle with base horizontal, base is at depth 1 m, vertex at depth $1 + 0.005 = 1.005 \text{ m}$)

To find: (i) Total pressure (force), (ii) Centre of pressure

Assumptions: Plate is vertical, base horizontal at top (since base is mentioned as 1 m below surface, vertex below base).

Solution:

Step 1 – Geometry

Triangle height = 0.005 m, base = 0.005 m.

For a triangle with base at top, centroid from base = $h/3 = 0.005/3 = 0.001667 \text{ m}$ downward.

Centroid depth $\bar{h} = 1 + 0.001667 = 1.001667 \text{ m}$

Step 2 – Area

$$A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 0.005 \times 0.005 = 1.25 \times 10^{-5} \text{ m}^2$$

Step 3 – Total force

$$\begin{aligned} F &= \gamma_{\text{oil}} \cdot \bar{h} \cdot A = 7848 \times 1.001667 \times 1.25 \times 10^{-5} \\ &= 7848 \times 1.2521 \times 10^{-5} = 0.0982 \text{ N} \end{aligned}$$

Step 4 – Centre of pressure

For triangle with base at top, I_G about centroidal horizontal axis = $\frac{bh^3}{36} = \frac{0.005 \times (0.005)^3}{36} =$

$$\frac{0.005 \times 1.25 \times 10^{-7}}{36} = \frac{6.25 \times 10^{-10}}{36} = 1.736 \times 10^{-11} \text{ m}^4$$

$$\begin{aligned} h^* &= \frac{I_G}{A\bar{h}} + \bar{h} = \frac{1.736 \times 10^{-11}}{(1.25 \times 10^{-5}) \times 1.001667} + 1.001667 \\ &= \frac{1.736 \times 10^{-11}}{1.2521 \times 10^{-5}} + 1.001667 = 1.386 \times 10^{-6} + 1.001667 \approx 1.001668 \text{ m} \end{aligned}$$

Final Answer:

$$F = 0.0982 \text{ N}, h^* = 1.00167 \text{ m (almost at centroid due to tiny plate)}$$

3. A rectangular plane surface is 1 m wide and 1.5 m deep, having a circular hole of 0.5 m diameter at the centre. The upper edge and lower edge are below free surface being 1 m and 2 m respectively. Calculate the magnitude, direction and location of the force acting upon one side of the plate due to water pressure.
- **Appeared in:** SUM25 (Q2c, 07 marks)
 - **Frequency Tag:** Most Repeated
 - **Input Variables:** width = 1 m, depth = 1.5 m, hole diameter = 0.5 m, upper edge depth = 1 m, lower edge depth = 2 m
 - **Diagram needed** (hole geometry)

Answer:

Given:

Rectangular plate: width $W = 1$ m, depth (height) $H_{\text{rect}} = 1.5$ m

Circular hole diameter $d = 0.5$ m at centre of plate

Upper edge depth = 1 m, lower edge depth = 2 m

Water ($\gamma = 9810 \text{ N/m}^3$)

To find: Magnitude, direction, location of force on one side of plate (net force after subtracting hole)

Solution:

Step 1 – Force on solid plate (without hole)

Centroid depth of rectangle $= \frac{1+2}{2} = 1.5$ m

Area of rectangle $A_{\text{rect}} = 1 \times 1.5 = 1.5 \text{ m}^2$

$$F_{\text{rect}} = \gamma \bar{h} A_{\text{rect}} = 9810 \times 1.5 \times 1.5 = 22,072.5 \text{ N}$$

Centre of pressure for rectangle: $I_G = \frac{WH^3}{12} = \frac{1 \times (1.5)^3}{12} = \frac{3.375}{12} = 0.28125 \text{ m}^4$

$$h_{\text{rect}}^* = \frac{I_G}{A_{\text{rect}} \bar{h}} + \bar{h} = \frac{0.28125}{1.5 \times 1.5} + 1.5 = \frac{0.28125}{2.25} + 1.5 = 0.125 + 1.5 = 1.625 \text{ m}$$

Step 2 – Force on hole (imaginary, same fluid would act on hole area)

Hole centroid at same depth as plate centroid = 1.5 m (since hole at centre)

Area of hole $A_{\text{hole}} = \frac{\pi}{4} (0.5)^2 = 0.19635 \text{ m}^2$

$$F_{\text{hole}} = \gamma \bar{h} A_{\text{hole}} = 9810 \times 1.5 \times 0.19635 = 2,888.6 \text{ N}$$

Centre of pressure for circle: $I_G = \frac{\pi d^4}{64} = \frac{\pi (0.5)^4}{64} = \frac{\pi \times 0.0625}{64} = 0.003068 \text{ m}^4$

$$h_{\text{hole}}^* = \frac{I_G}{A_{\text{hole}} \bar{h}} + \bar{h} = \frac{0.003068}{0.19635 \times 1.5} + 1.5 = \frac{0.003068}{0.2945} + 1.5 = 0.01042 + 1.5 = 1.5104 \text{ m}$$

Step 3 – Net force

$$\text{Net force} = F_{\text{rect}} - F_{\text{hole}} = 22,072.5 - 2,888.6 = 19,183.9 \text{ N}$$

Step 4 – Location of net force (moment equilibrium about free surface)

Let h_{net}^* be the depth of net force.

$$\begin{aligned} F_{\text{net}} \cdot h_{\text{net}}^* &= F_{\text{rect}} \cdot h_{\text{rect}}^* - F_{\text{hole}} \cdot h_{\text{hole}}^* \\ 19,183.9 \times h_{\text{net}}^* &= (22,072.5 \times 1.625) - (2,888.6 \times 1.5104) \\ &= 35,867.8 - 4,363.5 = 31,504.3 \end{aligned}$$

$$h_{\text{net}}^* = \frac{31,504.3}{19,183.9} = 1.642 \text{ m}$$

Direction: Perpendicular to plate (horizontal).

Final Answer:

$$F_{\text{net}} = 19.18 \text{ kN}, h_{\text{net}}^* = 1.642 \text{ m below free surface}$$

4. Panel ABC in the slanted side of a water tank is an isosceles triangle with vertex at A and base $BC = 2 \text{ m}$ as shown in figure. Find the water force on the panel and its line of action.
- **Appeared in:** WIN24 (Q2c, 07 marks)
 - **Frequency Tag:** Most Repeated
 - **Input Variables:** base $BC = 2 \text{ m}$
 - **Diagram needed**

Answer:

5. A tank of water has a gate of 5 m height and 3 m width in its vertical wall. The top edge of the gate is 2 m below the water surface. Calculate hydrostatic force on the gate and location of centre of pressure.
- **Appeared in:** WIN24 (Q2c alternate, 07 marks)
 - **Frequency Tag:** Most Repeated
 - **Input Variables:** gate height = 5 m, width = 3 m, top edge depth = 2 m

Answer:

Given:

Gate height $H = 5 \text{ m}$, width $W = 3 \text{ m}$

Top edge depth = 2 m below water surface

Vertical wall ($\theta=90^\circ$)

To find: Hydrostatic force and centre of pressure

Solution:

Step 1 – Centroid depth

Bottom edge depth = $2 + 5 = 7 \text{ m}$

Centroid depth $\bar{h} = \frac{2+7}{2} = 4.5 \text{ m}$

Step 2 – Area

$$A = W \times H = 3 \times 5 = 15 \text{ m}^2$$

Step 3 – Total force

$$F = \gamma_{\text{water}} \cdot \bar{h} \cdot A = 9810 \times 4.5 \times 15 = 9810 \times 67.5 = 662,175 \text{ N} = 662.2 \text{ kN}$$

Step 4 – Centre of pressure

$$\text{For rectangle: } I_G = \frac{WH^3}{12} = \frac{3 \times (5)^3}{12} = \frac{3 \times 125}{12} = \frac{375}{12} = 31.25 \text{ m}^4$$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{31.25}{15 \times 4.5} + 4.5 = \frac{31.25}{67.5} + 4.5 = 0.463 + 4.5 = 4.963 \text{ m}$$

Final Answer:

$$F = 662.2 \text{ kN}, h^* = 4.96 \text{ m below free surface}$$

6. A circular opening 2.5 m diameter, in a vertical side of a tank is closed by a disc of 2.5 m diameter which can rotate about a horizontal diameter. Determine: (i) the force on the disc, (ii) the torque required to maintain the disc in equilibrium in vertical position when the head of water above the horizontal diameter is 3.5 m.
- **Appeared in:** W25 (Q2c, 07 marks)
 - **Frequency Tag:** Most Repeated
 - **Input Variables:** disc diameter = 2.5 m, head above horizontal diameter = 3.5 m

Answer:

Given:

Circular opening diameter $D = 2.5 \text{ m}$ (disc same diameter)

Disc rotates about a horizontal diameter

Head of water above horizontal diameter = 3.5 m

Water $\gamma = 9810 \text{ N/m}^3$

To find: (i) Force on disc, (ii) Torque required to maintain disc in vertical position

Solution:

Step 1 – Force on disc

Centroid of disc = centre of disc. Horizontal diameter is at depth $\bar{h} = 3.5 \text{ m}$.

$$\text{Area } A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (2.5)^2 = \frac{\pi}{4} \times 6.25 = 4.9087 \text{ m}^2$$

$$F = \gamma \bar{h} A = 9810 \times 3.5 \times 4.9087 = 9810 \times 17.1805 = 168,540 \text{ N} = 168.54 \text{ kN}$$

Step 2 – Torque

When disc is vertical, the hydrostatic force acts at centre of pressure, which is offset from the

horizontal diameter axis. For a circle, $I_G = \frac{\pi D^4}{64} = \frac{\pi (2.5)^4}{64} = \frac{\pi \times 39.0625}{64} = 1.917 \text{ m}^4$

Distance from centroid to centre of pressure (vertical offset):

$$y_{cp} = \frac{I_G}{A \bar{h}} = \frac{1.917}{4.9087 \times 3.5} = \frac{1.917}{17.1805} = 0.1116 \text{ m}$$

This is the eccentricity from the horizontal diameter. The force creates a torque about the horizontal diameter:

$$T = F \times y_{cp} = 168,540 \times 0.1116 = 18,809 \text{ Nm}$$

Final Answer:

$$F = 168.54 \text{ kN}, T = 18.81 \text{ kNm}$$

Topic/Formula Group: Buoyancy & Metacentric Height

Marks Group: 4

Numerical Question:

1. A rectangular block of size 3 m long $\times$ 1.5 m wide $\times$ 1 m high floats in water such that its depth of immersion is 0.8 m. What is the weight of block? Determine metacentric height and comment on outcome.
 - **Appeared in:** 3141906(2) (Q3b, 04 marks)
 - **Frequency Tag:** Single/Unique

- **Input Variables:** $L = 3$ m, $B = 1.5$ m, $H = 1$ m, immersion depth = 0.8 m, fluid = water

Answer:

Given:

Block: length $L = 3$ m, width $B = 1.5$ m, height $H = 1$ m

Immersion depth $d = 0.8$ m

Fluid: water ($\rho = 1000$ kg/m³, $\gamma = 9810$ N/m³)

To find:

- (i) Weight of block W
- (ii) Metacentric height GM
- (iii) Comment on stability

Solution:

Step 1 – Weight of block

From buoyancy principle: Weight = weight of displaced water

Displaced volume $V_{\text{disp}} = L \times B \times d = 3 \times 1.5 \times 0.8 = 3.6$ m³

$$W = \gamma_{\text{water}} \times V_{\text{disp}} = 9810 \times 3.6 = 35,316 \text{ N} = 35.32 \text{ kN}$$

Step 2 – Metacentric height

$$GM = \frac{I}{V_{\text{disp}}} - BG$$

Where I = second moment of waterplane area about longitudinal axis,

BG = distance between centre of buoyancy (B) and centre of gravity (G).

Waterplane area = $L \times B = 3 \times 1.5 = 4.5$ m²

$$I = \frac{LB^3}{12} = \frac{3 \times (1.5)^3}{12} = \frac{3 \times 3.375}{12} = \frac{10.125}{12} = 0.84375 \text{ m}^4$$

$$\frac{I}{V_{\text{disp}}} = \frac{0.84375}{3.6} = 0.2344 \text{ m}$$

Centre of buoyancy from bottom = $d/2 = 0.4$ m

Centre of gravity from bottom = $H/2 = 0.5$ m

$$BG = 0.5 - 0.4 = 0.1 \text{ m}$$

$$GM = 0.2344 - 0.1 = 0.1344 \text{ m}$$

Comment: $GM > 0 \rightarrow$ block is **stable** in floating position.

Final Answer:

$$W = 35.32 \text{ kN}, GM = 0.1344 \text{ m (stable)}$$

Marks Group: 7

Numerical Question:

1. A cylinder block weighs 22 kN having diameter of 2 m and height 2.5 m is to float in sea water ($S = 1.025$). Show that it does not float vertically.
 - **Appeared in:** SUM25 (OR Q2c, 07 marks)
 - **Frequency Tag:** Single/Unique
 - **Input Variables:** weight = 22 kN, diameter = 2 m, height = 2.5 m, sea water $SG = 1.025$

Answer:

Given:

Weight $W = 22 \text{ kN} = 22,000 \text{ N}$

Diameter $D = 2 \text{ m}$, Height $H = 2.5 \text{ m}$

Sea water $SG = 1.025 \rightarrow \gamma_{\text{sea}} = 1.025 \times 9810 = 10,055.25 \text{ N/m}^3$

To find: Show that it does not float vertically

Solution:

For vertical floating, the block must be in equilibrium with buoyancy equal to weight. The block will sink to a draft d such that weight = buoyancy.

Step 1 – Volume of block

$$V_{\text{block}} = \frac{\pi}{4} D^2 H = \frac{\pi}{4} \times 4 \times 2.5 = \pi \times 2.5 = 7.854 \text{ m}^3$$

Step 2 – Draft if vertical

Let draft = d . Displaced volume = $\frac{\pi}{4} D^2 d = \frac{\pi}{4} \times 4 \times d = \pi d$

Buoyancy = $\gamma_{\text{sea}} \times \pi d = 10,055.25 \times \pi d$

Set equal to weight:

$$10,055.25 \times \pi d = 22,000$$

$$d = \frac{22,000}{10,055.25 \times \pi} = \frac{22,000}{31,590} = 0.696 \text{ m}$$

Draft < Height $\rightarrow$ would float vertically if stable.

Step 3 – Check stability (metacentric height)

For a cylinder floating vertically:

Metacentric height $GM = \frac{I}{V_{\text{disp}}} - BG$

$$I = \frac{\pi D^4}{64} = \frac{\pi \times 16}{64} = \frac{16\pi}{64} = 0.7854 \text{ m}^4$$

$$V_{\text{disp}} = \pi d = \pi \times 0.696 = 2.186 \text{ m}^3$$

$$\frac{I}{V_{\text{disp}}} = \frac{0.7854}{2.186} = 0.359 \text{ m}$$

Centre of buoyancy from bottom = $d/2 = 0.348 \text{ m}$

Centre of gravity from bottom = $H/2 = 1.25 \text{ m}$ (assuming uniform cylinder)

$$BG = 1.25 - 0.348 = 0.902 \text{ m}$$

$GM = 0.359 - 0.902 = -0.543 \text{ m} \rightarrow$ **Negative, unstable.**

Thus it will not float vertically; it will tilt.

Final Answer:

$$GM = -0.543 \text{ m (unstable)} \Rightarrow \text{Does not float vertically}$$

Assignment – 1 (Question Related to Unit 1&2)

Q-1: Fluid Properties (Viscosity): A flat plate with an area of 0.2 m^2 is pulled at a velocity of 0.5 m/s over a stationary base plate. The gap between the plates is 0.8 mm and is filled with a biodiesel blend acting as a lubricant. If the force required to maintain the motion is 15 N , determine the dynamic viscosity and kinematic

viscosity of the biodiesel blend. (Assume the specific gravity of the blend is 0.88).

Answer:

Given:

Area $A = 0.2 \text{ m}^2$

Velocity $u = 0.5 \text{ m/s}$

Gap $dy = 0.8 \text{ mm} = 0.0008 \text{ m}$

Force $F = 15 \text{ N}$

Specific gravity $SG = 0.88$

Density of water $\rho_w = 1000 \text{ kg/m}^3$

To Find:

Dynamic viscosity μ ($\text{Pa}\cdot\text{s}$ or $\text{N}\cdot\text{s/m}^2$)

Kinematic viscosity ν (m^2/s)

Formula:

Shear stress $\tau = \frac{F}{A} = \mu \frac{du}{dy}$

Solution:

$$\tau = \frac{15}{0.2} = 75 \text{ Pa}$$

$$\frac{du}{dy} = \frac{0.5}{0.0008} = 625 \text{ s}^{-1}$$

$$\mu = \frac{\tau}{du/dy} = \frac{75}{625} = 0.12 \text{ Pa}\cdot\text{s}$$

Density of biodiesel $\rho = 0.88 \times 1000 = 880 \text{ kg/m}^3$

$$\nu = \frac{\mu}{\rho} = \frac{0.12}{880} = 1.364 \times 10^{-4} \text{ m}^2/\text{s}$$

Final Answer:

$$\mu = 0.12 \text{ Pa}\cdot\text{s}, \nu = 1.364 \times 10^{-4} \text{ m}^2/\text{s}$$

Q-2: Capillarity: Calculate the capillary rise of water in a glass tube of 2.5 mm diameter when submerged vertically in water. Take the surface tension of water as 0.0725 N/m and the angle of contact as 0° . How would this value change if the fluid was an automotive brake fluid with a higher surface tension?

Answer:

Given:

Diameter $d = 2.5 \text{ mm} = 0.0025 \text{ m} \rightarrow$ radius $r = 0.00125 \text{ m}$

Surface tension $\sigma = 0.0725 \text{ N/m}$

Contact angle $\theta = 0^\circ$

Density of water $\rho = 1000 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$

To Find: Capillary rise h

Formula:

$$h = \frac{2\sigma \cos \theta}{\rho g r}$$

Solution:

$$h = \frac{2 \times 0.0725 \times \cos 0^\circ}{1000 \times 9.81 \times 0.00125}$$

$$h = \frac{0.145}{12.2625} = 0.01182 \text{ m} = 11.82 \text{ mm}$$

Effect of higher surface tension:

If the brake fluid has higher surface tension, capillary rise increases proportionally, assuming similar density and contact angle. Higher $\sigma \rightarrow$ stronger cohesive forces $\rightarrow$ greater meniscus curvature $\rightarrow$ higher rise.

Final Answer:

$$h = 11.82 \text{ mm}$$

Q-3: Pressure Measurement: A U-tube differential manometer connects two pressure pipes, A and B. Pipe A contains a coolant fluid (Specific Gravity = 1.05) at a pressure of $1.2 \times 10^5 \text{ N/m}^2$, and Pipe B contains oil (Specific Gravity = 0.9) at a pressure of $2.0 \times 10^5 \text{ N/m}^2$. The manometer liquid is mercury. Find the difference in the mercury level if pipe B is positioned 0.5 m higher than pipe A.

Answer:

Given:

Pipe A: Coolant $SG_1 = 1.05$, $P_A = 1.2 \times 10^5 \text{ Pa}$

Pipe B: Oil $SG_2 = 0.9$, $P_B = 2.0 \times 10^5 \text{ Pa}$

Manometer liquid: Mercury $SG_m = 13.6$

$z_B = z_A + 0.5 \text{ m}$ (B is higher)

To Find: Difference in mercury level h (in m)

Formula:

Equality of pressures at same horizontal level in the manometer.

Solution:

Let level of mercury in limb A be reference.

Pressure at A + pressure due to coolant column above mercury on left = Pressure at B + pressure due to oil column + pressure due to mercury difference.

But careful: Pipe B is higher by 0.5 m. Let's define:

Mercury level difference h (higher in limb A? We'll solve).

Equate pressures at mercury level in limb A:

$$P_A + \rho_1 g(0.5 + h) = P_B + \rho_2 g(0.5) + \rho_m g h$$

Where $\rho_1 = 1.05 \times 1000 = 1050 \text{ kg/m}^3$

$$\rho_2 = 900 \text{ kg/m}^3$$

$$\rho_m = 13600 \text{ kg/m}^3$$

Substitute:

$$\begin{aligned} 1.2 \times 10^5 + (1050)(9.81)(0.5 + h) &= 2.0 \times 10^5 + (900)(9.81)(0.5) + (13600)(9.81)h \\ 1.2 \times 10^5 + 5150.25 + 10300.5h &= 2.0 \times 10^5 + 4414.5 + 133416h \\ 125150.25 + 10300.5h &= 204414.5 + 133416h \\ 125150.25 - 204414.5 &= (133416 - 10300.5)h \\ -79264.25 &= 123115.5h \end{aligned}$$

Negative h means mercury level higher in limb B.

$$h = -\frac{79264.25}{123115.5} = -0.6437 \text{ m}$$

Magnitude $h = 0.644 \text{ m}$ (mercury higher in limb B)

Final Answer:

$$h = 0.644 \text{ m}$$

Q-4: Hydrostatic Forces: A circular plate of 2 m diameter is submerged vertically in a liquid of specific gravity 0.95 such that its top edge is 1 m below the free surface. Calculate the total hydrostatic pressure force on the plate and the depth of the center of pressure.

Answer:

Given:

Diameter $D = 2 \text{ m} \rightarrow$ radius $R = 1 \text{ m}$

Specific gravity $SG = 0.95 \rightarrow \rho = 950 \text{ kg/m}^3$

Top edge is 1 m below free surface: $h_c = 1 + R = 2 \text{ m}$ (center of circular plate from surface)

To Find: Total hydrostatic force F and depth of center of pressure h_p

Formula:

$$F = \rho g h_c A$$

$$A = \pi R^2 = \pi(1)^2 = \pi \text{ m}^2$$

$$I_G = \frac{\pi R^4}{4} = \frac{\pi(1)^4}{4} = \frac{\pi}{4} \text{ m}^4 \text{ for circle}$$

$$h_p = h_c + \frac{I_G}{h_c A}$$

Solution:

$$F = 950 \times 9.81 \times 2 \times \pi = 950 \times 9.81 \times 2 \times 3.1416$$

$$F = 950 \times 61.68 = 58596 \text{ N} \approx 58.6 \text{ kN}$$

$$h_p = 2 + \frac{\pi/4}{2 \times \pi} = 2 + \frac{1/4}{2} = 2 + 0.125 = 2.125 \text{ m}$$

Final Answer:

$$F = 58.6 \text{ kN}, h_p = 2.125 \text{ m}$$

Q-5: Buoyancy: A solid cylinder of diameter 3 m and height 2 m floats in water with its vertical axis perpendicular to the surface. If the specific gravity of the cylinder material is 0.65, determine its metacentric height and state whether its equilibrium is stable.

Answer:

Given:

Cylinder: $D = 3 \text{ m}, H = 2 \text{ m}$

$SG = 0.65 \rightarrow \rho_{cyl} = 650 \text{ kg/m}^3$

Floats in water ($\rho = 1000$)

To Find: Metacentric height GM and stability

Solution:

Let submerged depth = d .

Weight = Buoyancy:

$$650 \times 9.81 \times \frac{\pi D^2}{4} \times H = 1000 \times 9.81 \times \frac{\pi D^2}{4} \times d$$

Cancel common terms:

$$650 \times 2 = 1000 \times d \Rightarrow d = 1.3 \text{ m}$$

Center of buoyancy $OB = d/2 = 0.65 \text{ m}$ from bottom.

Center of gravity $OG = H/2 = 1.0 \text{ m}$ from bottom.

So $BG = 1.0 - 0.65 = 0.35 \text{ m}$.

Metacentric radius $BM = \frac{I}{V_{sub}}$

$$I = \frac{\pi D^4}{64} = \frac{\pi(81)}{64} = 3.976 \text{ m}^4$$

$$V_{sub} = \frac{\pi D^2}{4} \times d = \frac{\pi \times 9}{4} \times 1.3 = 9.186 \text{ m}^3$$

$$BM = \frac{3.976}{9.186} = 0.433 \text{ m}$$

$$GM = BM - BG = 0.433 - 0.35 = 0.083 \text{ m} > 0$$

Stability: Positive GM $\rightarrow$ stable equilibrium.

Final Answer:

$GM = 0.083 \text{ m, Stable}$
